

Prelim Examination 2006 / 2007
(Assessing Units 1 & 2)

MATHEMATICS
Advanced Higher Grade

Time allowed - 2 hours

Read Carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**
4. **This examination paper contains questions graded at all levels.**

All questions should be attempted

1. (a) Given $f(x) = e^{-2x}\tan 4x$, $0 < x < \frac{\pi}{8}$, obtain $f'(x)$. 3

(b) For $y = \frac{\ln 5x}{x-1}$, where $x > 1$, determine $\frac{dy}{dx}$ in its simplest form. 3

2. For what value of t does the system of equations

$$\begin{aligned}x + 2y - 3z &= -7 \\4x - y + 2z &= 9 \\3x - 2y + tz &= 13\end{aligned}$$

have no solution? 5

3. Verify that $1 - 3i$ is a solution of $z^4 - 4z^3 + 11z^2 - 14z - 30 = 0$.

Hence express $z^4 - 4z^3 + 11z^2 - 14z - 30$ in the form $(z + a)(z + b)(z^2 + cz + d)$, where a, b, c and d are real numbers. 5

4. Use the substitution $x = 3\cos\theta$ to show that

$$\int_{\frac{3}{2}}^3 \frac{dx}{\sqrt{9-x^2}} = \frac{\pi}{3} \quad \text{6}$$

5. Obtain the binomial expansion of $\left(3a^2 - \frac{4}{b}\right)^5$. 3

6. Use integration by parts to evaluate $\int_0^1 x^2 e^{-x} dx$. 5

7. Determine whether the function $f(x) = x^2 \cos x + x^3$ is odd, even or neither.

Justify your answer. 3

8. A spherical balloon is being inflated.

Its volume, $V \text{ cm}^3$, is increasing at the rate of $\frac{30\pi}{7} \text{ cm}^3$ per second.

Find the rate at which the radius is increasing with respect to time when the volume is $\frac{36\pi}{5} \text{ cm}^3$.

[Note: The volume of a sphere is given by $V = \frac{4}{3} \pi r^3$.] 5

9. Prove that if n is odd then $n^4 - 1$ is divisible by 8. 3

10. (a) Obtain partial fractions for

$$\frac{9}{x^2 - 9} \quad \text{2}$$

(b) Hence evaluate

$$\int_0^1 \frac{x^2}{x^2 - 9} dx. \quad \text{4}$$

11. The function f is defined by

$$f(x) = \frac{x^2}{x+3}, \quad x \neq -3.$$

(a) Obtain algebraically the asymptotes of the graph of f . 3

(b) Find the stationary points of f and justify their nature. 5

(c) Sketch the curve showing clearly the features found in (a) and (b). 2

(d) Write down the coordinates of the stationary points of the graph of $g(x) = 10 + |f(x)|$. 2

12. The first two terms of a series are $1 + \sqrt{2}$ and $1 + \frac{1}{\sqrt{2}}$.

(a) If the series is arithmetic, show that the common difference is $-\frac{1}{2}\sqrt{2}$.

Show also that the sum of the first ten terms is $\frac{5}{2}(4 - 5\sqrt{2})$. **4**

(b) If the series is geometric, show that the sum to infinity exists.

Show also that $S_{\infty} = 4 + 3\sqrt{2}$. **5**

13. A solid is formed by rotating the curve $y = x^2 + 4$ between $x = 1$ and $x = t$, $t > 1$, through 360° about the y -axis.

Find the value of t given that the volume of the solid formed is 40π units³. **6**

[END OF QUESTION PAPER]

**Marking Scheme - Advanced Higher Grade 2006/2007
Prelim (Assessing Units 1 & 2)**

	Give one mark for each •	Illustrations for awarding each mark
1(a)	ans: $f'(x) = 2e^{-2x}(2\sec^2 4x - \tan 4x)$ <p style="text-align: right;">3 marks</p> <ul style="list-style-type: none"> • knows to use product rule • differentiates e^{-2x} correctly • differentiates $\tan 4x$ 	<ul style="list-style-type: none"> • • $-2e^{-2x}$ • $4\sec^2 4x$
1(b)	ans: $\frac{dy}{dx} = \frac{x(1 - \ln 5x) - 1}{x(x-1)^2}$ <p style="text-align: right;">3 marks</p> <ul style="list-style-type: none"> • knows to use the quotient rule • differentiates correctly • correct simplification for $\frac{dy}{dx}$ 	<ul style="list-style-type: none"> • • $\frac{x-1}{x} - \ln 5x$ • $\frac{x-1 - x \ln 5x}{x(x-1)^2}$
2	ans: $t = \frac{31}{9}$ <p style="text-align: right;">5 marks</p> <ul style="list-style-type: none"> • correct augmented matrix • first modified system correct • second modified system correct • third modified system correct • solves for t 	<ul style="list-style-type: none"> • $\begin{pmatrix} 1 & 2 & -3 & -7 \\ 4 & -1 & 2 & 9 \\ 3 & -2 & t & 13 \end{pmatrix}$ • $\begin{pmatrix} 1 & 2 & -3 & -7 \\ 0 & -9 & 14 & 37 \\ 3 & -2 & t & 13 \end{pmatrix}$ • $\begin{pmatrix} 1 & 2 & -3 & -7 \\ 0 & -9 & 14 & 37 \\ 0 & -8 & t+9 & 34 \end{pmatrix}$ • $\begin{pmatrix} 1 & 2 & -3 & -7 \\ 0 & -9 & 14 & 37 \\ 0 & 0 & t - \frac{31}{9} & \frac{10}{9} \end{pmatrix}$ • $t = \frac{31}{9}$

	Give one mark for each •	Illustrations for awarding each mark
3	<p>ans: Proof, $(z - 3)(z + 1)(z^2 - 2z + 10)$</p> <p style="text-align: right;">5 marks</p> <ul style="list-style-type: none"> • verifies that $1 - 3i$ is a solution • knows that $1 + 3i$ is a solution • uses $1 + 3i$ for substitution or synthetic division • finds $z^2 - 2z - 3 = (z - 3)(z + 1)$ • finds $z^2 - 2z + 10$ factor 	<ul style="list-style-type: none"> • correct substitution or synthetic division • $1 + 3i$ is a solution • correct substitution or synthetic division • $z^2 - 2z - 3 = (z - 3)(z + 1)$ • $z^2 - 2z + 10$
4	<p>ans: Proof</p> <p style="text-align: right;">6 marks</p> <ul style="list-style-type: none"> • starts substitution • changes limits correctly • correct substitution • deals with denominator • correctly integrates • substitutes limits correctly 	<ul style="list-style-type: none"> • $dx = -3\sin\theta d\theta$ • $\frac{3}{2} \rightarrow \frac{\pi}{3}, 3 \rightarrow 0$ • $\int_{\frac{\pi}{3}}^0 \frac{-3\sin\theta d\theta}{\sqrt{9 - 9\cos^2\theta}}$ • $\int_{\frac{\pi}{3}}^0 \frac{-3\sin\theta d\theta}{3\sin\theta}$ • $- [\theta]_{\frac{\pi}{3}}^0$ • $\frac{\pi}{3}$
5	<p>ans:</p> $243a^{10} - \frac{1620a^8}{b} + \frac{4320a^6}{b^2} - \frac{5760a^4}{b^3} + \frac{3840a^2}{b^4} - \frac{1024}{b^6}$ <p style="text-align: right;">3 marks</p> <ul style="list-style-type: none"> • correct binomial expression • correct expansion • correct simplification 	<ul style="list-style-type: none"> • $\sum_{r=0}^5 \binom{5}{r} (3a^2)^{5-r} \left(\frac{-4}{b}\right)^r$ • $(3a^2)^5 + 5(3a^2)^4 \left(\frac{-4}{b}\right) + 10(3a^2)^3 \left(\frac{-4}{b}\right)^2$ • $10(3a^2)^2 \left(\frac{-4}{b}\right)^3 + 5(3a^2) \left(\frac{-4}{b}\right)^4 + \left(\frac{-4}{b}\right)^5$ • answer

	Give one mark for each •	Illustrations for awarding each mark
6	ans: $2 - 5e^{-1}$ 5 marks <ul style="list-style-type: none"> • uses integration by parts correctly • uses integration by parts for a second time • integrates correctly • substitutes limits correctly • correct evaluation 	<ul style="list-style-type: none"> • $\left[-x^2e^{-x}\right]_0^1 + \int_0^1 2xe^{-x} dx$ • $\left[-2xe^{-x}\right]_0^1 + \int_0^1 2e^{-x} dx$ • $\left[-2e^{-x}\right]_0^1$ • $-e^{-1} - 2e^{-1} - 2e^{-1} + 2e^0$ • $2 - \frac{5}{e}$
7	ans: Neither 3 marks <ul style="list-style-type: none"> • knows to find $f(-x)$ • finds $f(-x)$ correctly • correct conclusion 	<ul style="list-style-type: none"> • $f(-x) = (-x)^2 \cos(-x) + (-x)^3$ • $f(-x) = x^2 \cos x - x^3$ • Neither
8	ans: 0.35 [cm/s] 5 marks <ul style="list-style-type: none"> • knows how find $\frac{dr}{dt}$ • finds $\frac{dr}{dV}$ correctly • finds correct formula for $\frac{dr}{dt}$ • finds correct radius • evaluates $\frac{dr}{dt}$ correctly 	<ul style="list-style-type: none"> • $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ • $\frac{dr}{dV} = \frac{1}{4\pi r^2}$ • $\frac{dr}{dt} = \frac{15}{14r^2}$ • $r = 1.75$ • $\frac{dr}{dt} = 0.35$
9	ans: Proof 3 marks <ul style="list-style-type: none"> • knows how to start proof : $n = 2k \pm 1$ • continues proof : simplifies $n = 2k \pm 1$ • completes proof : common factor of 8 	<ul style="list-style-type: none"> • n is odd $\Rightarrow n = 2k \pm 1$ ($k \in \mathbb{Z}$) • $\Rightarrow n^4 - 1 = 16k^4 \pm 32k^3 + 24k^2 \pm 8k$ • $\Rightarrow n^4 - 1 = 8(2k^4 \pm 4k^3 + 3k^2 \pm k)$ which is divisible by 8

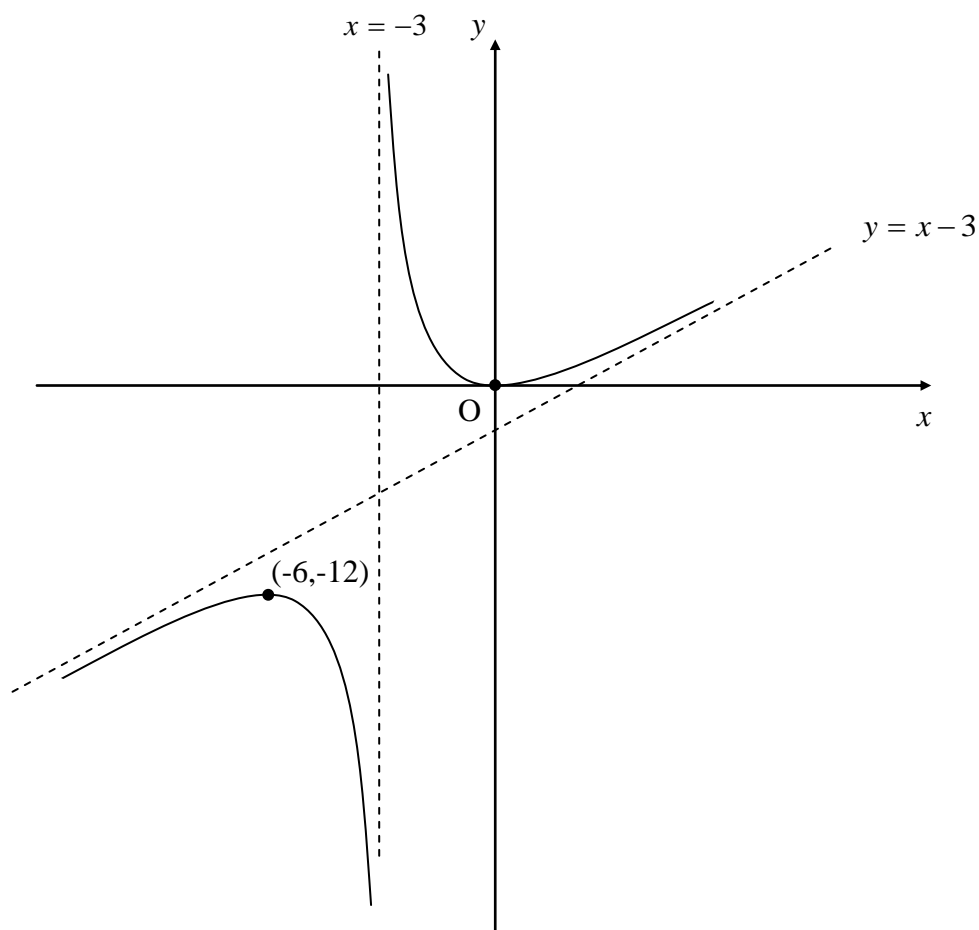
	Give one mark for each •	Illustrations for awarding each mark
10(a)	<p>ans: $\frac{3}{2(x-3)} - \frac{3}{2(x+3)}$</p> <p style="text-align: right;">2 marks</p> <ul style="list-style-type: none"> • first fraction • second fraction 	<ul style="list-style-type: none"> • $\frac{3}{2(x-3)}$ • $-\frac{3}{2(x+3)}$
10(b)	<p>ans: $1 + \frac{3}{2} \ln \frac{1}{2}$</p> <p style="text-align: right;">4 marks</p> <ul style="list-style-type: none"> • divides correctly • integrates correctly • substitutes limits correctly • evaluates correctly 	<ul style="list-style-type: none"> • $1 + \frac{9}{x^2 - 9}$ • $x + \frac{3}{2} \ln x-3 - \frac{3}{2} \ln x+3$ • $\left(1 + \frac{3}{2} \ln -2 - \frac{3}{2} \ln 4 \right) -$ • $\left(0 + \frac{3}{2} \ln -3 - \frac{3}{2} \ln 3 \right)$ • $1 + \frac{3}{2} (\ln 2 - \ln 4)$
11(a)	<p>ans: $x = -3$ & $y = x - 3$</p> <p style="text-align: right;">3 marks</p> <ul style="list-style-type: none"> • states equation of vertical asymptote • divides correctly • states equation of oblique asymptote 	<ul style="list-style-type: none"> • $x = -3$ • $f(x) = x - 3 + \frac{12}{x+3}$ • $y = x - 3$
11(b)	<p>ans: (0,0) → minimum turning point; (-6,-12) → maximum turning point</p> <p style="text-align: right;">5 marks</p> <ul style="list-style-type: none"> • differentiates correctly • finds x-coordinates of stationary points • finds y-coordinates of stationary points • finds second derivative or nature table • correct nature of both points 	<ul style="list-style-type: none"> • $f'(x) = \frac{x^2 + 6x}{(x+3)^3}$ • $f'(x) = 0 \Rightarrow x = 0, -6$ • (0,0) & (-6,-12) • $f''(x) = \frac{(2x+6)(x+3)^2 - 2(x^2+6x)(x+3)}{(x+3)^4}$ • $f''(0) > 0 \Rightarrow (0,0) \text{Min.T.P.}$ & $f''(-6) < 0 \Rightarrow (-6,-12) \text{Max.T.P.}$

	Give one mark for each •	Illustrations for awarding each mark
11(c)	ans: correct graph 2 marks <ul style="list-style-type: none"> • turning points shown • completes graph <p style="text-align: center;"><i>see graph on next page</i></p>	<ul style="list-style-type: none"> • correct turning points • correct behaviour at asymptotes
11(d)	ans: (0,10) & (-6,22) 2 marks <ul style="list-style-type: none"> • one point correct • second point correct 	<ul style="list-style-type: none"> • (0,10) • (-6,22)
12(a)	ans: Proof 4 marks <ul style="list-style-type: none"> • knows how to find common difference • simplifies correctly • knows how to find sum of first 10 terms • simplifies correctly 	<ul style="list-style-type: none"> • $1 + \frac{1}{\sqrt{2}} - (1 + \sqrt{2})$ • $\frac{1}{\sqrt{2}} - \sqrt{2} = \frac{1-2}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$ • $\frac{10}{2} \left[2(1 + \sqrt{2}) + (10-1) \left(\frac{-1}{\sqrt{2}} \right) \right]$ • $5 \left(2 + 2\sqrt{2} - \frac{9}{\sqrt{2}} \right) = \dots = \frac{5}{2} (4 - 5\sqrt{2})$
12(b)	ans: Proof 5 marks <ul style="list-style-type: none"> • knows to find common ratio • finds common ratio correctly • justifies that sum to infinity exists • knows how to find sum to infinity • simplifies correctly 	<ul style="list-style-type: none"> • $\frac{1 + \frac{1}{\sqrt{2}}}{1 + \sqrt{2}}$ • $\frac{1}{\sqrt{2}}$ • $-1 < \frac{1}{\sqrt{2}} < 1$ • $\frac{1 + \sqrt{2}}{1 - \frac{1}{\sqrt{2}}}$ • $\frac{\sqrt{2} + 2}{\sqrt{2} - 1} = \dots = 4 + 3\sqrt{2}$

	Give one mark for each •	Illustrations for awarding each mark
13	ans: $t = 3$ 6 marks <ul style="list-style-type: none"> • knows how to find volume of solid • finds limits of integration • integrates correctly • substitutes limits correctly • equates volumes • solves for t correctly 	<ul style="list-style-type: none"> • $V = \int \pi(y - 4)dy$ • $1 \rightarrow 5, t \rightarrow t^2 + 4$ • $\pi\left[\frac{y^2}{2} - 4y\right]$ • $\pi\left\{\left(\frac{(t^2 + 4)^2}{2} - 4(t^2 + 4)\right) - \left(\frac{5^2}{2} - 4(5)\right)\right\}$ • $40\pi = \pi\left(\frac{t^4}{2} - \frac{1}{2}\right)$ • $t = 3$

TOTAL MARKS = 74

Q11 (c)



Higher Still - 2006 / 2007

MATHEMATICS

Advanced Higher Grade – Mini Prelim (Unit 3 + Units 1/2 Revision)

Time allowed - 1 hour 20 minutes

Read Carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**
4. **This test contains questions graded at all levels.**

All questions should be attempted

1. Find the general solution of the differential equation

$$x \frac{dy}{dx} + (x-2)y = x^4. \quad 5$$

Given that $y = 5e^{-1}$ when $x = 1$, find the particular solution. 2

2. (a) Show that the matrix $A = \begin{pmatrix} 2 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 3 & 1 \end{pmatrix}$ is non-singular. 3

(b) Use elementary row operations to find A^{-1} . 5

(c) **Hence** solve the system of equations

$$\begin{aligned} 2x + y + 4z &= 2 \\ x + 2z &= 3 \\ 2x + 3y + z &= -6. \end{aligned} \quad 2$$

3. (a) Obtain the first five terms in the Maclaurin expansion of $(1+3x)^{\frac{5}{3}}$. 4

(b) For what values of x is this series valid? 2

(c) Use the expansion to find an approximation for $1 \cdot 9^{\frac{5}{3}}$. 2

4. (a) Express 458_6 in base 8. 3

(b) Prove by induction that $n(n+1)(n+2)$ is divisible by 6 for all positive integers n . 6

5. A function $y(x)$ is defined implicitly by $x^3 + 4xy = 3$.

Obtain formulae for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x and y only. 5

Hence evaluate $\frac{dy}{dx}$ at $(1, 0)$ and $\frac{d^2y}{dx^2}$ at $(2, -1)$. 2

6. (a) Find the point of intersection of the line L_1

$$\frac{x-6}{2} = \frac{y+2}{1} = \frac{z+7}{-3}$$

and the plane with equation $3x - y - 2z = 12$.

4

- (b) Find the point of intersection of the line L_1 and the line L_2

$$\frac{x-6}{-1} = \frac{y+7}{2} = \frac{z}{-2}.$$

4

7. Let $z = \frac{1}{\cos\theta + i\sin\theta}$.

- (a) Use de Moivre's theorem to express z^5 in the form $\cos p\theta - i\sin p\theta$, where p is a natural number.

2

- (b) Use the binomial theorem to express $\sin 5\theta$ in the form

$$q\sin\theta + r\sin^3\theta + t\sin^5\theta,$$

and state the values of q , r and t .

5

[END OF QUESTION PAPER]

Mini Prelim (Assessing Unit 3 + Revision)

	Give one mark for each •	Illustrations for awarding each mark
1	<p>ans: $y = x^3 - x^2 + \frac{Cx^2}{e^x}$ 5 marks</p> <p>$y = x^3 - x^2 + \frac{5x^2}{e^x}$ 2 marks</p> <ul style="list-style-type: none"> • knows to find integrating factor correctly • finds correct integrating factor • uses integrating factor correctly • uses integration by parts correctly • finds general solution correctly • substitutes conditions correctly • finds particular solution correctly 	<ul style="list-style-type: none"> • $e^{\int \frac{x-2}{x} dx}$ • $\frac{e^x}{x^2}$ • $\frac{e^x}{x^2} y = \int x e^x dx$ • $x e^x - \int e^x dx$ • $y = x^3 - x^2 + \frac{Cx^2}{e^x}$ • $5e^{-1} = 1 - 1 + \frac{C}{e}$ • $C = 5$
2(a)	<p>ans: $\det A = 3 \neq 0$ 3 marks</p> <ul style="list-style-type: none"> • knows how to find the determinant of a 3×3 matrix • finds determinant correctly • correct explanation 	<ul style="list-style-type: none"> • $\det A = \begin{vmatrix} 2 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 3 & 1 \end{vmatrix}$ • 3 • Since $A \neq 0$, A is non-singular
2(b)		

	<p>ans: $A^{-1} = \frac{1}{3} \begin{pmatrix} -6 & 11 & 2 \\ 3 & -6 & 0 \\ 3 & -4 & -1 \end{pmatrix}$</p> <p style="text-align: right;">5 marks</p> <ul style="list-style-type: none"> • correct augmented matrix • one correct row • a second correct row • the third row correct • identifies A^{-1} 	<ul style="list-style-type: none"> • $\begin{pmatrix} 2 & 1 & 41 & 0 & 0 \\ 1 & 0 & 20 & 1 & 0 \\ 2 & 3 & 10 & 0 & 1 \end{pmatrix}$ • $\begin{pmatrix} 0 & 1 & 01 & -2 & 0 \end{pmatrix}$ • $\begin{pmatrix} 0 & 0 & 11 & \frac{-4}{3} & \frac{-1}{3} \end{pmatrix}$ • $\begin{pmatrix} 1 & 0 & 0-2 & \frac{11}{3} & \frac{2}{3} \end{pmatrix}$ • $A^{-1} = \begin{pmatrix} -2 & \frac{11}{3} & \frac{2}{3} \\ 1 & -2 & 0 \\ 1 & \frac{-4}{3} & \frac{-1}{3} \end{pmatrix}$
2(c)	<p>ans: $x = 3, y = -4, z = 0$</p> <p style="text-align: right;">2 marks</p> <ul style="list-style-type: none"> • knows to pre-multiply both sides by A^{-1} • correct solution 	<ul style="list-style-type: none"> • $\frac{1}{3} \begin{pmatrix} -6 & 11 & 2 \\ 3 & -6 & 0 \\ 3 & -4 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$ • $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$
3(a)	<p>ans: $(1 + 3x)^{\frac{5}{3}} = 1 + 5x + 5x^2 - \frac{5}{3}x^3 + \frac{5}{3}x^4$</p> <p style="text-align: right;">4 marks</p> <ul style="list-style-type: none"> • evaluates $f(0)$ & $f'(0)$ correctly • evaluates $f''(0)$ correctly • evaluates $f'''(0)$ & $f^{iv}(0)$ correctly 	<ul style="list-style-type: none"> • $f(0) = 1$ & $f'(0) = 5$ • $f''(0) = 10$ • $f'''(0) = -10$ & $f^{iv}(0) = 40$

	<ul style="list-style-type: none"> • correct expansion 		<ul style="list-style-type: none"> • Correct expansion
3(b)	<p>ans: $x < \frac{1}{3}$</p> <ul style="list-style-type: none"> • knows range of validity • solves inequality 	2 marks	<ul style="list-style-type: none"> • $3x < 1$ • $x < \frac{1}{3}$
3(c)	<p>ans: 2.9185</p> <ul style="list-style-type: none"> • use expansion correctly • correct approximation 	2 marks	<ul style="list-style-type: none"> • $(1 + 3(0 \cdot 3))^{\frac{5}{3}} = 1 + 5(0 \cdot 3) + 5(0 \cdot 3)^2 -$ • $\frac{5}{3}(0 \cdot 3)^3 + \frac{5}{3}(0 \cdot 3)^4$ • 2.9185

4(a)	ans: 266_8 3 marks <ul style="list-style-type: none"> changes to base 10 repeated division by 8 correct answer in base 8 	<ul style="list-style-type: none"> $458_6 = 182$ $182 \div 8 = 22 \text{ r } 6, 22 \div 8 = 2 \text{ r } 6, 2 \div 8 = 0 \text{ r } 2$ 266_8
4(b)	ans: Proof 6 marks <ul style="list-style-type: none"> knows how to start proof; e.g. true for $n=1$ assume true for $n=k$ statement for $n=k+1$ continues proof : consider n odd continues proof : consider n even completes proof 	<ul style="list-style-type: none"> $n=1:1(1+1)(1+2) = 6$ which is divisible by 6 $n=k:k(k+1)(k+2) [=6L]$ is divisible by 6 $n=k+1:(k+1)(k+2)(k+3)$ is divisible by 6 <u>k odd</u> $[=2m+1]$- $(k+1)(k+2)(k+3)$ $=6L+3(k+1)(k+2)$ $=6L+3(2m+2)(2m+3)$ $=6[L+(m+1)(2m+3)]$ which is divisible by 6 <u>k even</u> $[=2m]$- $(k+1)(k+2)(k+3)$ $=6L+3(k+1)(k+2)$ $=6L+3(2m+1)(2m+2)$ $=6[L+(2m+1)(m+1)]$ which is divisible by 6 Since true for $n=1$ and $[\text{true for } n=k \Rightarrow \text{true for } n=k+1]$, the result is true for all positive integers n.
5	ans: $\frac{dy}{dx} = \frac{-3}{4}x - \frac{y}{x}$ 5 marks $\frac{d^2y}{dx^2} = \frac{2y}{x^2}$ $\frac{dy}{dx} = \frac{-3}{4}$ 2 marks $\frac{d^2y}{dx^2} = \frac{-1}{2}$ <ul style="list-style-type: none"> knows how to use implicit differentiation differentiates correctly knows how to find second derivative differentiates correctly finds simplified answer (in terms of x and y only) evaluates first derivative correctly evaluates second derivative correctly 	<ul style="list-style-type: none"> $3x^2 + 4y + 4x \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-3}{4}x - \frac{y}{x}$ $\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$ $\frac{-3}{4} - \left\{ -x^{-2}y + x^{-1} \frac{dy}{dx} \right\}$ $\frac{d^2y}{dx^2} = \frac{2y}{x^2}$ $\frac{-3}{4}$ $\frac{-1}{2}$

6(a)	ans: (2,-4,-1) 4 marks <ul style="list-style-type: none"> expresses x, y and z in terms of t substitutes in plane equation solves for t correct point 	<ul style="list-style-type: none"> $x = 2t + 6, y = t - 2, z = -3t - 7$ $3(2t + 6) - (t - 2) - 2(-3t - 7) = 12$ $t = -2$ (2,-4,-1)
6(b)	ans: (4,-3,-4) 4 marks <ul style="list-style-type: none"> correct system of equations integrates correct value for t correct value for s correct point 	<ul style="list-style-type: none"> $2t + s = 0, t - 2s = -5, -3t + 2s = 7$ $t = -1$ $s = 2$ (4,-3,-4)
7(a)	ans: $z^5 = \cos 5\theta - i \sin 5\theta$ 2 marks <ul style="list-style-type: none"> applies de Moivre's theorem correctly express answer in correct form 	<ul style="list-style-type: none"> $\cos(-5\theta) + i \sin(-5\theta)$ $\cos 5\theta - i \sin 5\theta$
7(b)	ans: $\sin 5\theta = 5 \sin \theta - 20 \sin^3 \theta + 16 \sin^5 \theta$ $q = 5, r = -20 \text{ \& } t = 16$ 5 marks <ul style="list-style-type: none"> uses the binomial theorem correctly equates imaginary parts substitutes correctly simplifies correctly correct values of $q, r \text{ \& } t$. 	$z^5 = (\cos \theta)^5 + 5(\cos \theta)^4(-i \sin \theta) +$ <ul style="list-style-type: none"> $10(\cos \theta)^3(-i \sin \theta)^2 + 10(\cos \theta)^2(-i \sin \theta)^3 + 5(\cos \theta)(-i \sin \theta)^4 + (-i \sin \theta)^5$ $\sin 5\theta = 5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta$ $5(1 - \sin^2 \theta)^2 \sin \theta - 10(1 - \sin^2 \theta) \sin^3 \theta + \sin^5 \theta$ $5 \sin \theta - 20 \sin^3 \theta + 16 \sin^5 \theta$ $q = 5, r = -20 \text{ \& } t = 16$

TOTAL MARKS = 56