

- (a) Show that  $x = 2$  is a root of the equation  $2x^3 + x^2 - 13x + 6 = 0$ .  
 (b) Hence find the other roots.

1  
3

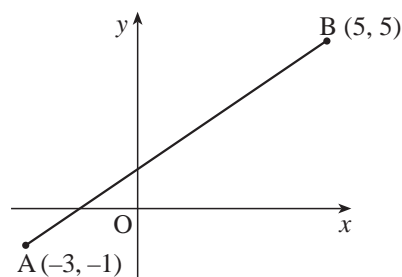
part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	1	2.1	1						2.1.1		Source 1999 P1 qu.1
(b)	3	2.1	3						2.1.2		

<p>•<sup>1</sup> <math>f(2) = 16 + 4 - 26 + 6 = 0</math>  <b>or</b>          the appearance of a '0' at the end of the 3rd line in the table below</p>	<p>•<sup>2</sup> <math display="block">\begin{array}{r rrrr} 2 &amp; 2 &amp; 1 &amp; -13 &amp; 6 \\ &amp; &amp; 4 &amp; 10 &amp; -6 \\ \hline &amp; 2 &amp; 5 &amp; -3 &amp; 0 \end{array}</math></p> <p>•<sup>3</sup> <math>2x^2 + 5x - 3</math>          •<sup>4</sup> <math>-3, \frac{1}{2}</math></p>
--	---

A and B are the points  $(-3, -1)$  and  $(5, 5)$ .

Find the equation of

- (a) the line AB  
 (b) the perpendicular bisector of AB.



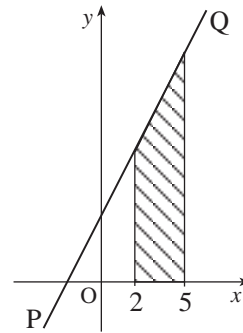
2  
3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	1.1					2		1.1.7		Source 1999 P1 qu.2
(b)	3	1.1					3		1.1.10		

<p>•<sup>1</sup> <math>m_{AB} = \frac{3}{4}</math>          •<sup>2</sup> <math>y - 5 = \frac{3}{4}(x - 5)</math> or <math>y - (-1) = \frac{3}{4}(x - (-3))</math></p>	<p>•<sup>3</sup> <math>m_{\perp} = -\frac{4}{3}</math>          •<sup>4</sup> midpoint = <math>(1, 2)</math>          •<sup>5</sup> <math>y - 2 = -\frac{4}{3}(x - 1)</math></p>
--	--

The line PQ has equation  $y = 2x + 4$ .

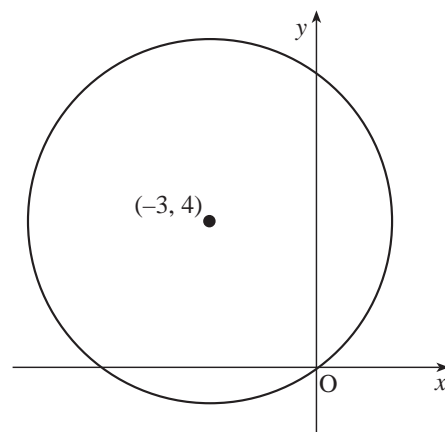
- (a) Find, without using calculus, the area of the shaded trapezium shown in the diagram. 2
- (b) Express the area of this trapezium as a definite integral. 1
- (c) Evaluate this integral. 2



part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.5
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	0.1	2						0.1		Source 1999 P1 qu.3
(b)	1	2.2	1						2.2.6		
(c)	2	2.2	2						2.2.5		

<ul style="list-style-type: none"> <li>•<sup>1</sup> evidence of e.g. triangle + rectangle</li> <li>•<sup>2</sup> area = 33</li> <li>•<sup>3</sup> <math>\int_2^5 (2x+4) dx</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>4</sup> <math>x^2 + 4x</math></li> <li>•<sup>5</sup> <math>45 - 12 = 33</math></li> </ul>
---	--

Find the equation of the circle with centre  $(-3, 4)$  and passing through the origin.



part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	2	2.4					2		2.4.3		Source 1999 P1 qu.4

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>r^2 = 25</math> stated or implied by •<sup>2</sup>.</li> <li>•<sup>2</sup> <math>(x+3)^2 + (y-4)^2 = 25</math></li> </ul>
--

Given  $f(x) = 3x^2(2x-1)$  find  $f'(-1)$ .

3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	3	1.3	3						1.3.4		Source 1999 P1 qu.5

- <sup>1</sup>  $6x^3 - 3x^2$
- <sup>2</sup>  $18x^2 - 6x$
- <sup>3</sup> 24

VABCD is a pyramid with rectangular base ABCD.

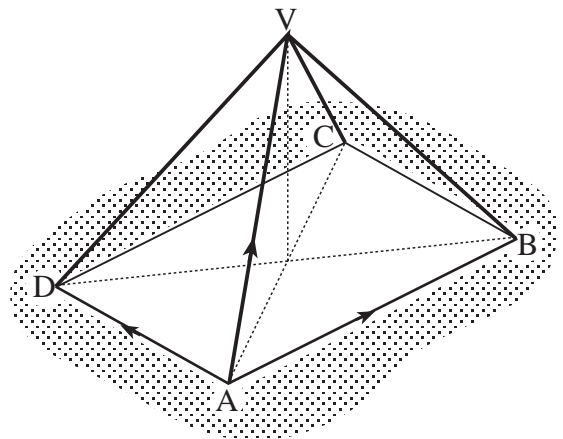
The vectors  $\vec{AB}$ ,  $\vec{AD}$  and  $\vec{AV}$  are given by

$$\vec{AB} = 8\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

$$\vec{AD} = -2\mathbf{i} + 10\mathbf{j} - 2\mathbf{k} \quad \text{and}$$

$$\vec{AV} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}.$$

Express  $\vec{CV}$  in component form.

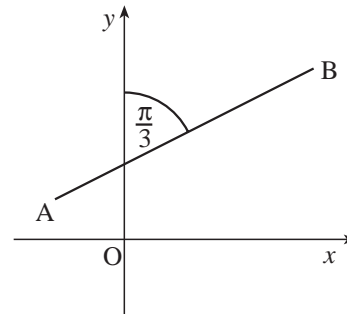


3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	3	3.1					3		3.1.8		Source 1999 P1 qu.6

- <sup>1</sup> pathway for  $\vec{CV}$ :  $\vec{CV} = \vec{CA} + \vec{AV}$
- <sup>2</sup> e.g.  $\vec{CB} = 2\mathbf{i} - 10\mathbf{j} + 2\mathbf{k}$   
 or  $\vec{BA} = -8\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$   
 or  $\vec{AC} = 6\mathbf{i} + 12\mathbf{j}$
- <sup>3</sup>  $\begin{pmatrix} -5 \\ -5 \\ 7 \end{pmatrix}$

The line AB makes an angle of  $\frac{\pi}{3}$  radians with the y-axis, as shown in the diagram. Find the exact value of the gradient of AB.



2

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	2	1.1						2	1.1.7		Source 1999 P1 qu.7

- <sup>1</sup> “correct angle” =  $\frac{\pi}{2} - \frac{\pi}{3}$
- <sup>2</sup>  $\frac{1}{\sqrt{3}}$

- (i) Write down the condition for the equation  $ax^2 + bx + c = 0$  to have no real roots. 1
- (ii) Hence or otherwise show that the equation  $x(x+1) = 3x - 2$  has no real roots. 2

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
(i)	1	2.1					1		2.1.6		Source
(ii)	2	2.1					2		2.1.6		1999 P1 qu.8

- <sup>1</sup>  $b^2 - 4ac = 0$
- <sup>2</sup>  $x^2 + 6x + 9 = 0$
- <sup>3</sup>  $b^2 - 4ac = 36 - 36 = 0$  OR •<sup>3</sup>  $(x+3)(x+3) = 0$  so roots are  $-3, -3$

The point P(-1, 7) lies on the curve with equation  $y = 5x^2 + 2$ . Find the equation of the tangent to the curve at P.

4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	4	1.3	4						1.3.9	1.1.7	Source 1999 P1 qu.9

<ul style="list-style-type: none"> <li>•1 <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•2 <math>10x</math></li> <li>•3 <math>-10</math></li> <li>•4 <math>y - 7 = -10(x - (-1))</math></li> </ul>
---

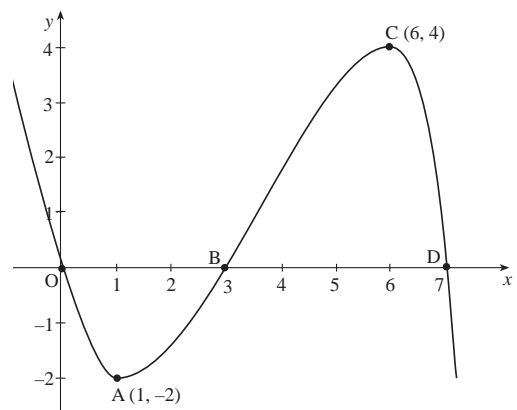
Part of the graph of  $y = f(x)$  is shown in the diagram.

On separate diagrams sketch the graphs of

(a)  $y = f(x+1)$

(b)  $y = -2f(x)$ .

Indicate on each graph the images of O, A, B, C and D.



1

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	1.2	2						1.2.4		Source
(b)	3	1.2	1	2					1.2.4		1999 P1 qu.10

<ul style="list-style-type: none"> <li>•1 translation of <math>\begin{pmatrix} -1 \\ 0 \end{pmatrix}</math></li> <li>•2 positions of images of A, B, C, D, O clear from the sketch</li> </ul>		<ul style="list-style-type: none"> <li>•3 reflect in x - axis</li> <li>•4 double y - coordinates</li> <li>•5 positions of images of A, B, C, D, O clear from the sketch</li> </ul>	
---	--	--	--

The graph of  $y = g(x)$  passes through the point  $(1,2)$ .

If  $\frac{dy}{dx} = x^3 + \frac{1}{x^2} - \frac{1}{4}$ , express  $y$  in terms of  $x$ .

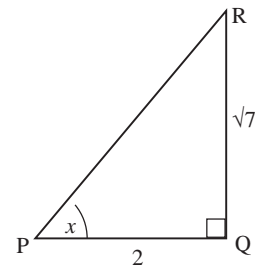
4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		<b>2.2</b>
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	4	2.2	4							2.2.8	Source <b>1999 P1 qu.11</b>

- <sup>1</sup>  $x^{-2}$  *stated or implied by* •<sup>2</sup> *or* •<sup>3</sup>
- <sup>2</sup>  $y = \int (x^3 + x^{-2} - \frac{1}{4})dx$  *or* the appearance of any term of  $\frac{1}{4}x^4 - \frac{1}{4}x - x^{-1}$
- <sup>3</sup> the remaining two terms
- <sup>4</sup>  $c = 3$

Using triangle PQR, as shown, find the exact value of  $\cos 2x$ .

3



part	marks	Unit	non-calc		calc		calc neut		Content Reference :		<b>2.3</b>
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	3	2.3	3							2.3.3	Source <b>1999 P1 qu.12</b>

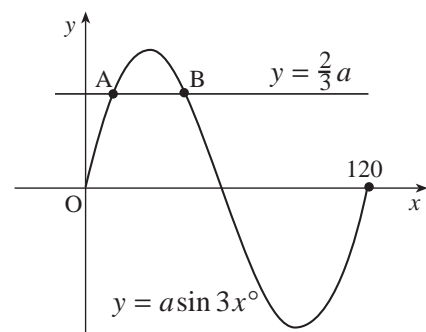
- <sup>1</sup>  $\cos x = \frac{2}{\sqrt{11}}$  *or*  $\sin x = \frac{\sqrt{7}}{\sqrt{11}}$
- <sup>2</sup>  $\cos 2x = 2 \times \left(\frac{2}{\sqrt{11}}\right)^2 - 1$
- <sup>3</sup>  $-\frac{3}{11}$

- (a) Show that  $f(x) = 2x^2 - 4x + 5$  can be written in the form  $f(x) = a(x+b)^2 + c$ . 3
- (b) Hence write down the coordinates of the stationary point of  $y = f(x)$  and state its nature. 2

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	3	1.2	3						1.2.8		Source
(b)	2	1.2	2						1.2.9		1999 P1 qu.13

- <sup>1</sup>  $2(x^2 - 2x) + 5$  stated or implied by •<sup>3</sup>
- <sup>2</sup>  $2(x-1)^2 + \dots$  stated or implied by •<sup>3</sup>
- <sup>3</sup>  $2(x-1)^2 + 3$
- <sup>4</sup> stationary pt at (1, 3)
- <sup>5</sup> stationary pt is minimum

The diagram shows part of the graph of  $y = a \sin 3x^\circ$  and the line with equation  $y = \frac{2}{3}a$ . Find the  $x$ -coordinates of A and B.



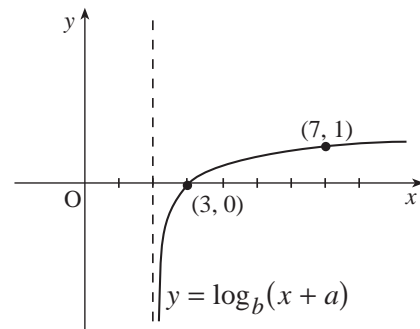
4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.3
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	4	2.3	4						2.3.1		Source
											1999 P1 qu.14

- <sup>1</sup>  $a \sin 3x = \frac{2}{3}a$  stated or implied by •<sup>2</sup>
- <sup>2</sup>  $\sin 3x = \frac{2}{3}$
- <sup>3</sup>  $3x = 41.8, 138.2$  (138.2 stated or implied by 46.1 in •<sup>4</sup>)
- <sup>4</sup> 13.9, 46.1

The diagram shows part of the graph of  $y = \log_b(x + a)$ .

Determine the values of  $a$  and  $b$ .



3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		<b>3.3</b>
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	3	3.3						3	3.3.1	1.2.5	Source <b>1999 P1 qu.15</b>

- |   |    |  |
|---|----|--|
| <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>a = -2</math></li> <li>•<sup>2</sup> <math>1 = \log_b(7 - 2)</math></li> <li>•<sup>3</sup> <math>b = 5</math></li> </ul> | OR | <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>1 = \log_b(7 + a)</math> <b>and</b> <math>0 = \log_b(a + 3)</math></li> <li>•<sup>2</sup> <math>7 + a = b</math> <b>and</b> <math>a + 3 = b^0</math></li> <li>•<sup>3</sup> <math>a = -2, b = 5</math></li> </ul> |
|---|----|--|

A curve has equation  $y = 2x^3 + 3x^2 + 4x - 5$ .

Prove that this curve has no stationary points.

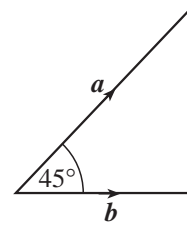
5

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		<b>1.3</b>
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	5	1.3	2	3					1.3.12	1.3.11	Source <b>1999 P1 qu.16</b>

- |   |    |  |
|---|----|--|
| <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>6x^2 + 6x + 4</math></li> <li>•<sup>3</sup> e.g. "<math>b^2 - 4ac</math>" = .....</li> <li>•<sup>4</sup> <math>-60</math> or <math>-15</math> (from <math>3x^2 + 3x + 2</math>)</li> <li>•<sup>5</sup> <math>\Delta</math> negative so no st. points</li> </ul> | OR | <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>6x^2 + 6x + 4</math></li> <li>•<sup>3</sup> e.g. complete square.....</li> <li>•<sup>4</sup> <math>S = 6\left(x + \frac{1}{2}\right)^2 + 2\frac{1}{2}</math></li> <li>•<sup>5</sup> <math>S \geq 2\frac{1}{2}</math> so no st. points</li> </ul> |
|---|----|--|



The diagram shows two vectors  $\mathbf{a}$  and  $\mathbf{b}$ , with  $|\mathbf{a}| = 3$  and  $|\mathbf{b}| = 2\sqrt{2}$ .  
These vectors are inclined at an angle of  $45^\circ$  to each other.



- (a) Evaluate
- (i)  $\mathbf{a} \cdot \mathbf{a}$
  - (ii)  $\mathbf{b} \cdot \mathbf{b}$
  - (iii)  $\mathbf{a} \cdot \mathbf{b}$
- (b) Another vector  $\mathbf{p}$  is defined by  $\mathbf{p} = 2\mathbf{a} + 3\mathbf{b}$ .  
Evaluate  $\mathbf{p} \cdot \mathbf{p}$  and hence write down  $|\mathbf{p}|$ .

2  
4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	3.1					2		3.1.9		Source
(b)	4	3.1					4		3.1.9		1999 P1 qu.17

• <sup>1</sup>	$\mathbf{a} \cdot \mathbf{a} = 9$ and $\mathbf{b} \cdot \mathbf{b} = 8$	• <sup>3</sup>	$(2\mathbf{a} + 3\mathbf{b}) \cdot (2\mathbf{a} + 3\mathbf{b})$
• <sup>2</sup>	$\mathbf{a} \cdot \mathbf{b} = 6$	• <sup>4</sup>	$4\mathbf{a} \cdot \mathbf{a} + 9\mathbf{b} \cdot \mathbf{b} + 12\mathbf{a} \cdot \mathbf{b}$
		• <sup>5</sup>	180
		• <sup>6</sup>	$\sqrt{180}$

Two sequences are defined by the recurrence relations

$$u_{n+1} = 0.2u_n + p, \quad u_0 = 1 \quad \text{and}$$

$$v_{n+1} = 0.6v_n + q, \quad v_0 = 1.$$

If both sequences have the same limit, express  $p$  in terms of  $q$ .

3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	3	1.4					3		1.4.5		Source
											1999 P1 qu.18

• <sup>1</sup>	" $L = 0.2L + p, \quad L = 0.6L + q$ " or use " $L = \frac{b}{1-a}$ "
• <sup>2</sup>	$\frac{p}{0.8}$ and $\frac{q}{0.4}$
• <sup>3</sup>	$p = \frac{0.8q}{0.4}$ or equivalent expression for $p$

Given  $f(x) = \cos^2 x - \sin^2 x$ , find  $f'(x)$ .

3

part marks	Unit	non-calc		calc		calc neut		Content Reference :		<b>3.2</b>
		C	A/B	C	A/B	C	A/B	Main	Additional	
3	3.2	1	2					3.2.2	3.2.1	Source <b>1999 P1 qu.19</b>

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(x) = \cos 2x</math></li> <li>•<sup>2</sup> <math>-\sin 2x</math></li> <li>•<sup>3</sup> <math>\times 2</math></li> </ul>	<p>For <math>\frac{d}{dx}(\cos^2 x)</math>     <b>OR</b>     For <math>\frac{d}{dx}(-\sin^2 x)</math></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2 \cos x</math></li> <li>•<sup>2</sup> <math>\times -\sin x</math></li> </ul> <p>For <math>\frac{d}{dx}(-\sin^2 x)</math></p> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>-2 \sin x \times \cos x</math></li> </ul>	<p>For <math>\frac{d}{dx}(-\sin^2 x)</math></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>-2 \sin x</math></li> <li>•<sup>2</sup> <math>\times \cos x</math></li> </ul> <p>For <math>\frac{d}{dx}(\cos^2 x)</math></p> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>2 \cos x \times -\sin x</math></li> </ul>
---	---	--

Find  $\int \frac{x^2 - 5}{x\sqrt{x}} dx$ .

4

part marks	Unit	non-calc		calc		calc neut		Content Reference :		<b>2.2</b>
		C	A/B	C	A/B	C	A/B	Main	Additional	
4	2.2	2	2					2.2.4		Source <b>1999 P1 qu.20</b>

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\left(\frac{x^2}{x\sqrt{x}}\right) x^{\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>\left(\frac{-5}{x\sqrt{x}}\right) -5x^{-\frac{3}{2}}</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>3</sup> <math>\frac{x^{\frac{3}{2}}}{\frac{3}{2}}</math></li> <li>•<sup>4</sup> <math>\frac{-5}{-\frac{1}{2}} x^{-\frac{1}{2}}</math></li> </ul>
--	---

A function  $f$  can be expressed as an infinite series by  $f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$

(a) Write down the series for  $f(2x)$  as far as the term in  $x^5$ .

1

The derivative of  $f(x)$  can be calculated as follows:

$$f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$$

$$\text{so } f'(x) = 0 + 1 + \frac{2x}{2} + \frac{3x^2}{6} + \frac{4x^3}{24} + \frac{5x^4}{120} + \dots$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

*i.e.*  $f'(x) = f(x)$

(b) If  $g(x) = f(2x)$  find  $g'(x)$  and express it in terms of  $f(2x)$ .

3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		0.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	1	0.1						1	0.1		Source <b>1999 P1 qu.21</b>
(b)	3	0.1					3	0.1	1.3.4		

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>1 + 2x + \frac{(2x)^2}{2} + \frac{(2x)^3}{6} + \frac{(2x)^4}{24} + \frac{(2x)^5}{120}</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>2</sup> <math>1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5</math></li> <li>•<sup>3</sup> <math>2 + 4x + 4x^2 + \frac{8}{3}x^3 + \frac{4}{3}x^4</math></li> <li>•<sup>4</sup> <math>2\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4\right)</math> <b>and</b> <math>2f(2x)</math></li> </ul>
--	---