Diagram 1 shows a circle with equation $x^2 + y^2 + 10x - 2y - 14 = 0$ and a straight line, l_1, with equation $y = 2x + 1$.
The line intersects the circle at A and B.

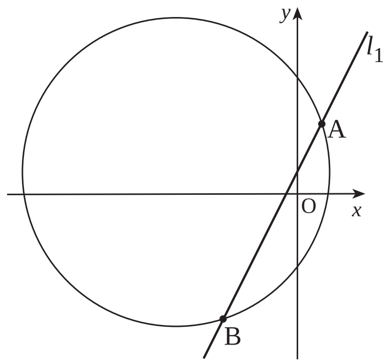


Diagram 1

(*a*) Find the coordinates of the points A and B. **(5)**

(*b*) Diagram 2 shows a second line, l_2, which passes through the centre of the circle, C, and is at right angles to line l_1.

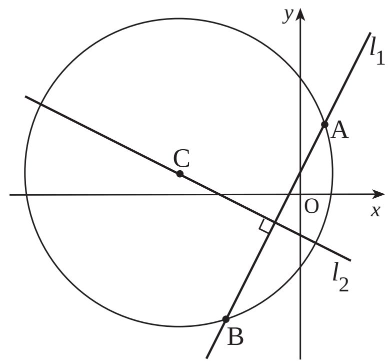


Diagram 2

(i) Write down the coordinates of C. **(1)**

(ii) Find the equation of the line l_2. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	**2.4**
(*a*)	5	2.4					5		2.4.4		Source
(*b*)i	1	2.4					1		2.4.2		**1997 Paper 2**
(*b*)ii	3	1.1					3		1.1.10	1.1.7	**Qu.1**

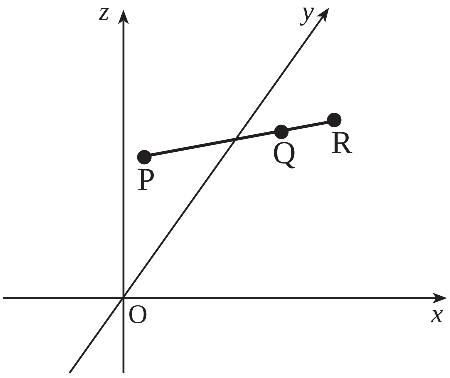
(*a*)

- •1 know to substitute
- •2 correct substitution
- •3 a “quadratic” = 0
- •4 $x = -3,\ 1$
- •5 $y = -5,\ 3$

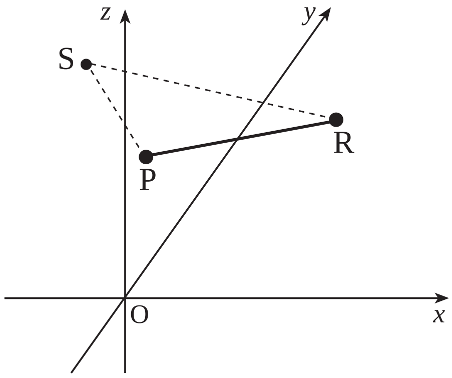
(*b*)

- •6 $m_{diameter} = 2$
- •7 $m_{perpendicular} = -\frac{1}{2}$
- •8 centre = $(-1,\ -1)$
- •9 equation: $y + 1 = -\frac{1}{2}(x + 1)$

Relative to the axes shown and with an appropriate scale, P(–1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.



(*a*) Find the coordinates of R. **(3)**



(*b*) Roads from P and R are built to meet at the point S (–2, 2, 5). Calculate the size of angle PSR. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
(*a*)	3	3.1			3				3.1.6		Source
(*b*)	7	3.1			7				3.1.11		**1997 Paper 2 Qu.2**

(*a*)

•1 $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$ •2 $\begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix}$

•3 $R = (7, -1, 6)$

(*b*)

•4 $\overrightarrow{SP}.\overrightarrow{SR} = |SP||SR|\cos P\hat{S}R$

•5 $\overrightarrow{SP} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$ •6 $\overrightarrow{SR} = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$

•7 $|SP| = \sqrt{11}$ •8 $|SR| = \sqrt{91}$

•9 $\overrightarrow{SP}.\overrightarrow{SR} = 3$

•10 $P\hat{S}R = 84 \cdot 6°$

The sum of £1000 is placed in an investment account on January 1st and, thereafter, £100 is placed in the account on the first day of each month.

- Interest at the rate of 0·5% per month is credited to the account on the last day of each month.
- This interest is calculated on the amount in the account on the first day of the month.

(*a*) How much is in the account on June 30th ? **(4)**

(*b*) On what date does the account first exceed £2000? **(2)**

(*c*) Find a recurrence relation which describes the amount in the account, explaining your notation carefully. **(3)**

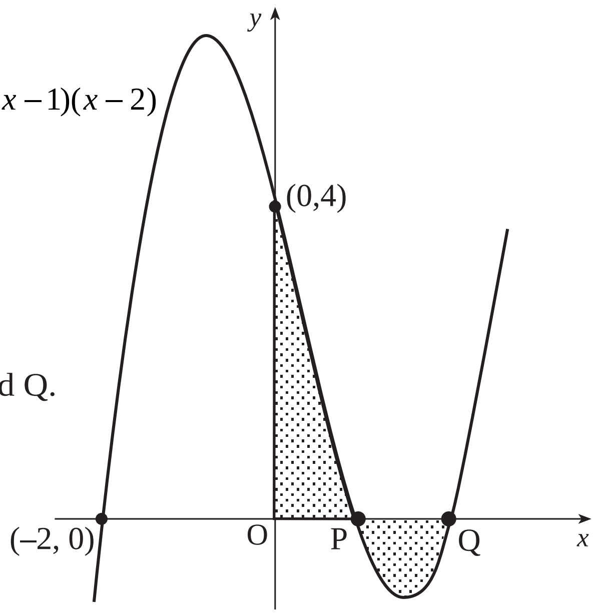
part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*)	4	1.4			4				1.4.1		Source
(*b*)	2	1.4			2				1.4.1		**1997 Paper 2**
(*c*)	3	1.4			3				1.4.3		**Qu.3**

(*a*)
- •1 1.005
- •2 £1000 + interest = £1005
- •3 £1005 + £100 + interest = £1110.525
- •4 £1537.93

(*b*)
- •5 complete another month
- •6 £2073.94 on Nov.1st

(*c*)
- •7 $u_{n+1} = 1.005u_n + 100$
- •8 u_n = amount on 1st day of each month
- •9 $u_0 = 1000$ (on 1st January)

The diagram shows a sketch of the graph of $y=(x+2)(x-1)(x-2)$. The graph cuts the axes at (–2, 0), (0, 4) and the points P and Q.



(*a*) Write down the coordinates of P and Q. **(2)**

(*b*) Find the total shaded area. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	2	2.1	2						2.1.2		Source
(*b*)	7	2.2	6	1					2.2.6		**1997 Paper 2 Qu.4**

(*a*)

•[1] $(1,0)$

•[2] $(2,0)$

(*b*)

•[3] $\int f(x)dx$

•[4] $\int_0^1 - \int_1^2$

•[5] $(x+2)\left(x^2-3x+2\right)$ or equiv.

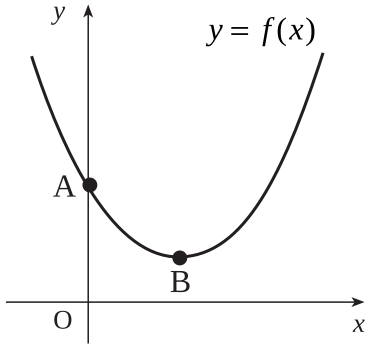
•[6] x^3-x^2-4x+4

•[7] $\frac{1}{4}x^4-\frac{1}{3}x^3-2x^2+4x$

•[8] $1\frac{11}{12}$ or $-\frac{7}{12}$

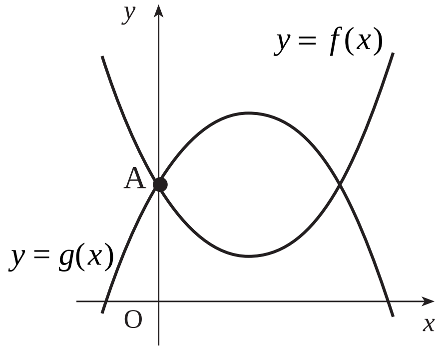
•[9] $2\frac{1}{2}$

The first diagram shows a sketch of part of the graph of $y = f(x)$ where $f(x) = (x-2)^2 + 1$.
The graph cuts the y-axis at A and has a minimum turning point at B.



(*a*) Write down the coordinates of A and B. **(3)**

(*b*) The second diagram shows the graphs of $y = f(x)$ and $y = g(x)$ where $g(x) = 5 + 4x - x^2$.
Find the area enclosed by the two curves. **(5)**



(*c*) $g(x)$ can be written in the form $m + n \times f(x)$ where m and n are constants.
Write down the values of m and n. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	3	1.2	3						1.2.9		Source
(*b*)	5	2.2	5						2.2.7		**1997 Paper 2**
(*c*)	3	0.1		2					0.1		**Qu.5**

(*a*)

- •1 $A = (0, 5)$
- •2 $x_B = 2$
- •3 $y_B = 1$

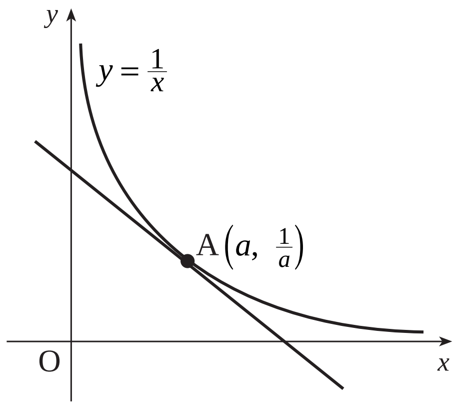
(*b*)

- •4 $\int_0^4$
- •5 $\int \left(\left(5 + 4x - x^2\right) - \left(x^2 - 4x + 5\right) \right) dx$
- •6 $8x - 2x^2$ *or equiv.*
- •7 $4x^2 - \frac{2}{3}x^3$ *or equiv.*
- •8 $\frac{64}{3}$

(*c*)

- •9 $n = -1$
- •10 $m = 10$

(*a*) A sketch of part of the graph of $y = \frac{1}{x}$ is shown in the diagram. The tangent at A $\left(a,\ \frac{1}{a}\right)$ has been drawn.
Find the gradient of this tangent. **(4)**



(*b*) Hence show that the equation of this tangent is $x + a^2 y = 2a$. **(2)**

(*c*) This tangent cuts the *y*-axis at B and the *x*-axis at C.

(i) Calculate the area of triangle OBC **(3)**

(ii) Comment on your answer to *c*(i). **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.3
(*a*)	4	1.3					4		1.3.7		Source
(*b*)	2	1.1					1	1	1.1.7		**1997 Paper 2**
(*c*)	4	0.1						4	0.1		**Qu.6**

(*a*)	•1	$\frac{1}{x} = x^{-1}$
	•2	$\frac{dy}{dx} = \ldots\ldots$
	•3	$\frac{dy}{dx} = -x^{-2}$
	•4	gradient = $-a^{-2}$
(*b*)	•5	use $y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$
	•6	$a^2 y - a = -(x - a)$ and completes proof
(*c*)	•7	$y_B = \frac{2a}{a^2}$
	•8	$x_A = 2a$
	•9	2
	•10	independent of *a*

In certain topics in Mathematics, such as calculus, we often require to write an expression such as $\frac{8x+1}{(2x+1)(x-1)}$ in the form $\frac{2}{2x+1}+\frac{3}{x-1}$.

$\frac{2}{2x+1}+\frac{3}{x-1}$ are called **Partial Fractions** for $\frac{8x+1}{(2x+1)(x-1)}$.

The worked example shows you how to find partial fractions for the expression $\frac{6x+2}{(x+2)(x-3)}$.

Worked Example

Find partial fractions for $\frac{6x+2}{(x+2)(x-3)}$.

Let $\frac{6x+2}{(x+2)(x-3)}=\frac{A}{x+2}+\frac{B}{x-3}$ where A and B are constants

$$=\frac{A(x-3)}{(x+2)(x-3)}+\frac{B(x+2)}{(x-3)(x+2)}$$

i.e. $\frac{6x+2}{(x+2)(x-3)}=\frac{A(x-3)+B(x+2)}{(x+2)(x-3)}$

Hence $6x+2=A(x-3)+B(x+2)$ for all values of x.

A and B can be found as follows:

Select a value of x that makes the first bracket zero

Let $x = 3$ (this eliminates A)

$18+2=A\times 0 \quad + \quad B\times 5$

$20=5B$

$\underline{B=4}$

Select a value of x that makes the second bracket zero

Let $x = -2$ (this eliminates B)

$-12+2=A\times(-5) \quad + \quad B\times 0$

$-10=-5A$

$\underline{A=2}$

Therefore $\frac{6x+2}{(x+2)(x-3)}=\frac{2}{x+2}+\frac{4}{x-3}$.

Find partial fractions for $\frac{5x+1}{(x-4)(x+3)}$. **(6)**

1997 Paper 2 Qu.7

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	4
	6	0.1					6		0.1		Source **1997 Paper 2 Qu.7**

- •[1] $\frac{A}{x-4}+\frac{B}{x+3}$
- •[2] $\frac{A(x+3)+B(x-4)}{(2x-1)(x+3)}$
- •[3] $5x+1=A(x+3)+B(x-4)$
- •[4] choose to let $x=-3$ and 4 in turn
- •[5] $A=3$
- •[6] $B=2$

The radioactive element carbon-14 is sometimes used to estimate the age of organic remains such as bones, charcoal, and seeds.

Carbon-14 decays according to a law of the form $y = y_0 e^{kt}$ where y is the amount of radioactive nuclei present at time t years and y_0 is the initial amount of radioactive nuclei.

(*a*) The half-life of carbon-14, i.e. the time taken for half the radioactive nuclei to decay, is 5700 years. Find the value of the constant k , correct to 3 significant figures. **(3)**

(*b*) What percentage of the carbon-14 in a sample of charcoal will remain after 1000 years? **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.3
(*a*)	3	3.3			1	2			3.3.7		Source **1997 Paper 2 Qu.8**
(*b*)	3	3.3				3			3.3.7		

(*a*)

•1 $\frac{1}{2} y_0 = y_0 e^{5700k}$

•2 $\ln \frac{1}{2} = 5700k$

•3 $k = -0.000122$

(*b*)

•4 $y = y_0 e^{-0.000122 \times 1000}$

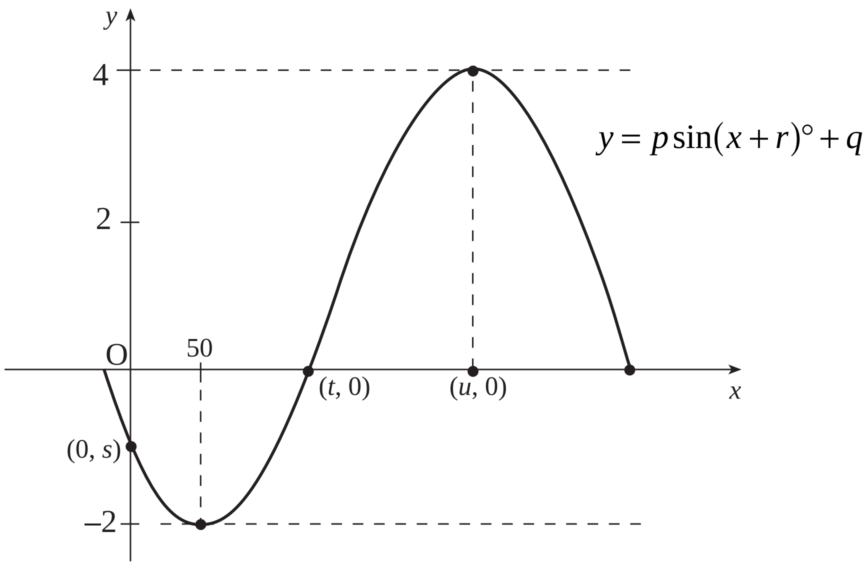
•5 $\frac{y}{y_0} = \ldots$

•6 88.5%

The sketch represents part of the graph of a trigonometric function of the form $y = p\sin(x + r)° + q$. It crosses the axes at (0, s) and (t,0), and has turning points at (50, –2) and (u, 4).

(i) Write down values for p, q, r and u. **(4)**

(ii) Find the values for s and t. **(4)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(a)	4	1.2			2	2			1.2.3		Source **1997 Paper 2 Qu.9**
(b)	4	2.3				4			2.3.1		

(a)

•1 $p = -3$

•2 $q = 1$

•3 $r = 40$ *or* -320

•4 $u = 230$

(b)

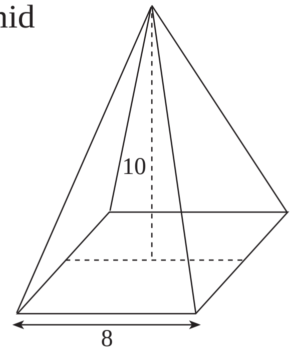
•5 replace x by 0

•6 –0.928

•7 replace y by 0

•8 120.5

A cuboid is to be cut out of a right square-based pyramid. The pyramid has a square base of side 8 cm. and a vertical height of 10cm.

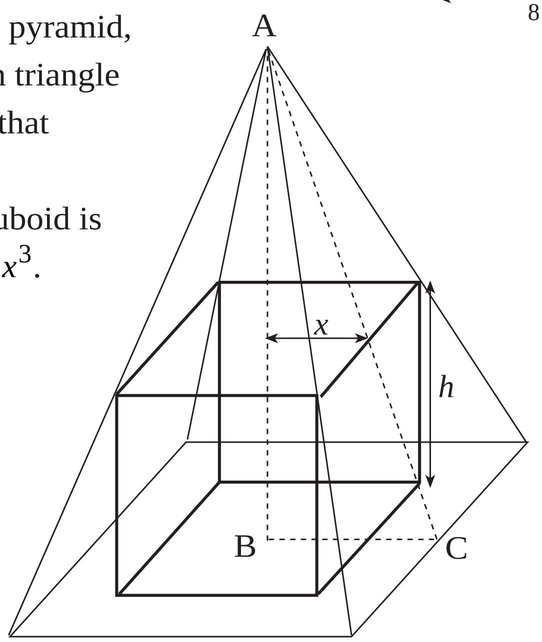


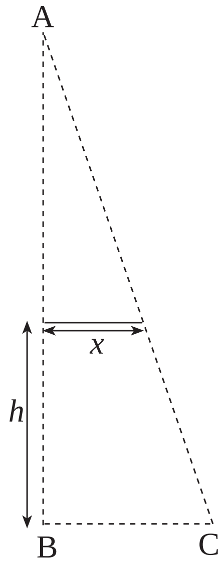
(*a*) The cuboid has a square base of side $2x$ cm and a height of h cm.

If the cuboid is to fit into the pyramid, use the information shown in triangle ABC, or otherwise, to show that

(i) $h = 10 - \frac{5}{2}x$. **(3)**

(ii) the volume, V, of the cuboid is given by $V = 40x^2 - 10x^3$. **(1)**





(*b*) Hence find the dimensions of the square-based cuboid with the greatest volume which can be cut from the pyramid. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.3
(*a*)	4	0.1					1	3	0.1		Source
(*b*)	6	1.3					3	3	1.3.15		**1997 Paper 2 Qu.10**

(*a*)

•1 strategy: e.g. equate ratios from similar triangles

•2 $\frac{10}{4} = \frac{10-h}{x}$ or equivalent

•3 complete proof

•4 $V = 40x^2 - 10x^3$

(*b*)

•5 $\frac{dV}{dx} =$

•6 $80x - 30x^2$

•7 $\frac{dV}{dx} = 0$ for stationary points

•8 $0, \frac{8}{3}$

•9

x	…	$\frac{8}{3}$	…
$\frac{dV}{dx}$	+	0	–
		max	

•10 $\frac{16}{3}$ and $\frac{10}{3}$

Two identical coins, radius 1 unit, are supported by horizontal and vertical plates at B and C. Diagram 1 shows the coins touching each other and the line of centres is inclined at p radians to the vertical.

Let d be the length of BC.

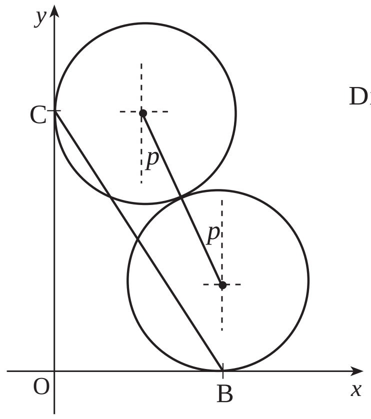


Diagram 1

(*a*) (i) Show that $\text{OB} = 1 + 2\sin p$ **(1)**

(ii) Write down a similar expression for OC and hence show that $d^2 = 6 + 4\cos p + 4\sin p$. **(2)**

(*b*) (i) Express d^2 in the form $6 + k\cos(p - \alpha)$ **(4)**

(ii) Hence write down the exact maximum value of d^2 and the value of p for which this occurs. **(2)**

(*c*) Diagram 2 shows the special case where $p = \frac{\pi}{4}$.

(i) Show that $\text{OB} = 1 + \sqrt{2}$ and find the exact length of BD. **(2)**

(ii) Using your answer to (*b*)(ii) find the exact value of $\sqrt{6 + 4\sqrt{2}}$. **(2)**

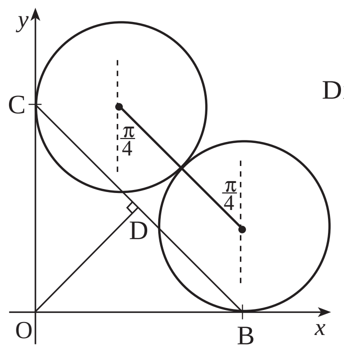


Diagram 2

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	3.4
(*a*)	3	0.1	2	1					0.1		Source
(*b*)	6	3.4	4	2					3.4.1	3.4.3	**1997 Paper 2**
(*c*)	4	0.1	1	3					0.1		**Qu.11**

(*a*)

•1 $\sin p = \frac{\text{“}hor\text{”}}{2}$ *and* $\text{OB} = 1 + \text{“}hor\text{”}$

•2 $\text{OC} = 1 + 2\cos p$

•3 $d^2 = (1 + 2\cos p)^2 + (1 + 2\sin p)^2$ and completes proof

(*b*)

•4 $k\cos(p - \alpha) = k\cos p\cos\alpha + k\sin p\sin\alpha$

•5 $k\cos\alpha = 4$ *and* $k\sin\alpha = 4$

•6 $k = 4\sqrt{2}$

•7 $\alpha = \frac{\pi}{4}$

•8 maximum value $= 6 + 4\sqrt{2}$

•9 occurs when $p = \frac{\pi}{4}$

(*c*)

•10 $OB = 1 + 2\sin\frac{\pi}{4}$ and completes proof

•11 $BD = (1 + \sqrt{2}) \times \frac{1}{\sqrt{2}}$

•12 $BC = 2 + \sqrt{2}$

•13 $6 + 4\sqrt{2} = (2 + \sqrt{2})^2$ *so* $\sqrt{6 + 4\sqrt{2}} = 2 + \sqrt{2}$