1. Find the derivatives of the following functions from first principles :

(a) $f(x)=2x^2$ (b) $f(x)=\frac{2}{x^2}$ (c) $f(x)=e^x$

2. Differentiate these functions with respect to x :-

(a) $\frac{1}{(x^2+3x+5)^2}$ (b) $\tan^3 x$ (c) $\sin^2\left(2x-\frac{\pi}{6}\right)$

(d) $\sqrt{x}\ln x$ (e) x^4e^{3x} (f) $\frac{x^2}{2x+3}$

(g) $\frac{x^2\ln x}{x+1}$ (h) $x^2e^{\cos x}$ (i) $\frac{\sin x}{x^2}$

3. (a) Write down the definitions of sec$x$, cosec$x$ and cot$x$.

(b) Use either the chain or quotient rule to differentiate sec$x$, cosec$x$ and cot$x$.

(c) Hence find the derivatives of :-

(i) $\cot 3x$ (ii) $\text{cosec}^2 x$ (iii) $2x^2\sec x$

4. If $y=\frac{\sin x}{x^2}$, prove that $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+(x^2+2)y=0$.

5. Part of a journey an object made was observed.
The displacement, s metres, of the object travelling in a straight line at time t seconds is given by :-

$$s=\frac{t^3}{3}+t^2-8t+10$$

(a) How far from its origin was the object when the observation was started ?

(b) At what time was the object stationary ?

(c) Comment on the motion of the object when t=5 secs.

(d) Does the object ever reach a constant velocity or decelerate during its journey ? Justify your answer.

6. A farmer ploughed a square field, ABCD, of side 132 metres.

There is a path along the perimeter of the field which the farmer can walk along at a speed of 8km/h. He can walk across the ploughed field at a speed of 5 km/h.

In order to get from A to the opposite corner C, the farmer starts walking along the perimeter path from A to B. When he reaches a point P he leaves the path and heads directly for C, across the ploughed field.

What is the distance, AP, if he takes the least possible time in getting from A to C by the route described ?

7. Determine the stationary points of $y = x^3e^{-x}$. Use the second derivative to help determine the nature of the stationary points.