

X056/302

NATIONAL
QUALIFICATIONS
2001

THURSDAY, 17 MAY
9.00 AM – 10.10 AM

**MATHEMATICS
HIGHER**

Units 1, 2 and Statistics
Paper 1
(Non-calculator)

Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Statistics:

Sample standard deviation: $s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1} \left(\sum x_i^2 - \frac{1}{n} (\sum x_i)^2 \right)}$ where n is the sample size.

Sums of squares and products: $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{1}{n} (\sum x_i)^2$

$$S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{1}{n} (\sum y_i)^2$$

$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{1}{n} \sum x_i \sum y_i$$

Linear regression:

The equation of the least squares regression line of y on x is given by $y = \alpha + \beta x$, where estimates for α and β , a and b , are given by:

$$a = \bar{y} - b\bar{x}$$

$$b = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}}$$

Product moment correlation coefficient: $r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

ALL questions should be attempted.

Marks

1. Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point $(2, -1)$. **3**

2. For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots? **3**

3. A fish farm had a pond from which a sample of 24 fish were netted, measured and returned to the pond. The measurements of their lengths are shown in the stem-and-leaf diagram below.

Length of fish in cm	
3	6
4	5 6 7 8
5	1 1 2 3 4 4 6 6 7 7 7 8
6	0 0 1 2 2 3
7	0
$n = 24$	7 0 represents 70 cm

- (a) Identify any outliers and illustrate the data with a box plot. **4**
 - (b) An adjacent pond held fish spawned at the same time but fed on a different diet. A sample of 24 fish from this pond had length measurements with a median of 58 cm and a semi-interquartile range of 3 cm. Comment on the two sets of results. **2**
-
4. Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$. **2**

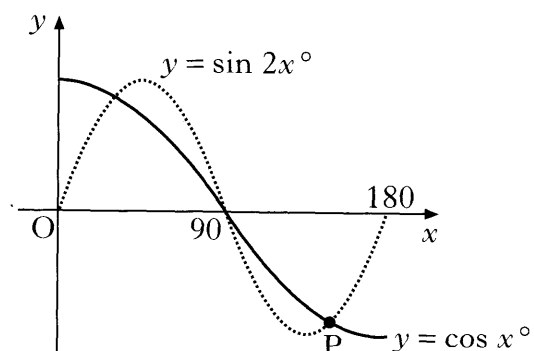
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5. (a) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$.

4

- (b) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (a) to write down the coordinates of the point P.



1

6. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \leq x \leq 12$.

Find the value of x which gives the maximum profit.



5

7. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(a) Find expressions for:

- (i) $f(h(x))$;
 (ii) $g(h(x))$.

2

(b) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$.

(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$.

5

8. The random variable X represents the number of faulty components in a circuit board. X has the following probability distribution:

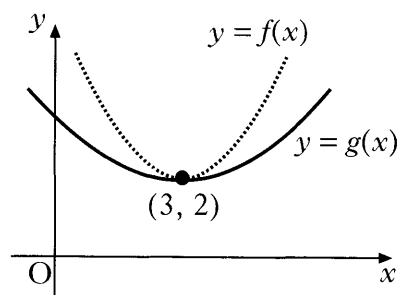
x	0	1	2	3
$P(X = x)$	k	$\frac{1}{2}k$	$\frac{1}{4}k$	$\frac{1}{4}k$

where k is a constant.

- (a) Find the value of k . 2
 (b) Find the expected value of X . 2

9. The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at $(3, 2)$.

Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$.



2

10. Two unbiased dice are numbered “0, 1, 2, 3, 4, 5”. An experiment consists of rolling the two dice and noting the total score.

A calculator was used to produce the following list of random numbers.

0.203 0.974 0.709 0.624 0.976 0.193 0.653 0.214 0.919 0.743
 0.363 0.342 0.221 0.696 0.327 0.253 0.313 0.060 0.591 0.459

- (a) Using digits from these random numbers, simulate 12 trials of the above experiment. 2
 (b) Using the results of your simulation, estimate the probability of a total score of 8. 1

11. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

- (a) (i) Show that the radius of circle P is $4\sqrt{2}$.
 (ii) Hence show that circles P and Q touch. 4
 (b) Find the equation of the tangent to circle Q at the point $(-4, 1)$. 3
 (c) The tangent in (b) intersects circle P in two points. Find the x -coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$. 3

[END OF QUESTION PAPER]

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THURSDAY, 17 MAY
10.30 AM – 12.00 NOON

**MATHEMATICS
HIGHER**
Units 1, 2 and Statistics
Paper 2

Read Carefully

- 1 **Calculators may be used in this paper.**
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ALL questions should be attempted.

Marks

1. (a) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k . 3
 (b) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. 2

2. A curve has equation $y = x - \frac{16}{\sqrt{x}}$, $x > 0$.
 Find the equation of the tangent at the point where $x = 4$. 6

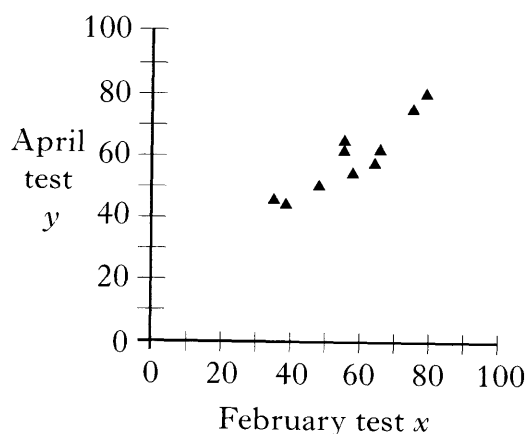
3. On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1.5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

(a) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

Let u_n and u_{n+1} represent the amounts that he owes at the starts of two successive months. Write down a recurrence relation involving u_{n+1} and u_n . 2

(b) Find the date and the amount of the final payment. 4

4. A group of ten students sat tests in February and April in preparation for their national examination in May. The scattergraph of their percentage marks shows that a linear model is appropriate.



The following summary statistics were calculated:

$$n = 10, \quad \sum x = 574, \quad \sum x^2 = 34836, \quad \sum y = 598, \quad \sum y^2 = 36848 \quad \text{and} \quad \sum xy = 35613.$$

- (a) Determine the equation of the least squares regression line of y on x . 4
 (b) Two other students were absent on the day of the test in April. In the February test, student A scored 63% and student B scored 85%. Use the regression equation to predict the expected percentage mark in the April test of each of these students and comment on the reliability of your predictions. 2

5. Find $\int \frac{(x^2 - 2)(x^2 + 2)}{x^2} dx, x \neq 0$

4

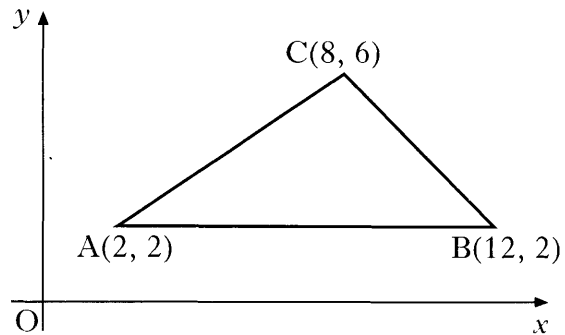
6. Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6).

(a) Write down the equation of l_1 , the perpendicular bisector of AB.

(b) Find the equation of l_2 , the perpendicular bisector of AC.

(c) Find the point of intersection of lines l_1 and l_2 .

(d) Hence find the equation of the circle passing through A, B and C.



1

4

1

2

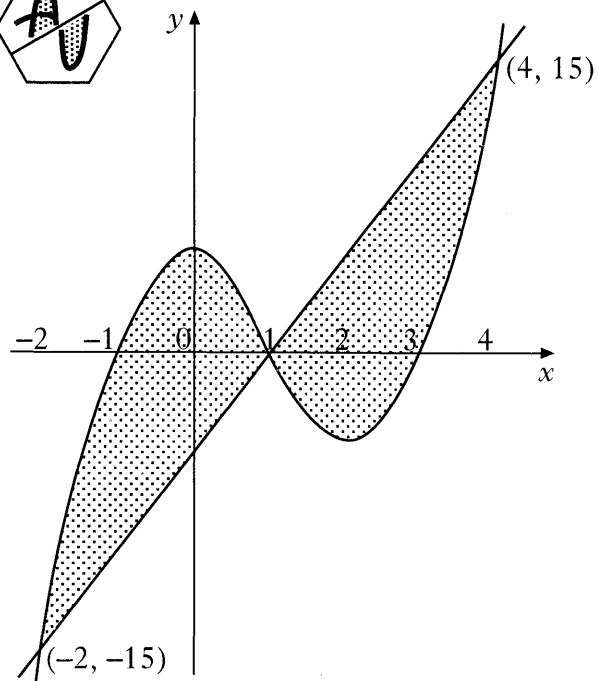
7. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.



A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation $y = (x + 1)(x - 1)(x - 3)$ and the straight line has equation $y = 5x - 5$. The point (1, 0) is the centre of half-turn symmetry.

Calculate the total shaded area.



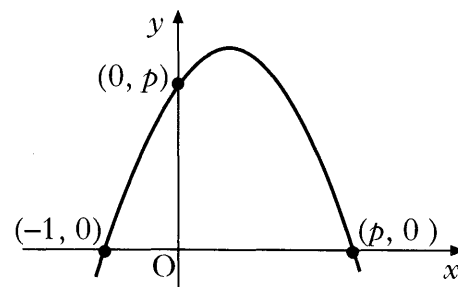
7

8. The duration, in days, of a certain viral infection in humans is a continuous random variable T whose probability density function is

$$f(t) = \begin{cases} \frac{1}{108}t^2(6-t) & 0 \leq t \leq 6 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Find $P(T < 2)$. 3
- (b) Calculate the modal number of days that the infection lasts. 3
- (c) (i) Show that the mean of this distribution is 3.6 days.
- (ii) Calculate the variance of this distribution. 6

9. The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.



- (a) Show that the equation of the parabola is $y = p + (p - 1)x - x^2$. 3
- (b) For what value of p will the line $y = x + p$ be a tangent to this curve? 3

[END OF QUESTION PAPER]