

1) Given that,  $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & -1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ ,  $C = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$  and  $D = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ , find, where possible:

- (i)  $A+B$       (ii)  $A+C$       (iii)  $C+D$       (iv)  $CA$   
 (v)  $AB$       (vi)  $AC$       (vii)  $A'C'$       (viii)  $C^{-1}$

2) Find the set of all  $2 \times 2$  matrices  $X$  such that  $X^2 = 0$ . Give one example.

3) Use elementary row operations to find the inverse of the matrix  $\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \\ 2 & -1 & 2 \end{pmatrix}$  and hence solve the equations:

- (i)  $x - y + z = 1$   
 (ii)  $x + y + 2z = 0$   
 (iii)  $2x - y + 3z = 2$

4) If  $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ , show that the most general  $2 \times 2$  matrix  $X$  such that  $AX = XB$  is of the form  $\begin{pmatrix} x & y \\ x & -y \end{pmatrix}$ . Hence, find a  $2 \times 2$  matrix  $P$  such that  $P^{-1}AP = B$  and  $P^{-1}P = I$ , where  $I$  is the unit  $2 \times 2$  matrix.

5) What transformations are represented by the following matrices? Illustrate each transformation by means of an example.

- (i)  $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$       (ii)  $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$       (iii)  $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$       (iv)  $\begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}$       (v)  $\begin{pmatrix} -\sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{pmatrix}$

6) Solve for  $x$ :

$$\begin{vmatrix} 2-x & -3 & 5 \\ 0 & 5-x & 2 \\ 8 & -6 & 8-x \end{vmatrix} = 0$$