H

National
Qualifications
EXEMPLAR PAPER ONLY

EP30/H/01

Mathematics
Paper 1
(Non-Calculator)

Date — Not applicable

Duration — 1 hour and 10 minutes

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

MARKS

Total marks — 60

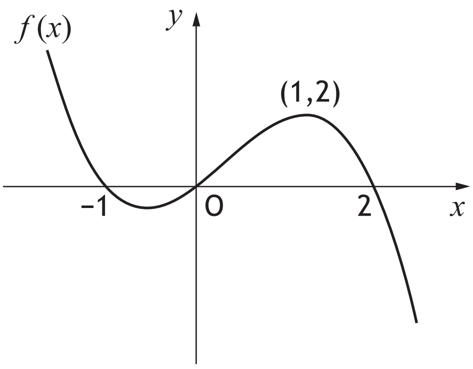
Attempt ALL questions

1. The point P (5,12) lies on the curve with equation $y = x^2 - 4x + 7$.

Find the equation of the tangent to this curve at P. **3**

2. The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$.

The curve passes through (-1,0), (0,0), (1,2) and (2,0).



Find the values of a, b and k. **3**

3. Evaluate $\int_1^2 \frac{1}{6}x^{-2}\,dx$. **3**

4. For the function $f(x) = 2 - 3\sin\left(x - \frac{\pi}{3}\right)$ in the interval $0 \leq x < 2\pi$, determine which two of the following statements are true **and justify your answer**. **3**

Statement A The maximum value of $f(x)$ is -1.

Statement B The maximum value of $f(x)$ is 5.

Statement C The maximum value occurs when $x = \frac{5\pi}{6}$.

Statement D The maximum value occurs when $x = \frac{11\pi}{6}$.

5. For the polynomial, $x^3 - 4x^2 + ax + b$

- $x - 1$ is a factor
- -12 is the remainder when it is divided by $x - 2$

(a) Determine the values of a and b. **4**

(b) Hence solve $x^3 - 4x^2 + ax + b = 0$. **4**

6. (a) Find the equation of l_1, the perpendicular bisector of the line joining P (3,−3) and Q (−1,9). **4**

(b) Find the equation of l_2 which is parallel to PQ and passes through R (1,−2). **2**

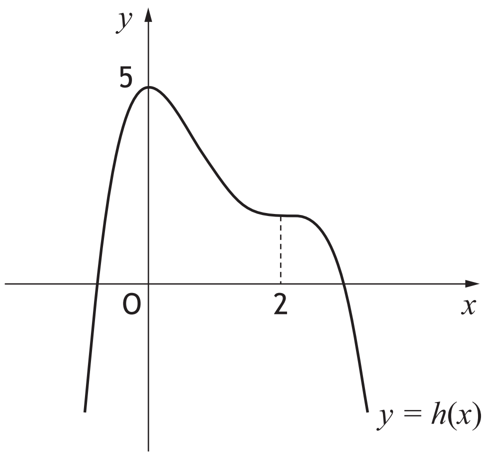
(c) Find the point of intersection of l_1 and l_2. **3**

(d) Hence find the shortest distance between PQ and l_2. **2**

7. (a) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. **5**

(b) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$. **2**

8. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



On separate diagrams sketch the graphs of:

(a) $y = 2 - h(x)$. **3**

(b) $y = h'(x)$. **3**

MARKS

9. The expression $\cos 4x - \sqrt{3}\sin 4x$ can be written in the form $k\cos(4x + a)$ where $k > 0$ and $0 \leq a \leq 2\pi$.

(a) Calculate the values of k and a. **4**

(b) Find the points of intersection of the graph of $y = \cos 4x - \sqrt{3}\sin 4x$ with the x axis, in the interval $0 \leq x \leq \frac{\pi}{2}$. **3**

10. The gradient of a tangent to a curve is given by $\frac{dy}{dx} = 3\cos 2x$.

The curve passes through the point $\left(\frac{7\pi}{6}, \sqrt{3}\right)$.

Find y in terms of x. **4**

11. Functions f and g are defined on suitable domains by $f(x) = x^3 - 1$ and $g(x) = 3x + 1$.

(a) Find an expression for $k(x)$, where $k(x) = g(f(x))$. **2**

(b) If $h(k(x)) = x$, find an expression for $h(x)$. **3**

[END OF EXEMPLAR QUESTION PAPER]

H

National
Qualifications
EXEMPLAR PAPER ONLY

EP30/H/02

Mathematics
Paper 2

Date — Not applicable

Duration — 1 hour and 30 minutes

Total marks — 70

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

EP30H02

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product:

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Total marks — 70 **MARKS**

Attempt ALL questions

1. A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.

(a) Determine the values of u_1 and u_2. **1**

(b) A second sequence is given by 4, 5, 7, 11, ...

It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.

Find the values of p and q. **3**

(c) Either the sequence in (a) or the sequence in (b) has a limit.

(i) Calculate this limit.

(ii) Why does this other sequence not have a limit? **3**

2. (a) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

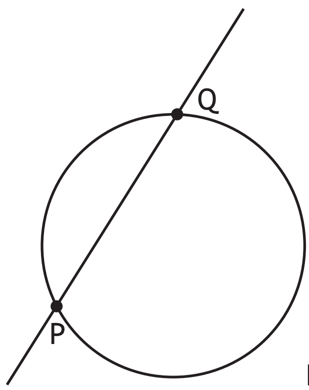


Diagram 1

Find the coordinates of P and Q. **6**

(b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.

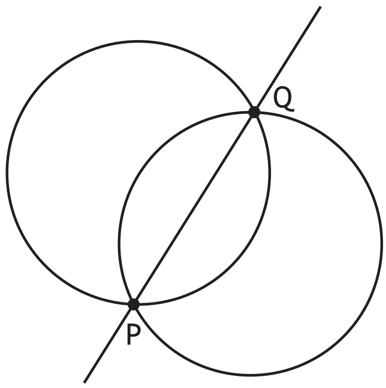


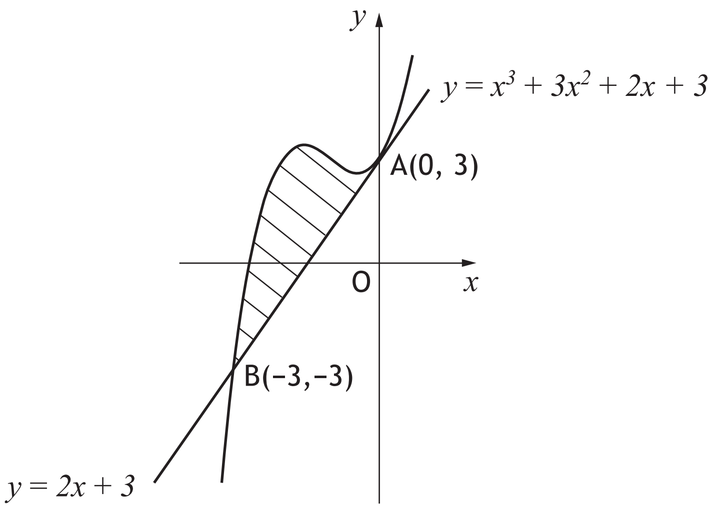
Diagram 2

Determine the equation of this second circle. **6**

MARKS

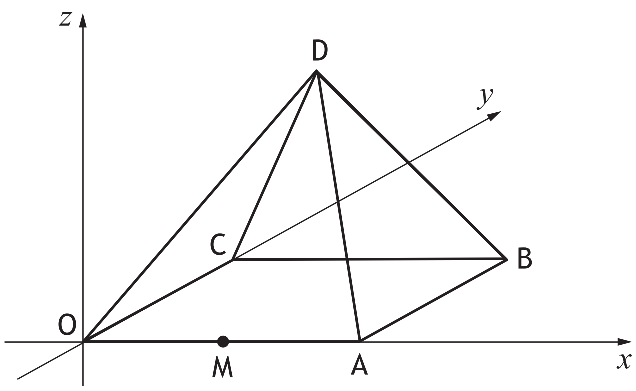
3. Find the value of p such that the equation $x^2+(p+1)x+9=0$ has no real roots. **4**

4. The line with equation $y=2x+3$ is a tangent to the curve with equation $y=x^3+3x^2+2x+3$ at A (0, 3), as shown.



The line meets the curve again at B (−3,−3). Find the area enclosed by the line and the curve. **5**

5. D,OABC is a square-based pyramid as shown.



O is the origin and OA = 4 units.

M is the mid-point of OA.

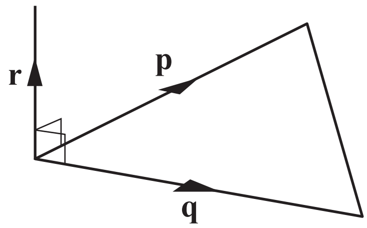
$\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

(a) Express $\overrightarrow{OB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$ and $\mathbf{k}$. **1**

(b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(c) Find the size of angle BDM. **5**

6. An equilateral triangle with sides of length 3 units is shown.



Vector $\mathbf{r}$ is 2 units long and is perpendicular to both vectors $\mathbf{p}$ and $\mathbf{q}$.

Calculate the value of the scalar product $\mathbf{p.(p+q+r)}$. 4

7. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation.

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

(a) If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. 4

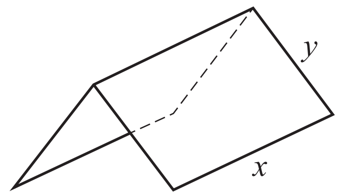
On all *Xpesto* packaging, the manufacturer states that 80 days after application the concentration of *Xpesto* in the soil will have decreased by over 90%.

(b) Is this statement correct? Justify your answer. 4

8. Given that $\int_{\frac{\pi}{8}}^{a} 5\sin(4x-\frac{\pi}{2})dx = \frac{10}{4}$, $0 \leq a < \frac{\pi}{2}$, calculate the value of a. 6

MARKS

9. A manufacturer is asked to design an open-ended shelter, as shown:



The frame of the shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.

The total length, L metres, of the rods used in a shelter is given by:

$$L = 3x + \frac{48}{x}$$

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(a) Find the value of x for which L is a minimum. **5**

The rods used for the frame cost £8·25 per metre.

The manufacturer claims that the minimum cost of a frame is less than £195.

(b) Is this claim correct? Justify your answer. **2**

MARKS

10. Acceleration is defined as the rate of change of velocity.

An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.

(a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. **3**

(b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. **2**

(c) Velocity is defined as the rate of change of displacement.

Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$. **3**

[END OF EXEMPLAR QUESTION PAPER]