Outcome 5 HOMEWORK

1. Prove that if n is odd then n^2 is also odd.

2. a) Prove by induction that if $0 < a < b$ then $a^n < b^n$ for $n > 0$.

   b) Show by counterexample that this result is not true if $n < 0$.

3. Prove by induction that $(1\times 2) + (2\times 3) + (3\times 4) + \ldots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$, for $n \in N$.

4. A pupil testing the formula $n^2 + n + 11$ produces a prime number every time for $n = 1,2,3\ldots$ He therefore predicts that any number of the form $n^2 + n + 11$ where n is a positive integer is prime.

   Prove by counterexample that this is not so.

5. Prove that if x and y are rational numbers then $x + y$ is also a rational number.

6. A type of Fibonnacci sequence is defined as $u_n = u_{n-1} + u_{n-2}$ where $u_1 = 1$, $u_2 = 2$, $u_3 = 3$, etc.

   Prove that $u_1 + u_2 + \ldots + u_n = u_{n+2} - 2$.

7. Use induction to prove that if $x > 0$ then for any $n \in N$

$$(1+x)^{n+1} > 1 + (n+1)x.$$

8. Prove each of the statements below and then use a counter example in each case to show that the converse of each statement is false.

   a) If $n > 2$ and n is a prime number then n is an odd number.

   b) If a and b are odd numbers, then $a + b$ is even.

9. Prove that $\sum_{k=1}^{n}(-1)^{r-1}r^2 = \frac{1}{2}(-1)^{n-1}n(n+1)$.

10. Evaluate the sum $\sum_{k=1}^{n}\frac{r}{(r+1)!}$ for $n = 1,2,3$ and 4.

Using this, conjecture a formula for the sum and use induction to prove your conjecture.

11. Prove by contradiction that $\log_{10} 5$ is irrational.