1) Use Maclaurin's Theorem to **derive** the series expansion for $\cos(x)$, giving the first three non-zero terms. Hence, obtain the first three non-zero terms of the series expansion for:

(i) $\cos 2x$
(ii) $\cos^2 x$

2) State the Maclaurin expansion for $\ln(1+x)$ and $\sin(x)$ and use them to obtain a power series expansion for $\ln(1+\sin(x))$ as far as the term in x^4. What is the domain of validity of this expansion?

3) Use the exponential series to calculate accurately to 3 decimal places:

(i) $\sqrt{e}$
(ii) $\dfrac{1}{e}$
(iii) $e^{0.1}$

4) Derive the Maclaurin series expansion for

$$f(x) = \frac{1}{1+x-2x^2}.$$

[Hint: use partial fractions]

5) Derive the power series

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

[Hint: for $f(x) = \tan^{-1}(x)$, $f'(x) = \dfrac{1}{1+x^2} = (1+x^2)^{-1}$;]

Deduce that $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{9} + \ldots$

6) Show that the equation $x^3 - x^2 - 6 = 0$ can be rearranged in the following different ways,

(a) $x = (x^2 + 6)^{\frac{1}{3}}$
(b) $x = (x^3 - 6)^{\frac{1}{2}}$
(c) $x = \left(x + \dfrac{6}{x}\right)^{\frac{1}{2}}$

and determine which of them give(s) a convergent iterative procedure for finding the root near $x = 2$. Find this root accurate to 3 decimal places.