

X100/301

NATIONAL
QUALIFICATIONS
2007

TUESDAY, 15 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
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ALL questions should be attempted.

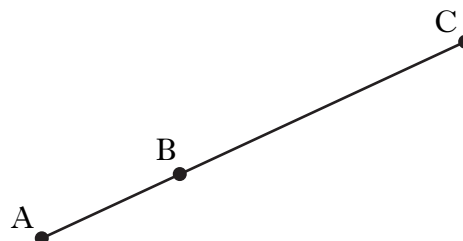
Marks

1. Find the equation of the line through the point $(-1, 4)$ which is parallel to the line with equation $3x - y + 2 = 0$. 3

2. Relative to a suitable coordinate system A and B are the points $(-2, 1, -1)$ and $(1, 3, 2)$ respectively.

A, B and C are collinear points and C is positioned such that $BC = 2AB$.

Find the coordinates of C.



4

3. Functions f and g , defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.

Find:

(a) $g(f(x))$;

2

(b) $g(g(x))$.

2

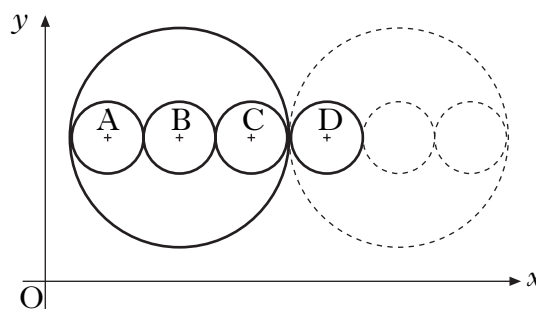
4. Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots. 4

5. The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x -axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.



5

[Turn over

6. Solve the equation $\sin 2x^\circ = 6\cos x^\circ$ for $0 \leq x \leq 360$. 4

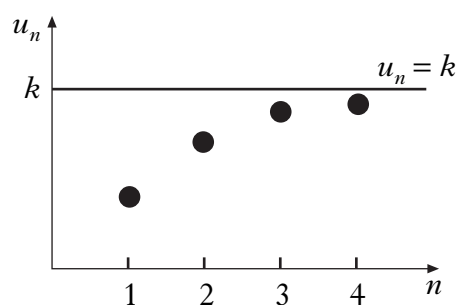
7. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16, \quad u_0 = 0.$$

(a) Calculate the values of u_1, u_2 and u_3 . 3

Four terms of this sequence, u_1, u_2, u_3 and u_4 are plotted as shown in the graph.

As $n \rightarrow \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence.



(b) (i) Give a reason why this sequence has a limit.

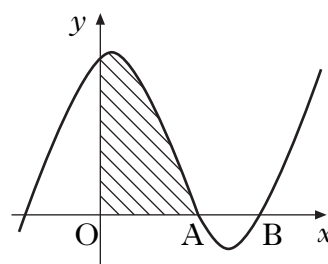
(ii) Find the exact value of k . 3

8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.

(a) Show that the graph cuts the x -axis at $(3, 0)$. 1

(b) Hence or otherwise find the coordinates of A. 3

(c) Find the shaded area. 5



9. A function f is defined by the formula $f(x) = 3x - x^3$.
 (a) Find the exact values where the graph of $y = f(x)$ meets the x - and y -axes. 2

(b) Find the coordinates of the stationary points of the function and determine their nature. 7

(c) Sketch the graph of $y = f(x)$. 1

10. Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$. 3

11. (a) Express $f(x) = \sqrt{3} \cos x + \sin x$ in the form $k \cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$. 4

(b) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$. 4

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X100/303

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2007

TUESDAY, 15 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
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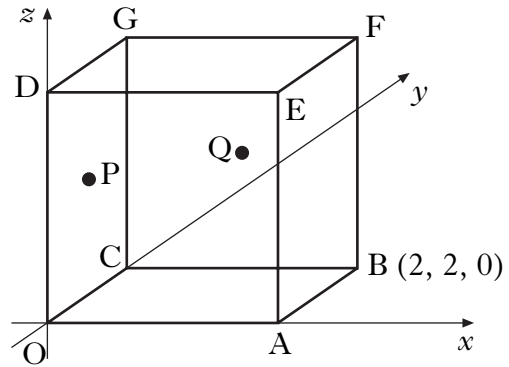
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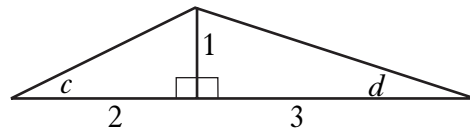
Marks

1. OABCDEFG is a cube with side 2 units, as shown in the diagram.
 B has coordinates (2, 2, 0).
 P is the centre of face OCGD and Q is the centre of face CBFG.



- (a) Write down the coordinates of G. 1
 (b) Find \mathbf{p} and \mathbf{q} , the position vectors of points P and Q. 2
 (c) Find the size of angle POQ. 5

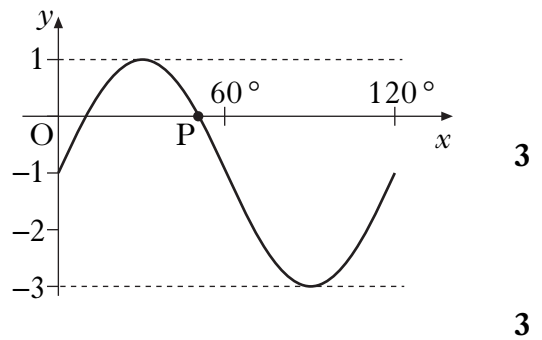
2. The diagram shows two right-angled triangles with angles c and d marked as shown.



- (a) Find the exact value of $\sin(c + d)$. 4
 (b) (i) Find the exact value of $\sin 2c$. 4
 (ii) Show that $\cos 2d$ has the same exact value. 4

3. Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. 6

4. The diagram shows part of the graph of a function whose equation is of the form $y = a \sin(bx^\circ) + c$.

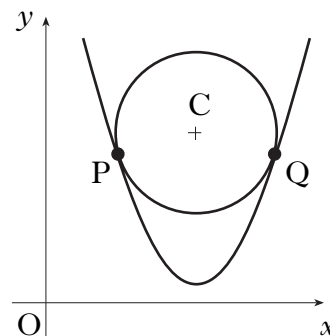


- (a) Write down the values of a , b and c . 3
 (b) Determine the exact value of the x -coordinate of P, the point where the graph intersects the x -axis as shown in the diagram. 3

[Turn over

5. A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q .

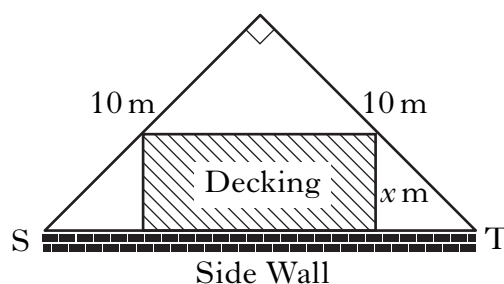
- (a) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q .
- (b) Find the coordinates of P .
- (c) Find the coordinates of C , the centre of the circle.



5
2
2

6. A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



- (a) (i) Find the exact value of ST .
- (ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = (10\sqrt{2})x - 2x^2.$$

3
5

- (b) Find the dimensions of the decking which maximises its area.

7. Find the value of $\int_0^2 \sin(4x + 1) dx$.

4

8. The curve with equation $y = \log_3(x - 1) - 2 \cdot 2$, where $x > 1$, cuts the x -axis at the point $(a, 0)$.

Find the value of a .

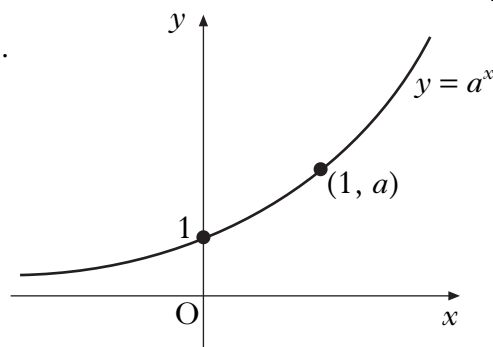
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9. The diagram shows the graph of $y = a^x$, $a > 1$.

On separate diagrams, sketch the graphs of:

(a) $y = a^{-x}$;

(b) $y = a^{1-x}$.

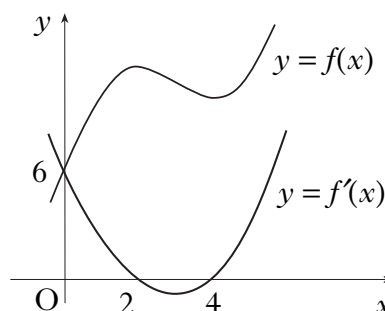


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10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point $(0, 6)$.

The graph of $y = f'(x)$ also passes through the points $(2, 0)$ and $(4, 0)$.



(a) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:

(i) write down the values of a and b ;

(ii) find the value of k .

3

(b) Find the equation of the graph of the cubic function $y = f(x)$.

4

11. Two variables x and y satisfy the equation $y = 3 \times 4^x$.

(a) Find the value of a if $(a, 6)$ lies on the graph with equation $y = 3 \times 4^x$.

1

(b) If $(-\frac{1}{2}, b)$ also lies on the graph, find b .

1

(c) A graph is drawn of $\log_{10}y$ against x . Show that its equation will be of the form $\log_{10}y = Px + Q$ and state the gradient of this line.

4

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