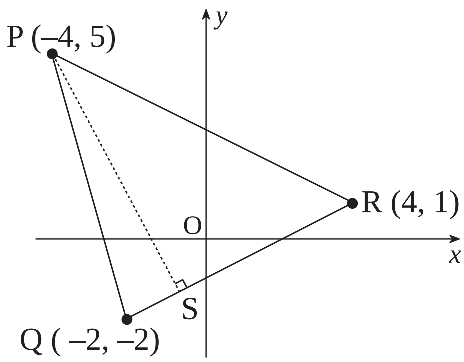
P(–4, 5), Q(–2, –2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. 3



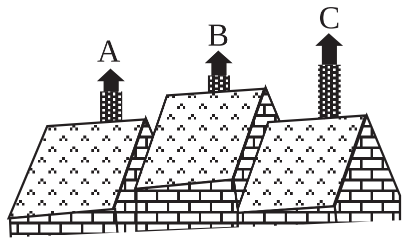
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.1
.	3	1.1					3		1.1.1	1.1.9, 1.1.7	Source **1997 P1 qu.1**

•1 $m_{QR} = \frac{1}{2}$

•2 $m_{PN} = -2$

•3 $PN: \; y - 4 = -2(x + 3)$

Relative to a suitable set of axes, the tops of three chimneys have coordinates given by A(1, 3, 2), B(2, –1,4) and C(4, –9, 8).
Show that A, B and C are collinear. 3



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	3.1
.	3	3.1					3		3.1.7		Source **1997 P1 qu.2**

•1 $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$

•2 $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -8 \\ 4 \end{pmatrix}$ **AND** $\overrightarrow{BC} = 2 \times \overrightarrow{AB}$

•3 $\overrightarrow{AB} \,||\, \overrightarrow{BC}$ & B is common hence A, B, C collinear

Functions f and g, defined on suitable domains, are given by $f(x) = 2x$ and $g(x) = \sin x + \cos x$.

Find $f(g(x))$ and $g(f(x))$. 4

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.2
.	4	1.2	4						1.2.6		Source: **1997 P1 qu.3**

•[1] $f(\sin x + \cos x)$

•[2] $2(\sin x + \cos x)$

•[3] $g(2x)$

•[4] $\sin 2x + \cos 2x$

The position vectors of the points P and Q are $\boldsymbol{p} = -\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{q} = 7\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k}$ respectively.

(*a*) Express $\overrightarrow{PQ}$ in component form. 2

(*b*) Find the length of PQ. 1

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(a)	2	3.1					2		3.1.8	3.1.1	Source: **1997 P1 qu.4**
(b)	1	3.1					1		3.1.3		

•[1] $q - p = 8\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$

or $p = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}$, $q = \begin{pmatrix} 7 \\ -1 \\ 5 \end{pmatrix}$

•[2] $\overrightarrow{PQ} = \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix}$

•[3] 9

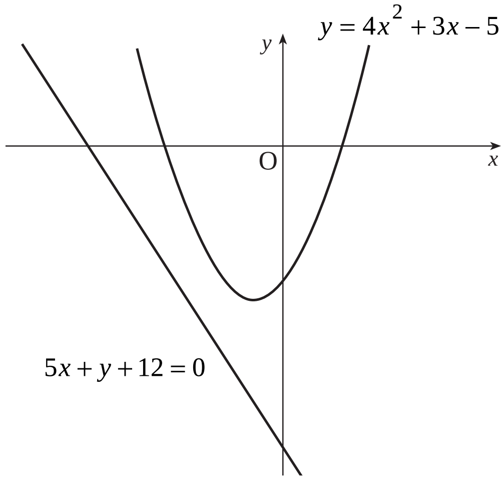
(*a*) Find a real root of the equation $2x^3 - 3x^2 + 2x - 8 = 0$. 2

(*b*) Show algebraically that there are no other real roots. 3

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(a)	2	2.1	2						2.1.2		Source
(b)	3	2.1	3						2.1.7		1997 P1 qu.5

- •1 looking for $f(x) = \ldots\ldots = 0$
- •2 $x = 2$ **explicitly stated**
- •3 $2x^2 + x + 4$
- •4 $b^2 - 4ac = 1 - 4 \times 2 \times 4$
- •5 $b^2 - 4ac < 0$ means no real roots

The diagram below shows a parabola with equation $y = 4x^2 + 3x - 5$ and a straight line with equation $5x + y + 12 = 0$.
A tangent to the parabola is drawn parallel to the given straight line.
Find the *x*-coordinate of the point of contact of this tangent. 5



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
.	5	1.3	5						1.1.8	1.3.7 1.3.1	Source 1997 P1 qu.6

- •1 equate gradients
- •2 $m = -5$
- •3 $\frac{dy}{dx} = \ldots$
- •4 $\frac{dy}{dx} = 8x + 3$
- •5 $x = -1$

If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x+30)°$ is $\frac{4\sqrt{3}+3}{10}$. 3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	2.3
			C	A / B	C	A / B	C	A / B	Main Additional	Source
.	3	2.3	3						2.3.2	**1997 P1 qu.7**

- •[1] $\sin x° \cos 30° + \cos x° \sin 30°$
- •[2] $\sin x° = \frac{4}{5}$ & $\cos x° = \frac{3}{5}$
- •[3] $\frac{4}{5} \cdot \frac{\sqrt{3}}{2} + \frac{3}{5} \cdot \frac{1}{2}$ and completes proof

Given that $y = 2x^2 + x$, find $\frac{dy}{dx}$ and hence show that $x\left(1 + \frac{dy}{dx}\right) = 2y$. 3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	1.3
			C	A / B	C	A / B	C	A / B	Main Additional	Source
.	3	1.3	3						1.3.4	**1997 P1 qu.8**

- •[1] $\frac{dy}{dx} = 4x + 1$
- •[2] $LHS = x(1 + 4x + 1)$ *or* $RHS = 2\left(2x^2 + x\right)$
- •[3] completes proof

(*a*) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x+b)^2 + c$ where *a*, *b* and *c* are constants. 3

(*b*) Hence, or otherwise, find the coordinates of the turning point of the function *f*. 1

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A / B	C	A / B	C	A / B	Main	Additional	Source
(a)	3	1.2	3						1.2.8		**1997 P1 qu.9**
(b)	1		1						1.2.9		

- •[1] $a = 2$
- •[2] $b = 2$
- •[3] $c = -11$
- •[4] $(-2, 11)$

Find the value of $\int_1^4 \sqrt{x}\, dx$. 4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A / B	C	A / B	C	A / B	Main	Additional	Source
.	4	2.2	4						2.2.5		**1997 P1 qu.10**

- •[1] $x^{\frac{1}{2}}$
- •[2] $x^{\frac{3}{2}} \div \frac{3}{2}$
- •[3] $\frac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)$
- •[4] $\frac{14}{3}$

Express $2\sin x° - 5\cos x°$ in the form $k\sin(x-\alpha)°$, $0 \le \alpha < 360$ and $k > 0$. 4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A / B	C	A / B	C	A / B	Main	Additional	Source
.	4	3.4			4				3.4.1		**1997 P1 qu.11**

•1 $k\sin(x-a) = k\sin x\cos a - k\cos x\sin a$ **stated explicitly**

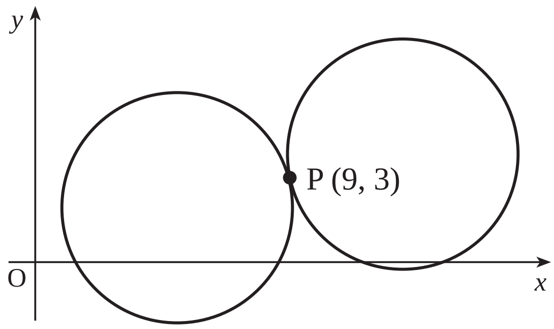
•2 $k\cos a = 2$ **and** $k\sin a = 5$

•3 $k = \sqrt{29}$

•4 $a = 68 \cdot 2$

Two identical circles touch at the point P (9, 3) as shown in the diagram. One of the circles has equation $x^2 + y^2 - 10x - 4y + 12 = 0$.

Find the equation of the other circle. 5



part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.4
			C	A / B	C	A / B	C	A / B	Main	Additional	Source
.	5	2.4					5		2.4.2	(3.1.6)	**1997 P1 qu.12**

•1 use P as midpoint of C_1C_2

•2 $C_1 = (5, 2)$

•3 $C_2 = (13, 4)$

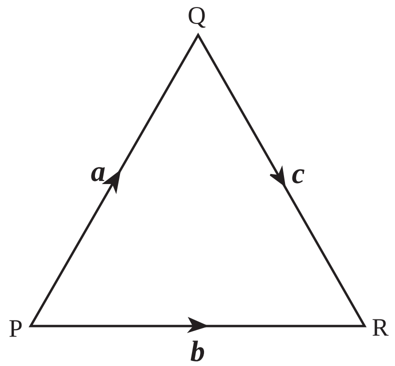
•4 radius $= \sqrt{17}$

•5 $(x-13)^2 + (y-4)^2 = 17$

PQR is an equilateral triangle of side 2 units.

$\overrightarrow{PQ} = \boldsymbol{a}$, $\overrightarrow{PR} = \boldsymbol{b}$ and $\overrightarrow{QR} = \boldsymbol{c}$.

Evaluate $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$ and hence identify two vectors which are perpendicular. 4



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	3.1
.	4	3.1					1	3	3.1.9	3.1.1	Source **1997 P1 qu.13**

- •[1] $\boldsymbol{a.b} + \boldsymbol{a.c}$
- •[2] $\boldsymbol{a.b} = 2 \times 2 \times \frac{1}{2}$
- •[3] $\boldsymbol{a.c} = 2 \times 2 \times -\frac{1}{2}$
- •[4] 0 **and** $\boldsymbol{a}$ is perpendicular to $(\boldsymbol{b}+\boldsymbol{c})$

For what range of values of c does the equation $x^2 + y^2 - 6x + 4y + c = 0$ represent a circle ? 3

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.4
.	3	2.4					2	1	2.4.2		Source **1997 P1 qu.14**

- •[1] $g^2 + f^2 - c > 0$
- •[2] $r^2 = 9 + 4 - c$
- •[3] $c < 13$

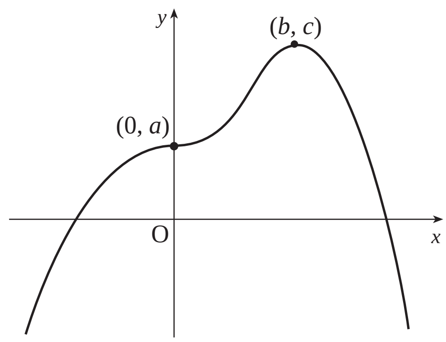
The curve $y = f(x)$ passes through the point $\left(\frac{\pi}{12}, 1\right)$ and $f'(x) = \cos 2x$.

Find $f(x)$. 3

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.2
.	3	3.2		3					3.2.4		Source **1997 P1 qu.15**

- •[1] $\frac{1}{2}\sin 2x$
- •[2] $1 = \frac{1}{2}\sin\frac{\pi}{6} + c$
- •[3] $c = \frac{3}{4}$

The diagram shows a sketch of part of the graph of $y = f(x)$. The graph of has a point of inflection at $(0, a)$ and a maximum turning point at (b, c).



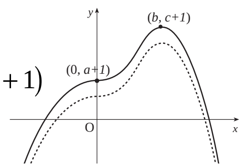
(*a*) Make a copy of this diagram and on it sketch the graph of $y = g(x)$ where $g(x) = f(x) + 1$. 2

(*b*) On a separate diagram sketch the graph of $y = f'(x)$. 2

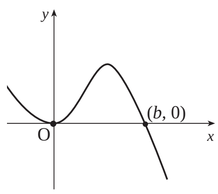
(*c*) Describe how the graph of $y = g'(x)$ is related to the graph of $y = f'(x)$. 1

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.2
(a)	2	1.2	2						1.2.4		Source
(b)	2	1.2	2						1.2.4		**1997 P1 qu.16**
(c)	1	0.1		1					0.1		

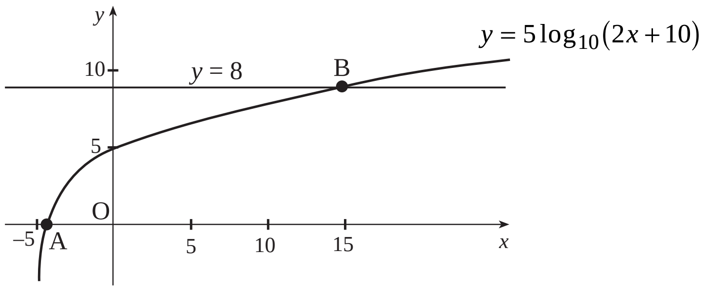
- •[1] translation $\begin{pmatrix}0\\1\end{pmatrix}$
- •[2] annotate $(0, a+1)$ & $(b, c+1)$



- •[3] roots at $(0,0)$ & $(b,0)$
- •[4] $y' > 0$ for $x < b$, $y' < 0$ for $x > b$
- •[5] they coincide



Part of the graph of $y = 5\log_{10}(2x+10)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B.



Find algebraically the x-coordinates of A and B. 4

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.3
.	4	3.3				4			3.3.4		Source **1997 P1 qu.17**

- •1 $x_A = -4\cdot 5$
- •2 $5\log_{10}(2x+10) = 8$
- •3 $2x + 10 = 10^{\frac{8}{5}}$
- •4 $x = 14\cdot 9$

(*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. 2

(*b*) **Hence** solve the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in the interval $0 \le x < 360$. 4

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(a)	2	2.3			1	1			2.3.3		Source
(b)	4	2.3			1	3			2.3.5		**1997 P1 qu.18**

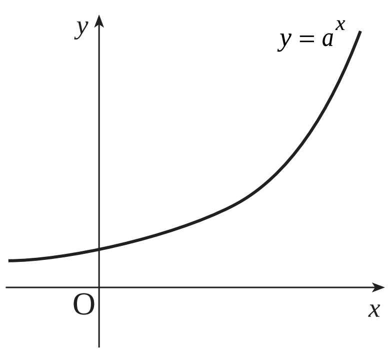
- •1 substitute $1 - 2\sin^2 x°$ for $\cos 2x°$
- •2 substitute $1 - \sin^2 x°$ for $\cos^2 x°$
- •3 $3\sin^2 x° + 2\sin x° - 1 = 0$
- •4 $(3\sin x° - 1)(\sin x° + 1) = 0$
- •5 $\sin x° = \frac{1}{3},\ -1$
- •6 $19\cdot 5,\ 160\cdot 5,\ 270$

The diagram shows a sketch of part of the graph of $y = a^x$, $a > 1$.

(*a*) If (1, *t*) and (*u*, 1) lie on this curve, write down the values of *t* and *u*. 2

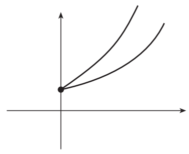
(*b*) Make a copy of this diagram and on it sketch the graph of $y = a^{2x}$. 2

(*c*) Find the coordinates of the point of intersection of $y = a^{2x}$ with the line $x = 1$. 1

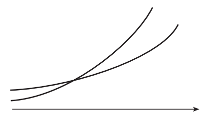


part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.3
											Source
(a)	2	3.3					2		3.3.4		
(b)	2	1.2						2	1.2.4		**1997 P1 qu.19**
(c)	1	0.1					1		0.1		

- •[1] $t = a$
- •[2] $u = 0$
- •[3] both passing thr' same point on y – axis
- •[4] $y = a^{2x}$ starting below $y = a^x$ and finishing above
- •[5] $(1, a^2)$



For mark 3



For mark 4

Diagram 1 shows 5 cars travelling up an incline on a roller-coaster. Part of the roller-coaster rail follows the curve with equation $y = 8 + 5\cos\frac{1}{2}x$.

Diagram 2 shows an enlargement of the last car and its position relative to a suitable set of axes. The floor of the car lies parallel to the tangent at P, the point of contact. Calculate the acute angle ***a*** between the floor of the car and the horizontal when the car is at the point where $x_P = \frac{7\pi}{3}$.

Express your answer in degrees. 4

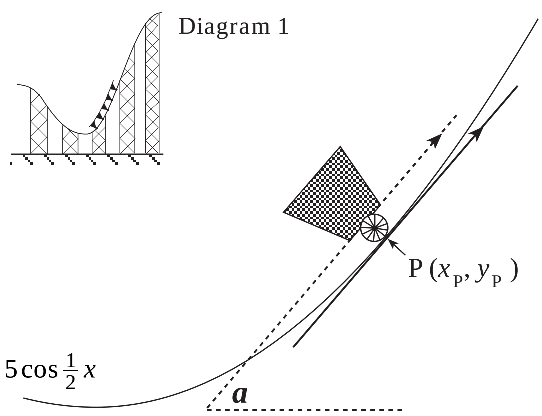


Diagram 2

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
											Source
.	4	1.3			1	3			1.3.7	1.3.9, 1.1.3	**1997 P1 qu.20**

- •[1] $\frac{dy}{dx} = \ldots$
- •[2] $5 \times \left(-\frac{1}{2}\sin\frac{1}{2}x\right)$
- •[3] $m = \frac{5}{4}$
- •[4] $\theta = 51.3°$