Outcome 4 HOMEWORK

1. The sum of the first n terms of an arithmetic series is $n(n+5)$.

   Find the first three terms of the series.

2. a) In an arithmetic series of 9 terms, the first term is 5 and the last is 23.

   Find the sum of the 9 terms.

   b) Find the sum of the first seven terms of a geometric series that has an eighth term of $\frac{2}{3}$ and a fifth term of 18.

3. For each of the following geometric series state whether a sum to infinity exists, and if so find it.

   a) $84 - 42 + 21 - 10\frac{1}{2} + 5\frac{1}{4} - \ldots$

   b) $8 + 12 + 18 + 27 + 40\frac{1}{2} + \ldots$

   c) $64 - 16 + 4 - 1 + \frac{1}{4} - \ldots$

4. The 3rd term of an arithmetic series is 17 and the 7th term is 33.

   Find a formula for the sum of n terms and find the sum of 20 terms.

5. Find the values of x for which $x-6$, $2x$ and $8x+20$ are consecutive terms of a geometric series.

6. Find a) $\sum_{k=11}^{20}(2k+1)$ b) $\sum_{k=1}^{\infty}\frac{1}{2^{k-1}}$

7. Expand the following as geometric series and state the necessary condition on x for each series to be valid.

   a) $\frac{1}{1+x}$ b) $\frac{1}{4-x}$ c) $\frac{1}{3+x}$

8. By first expressing $\dfrac{1}{x^2+3x+2}$ in partial fractions, use the fact that for $-1<r<1$

$$\frac{1}{1-r}=1+r+r^2+\ldots=\sum_{r=0}^{\infty} r^k$$

to express $\dfrac{1}{x^2+3x+2}$ in the form $\displaystyle\sum_{k=0}^{\infty} a_k x^k$.

Give an expression for the coefficient a_k, $k=0,1,2,\ldots$, and state the real values of x for which the series is valid.

9. Find $\displaystyle\sum_{k=1}^{n} k$ and $\displaystyle\sum_{k=1}^{n}(2k-1)$.

For all positive integral values of *n*, the sum of the first *n* terms of a series is $3n^2+2n$.

Find the *n*th term in its simplest form.

10. The population of a colony of insects increases in such a way that if it is *N* at the beginning of a week, then at the end of the week it is $a+bN$, where *a* and *b* are constants and $0<b<1$.

a) Starting from the beginning of the week when the population is *N*, write down an expression for the population at the end of one, two, three and four weeks.

Show that at the end of *n* consecutive weeks the population is

$$a\left(\frac{1-b^n}{1-b}\right)+b^n N.$$

b) When a = 2000 and b = 0.2, it is known that the population takes about 4 weeks to increase from *N* to 2*N*.

Estimate a value for *N* from this information.