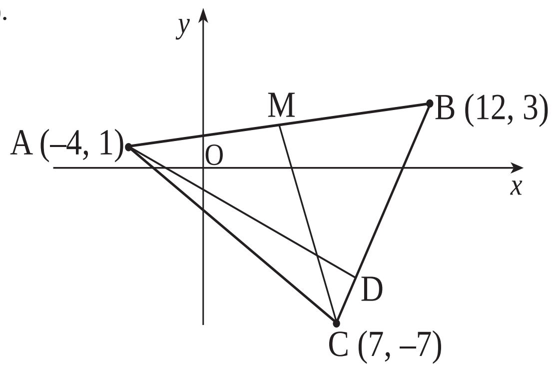
A triangle ABC has vertices A (–4, 1), B (12, 3) and C (7, –7).



(*a*) Find the equation of the median CM. **3**

(*b*) Find the equation of the altitude AD. **3**

(*c*) Find the coordinates of the point of intersection of CM and AD. **3**

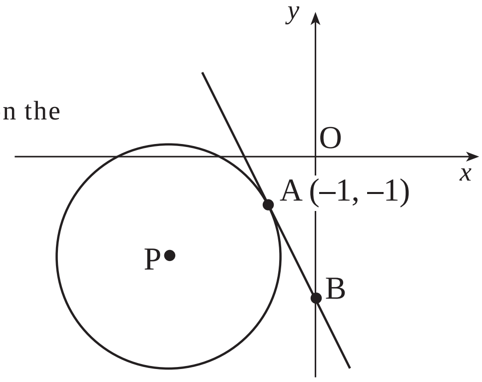
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.1
(*a*)	3	1.1					3		1.1.7		Source
(*b*)	3	1.1					3		1.1.7	1.1.9	**1999 Paper 2**
(*c*)	3	0.1					3		0.1		**Qu. 1**

(*a*)
- •1 midpoint $= (4, 2)$
- •2 $m_{MC} = -3$
- •3 $y - 2 = -3(x - 4)$ ***or*** $y - (-7) = -3(x - 7)$

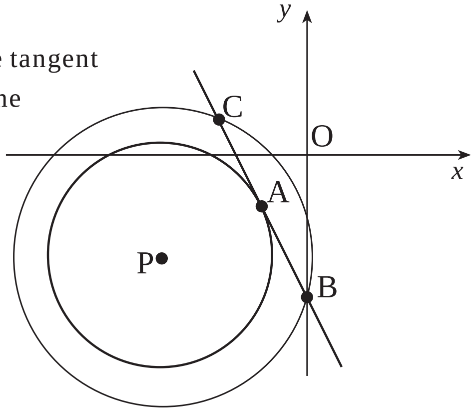
(*b*)
- •4 $m_{BC} = 2$
- •5 $m_{\perp} = -\frac{1}{2}$
- •6 $y - 1 = -\frac{1}{2}(x - (-4))$

(*c*)
- •7 *e.g.* $3x + y = 14$ *and* $x + 2y = -2$
- •8 attempt to eliminate a variable
- •9 $(6, -4)$

(*a*)	The diagram shows a circle, centre P, with equation $x^2 + y^2 + 6x + 4y + 8 = 0$. Find the equation of the tangent at the point A(–1, –1) on the circle.		**4**
(*b*)	The tangent crosses the y-axis at B. Find the coordinates of B.		**1**



(*c*)	Another circle, centre P, is drawn passing through B. The tangent at A meets the second circle at the point C, as shown in the diagram. Write down the coordinates of C.		**1**
(*d*)	Find the equation of the circle with BC as diameter.		**2**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.4
(*a*)	4	2.4					4		2.4.2	1.1.9	Source
(*b*)	1	0.1					1		0.1		**1999 Paper 2**
(c)	1	0.1					1		0.1		**Qu. 2**
(d)	2	2.4					2		2.4.4		

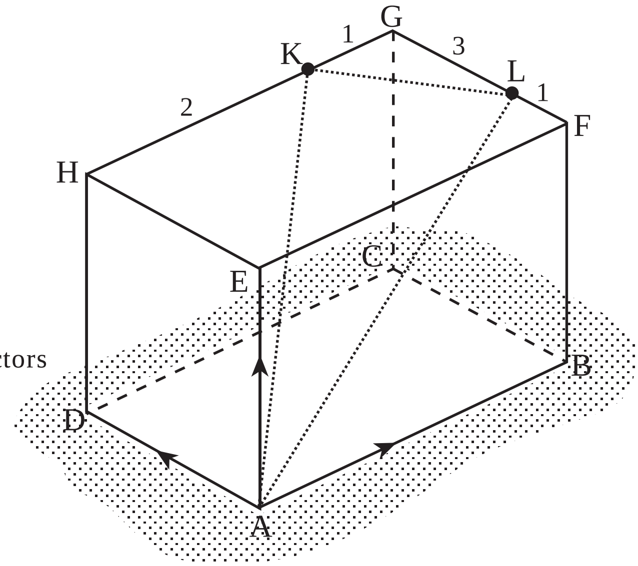
(*a*)
- •1 $centre = (-3, -2)$
- •2 $m_{rad} = \frac{1}{2}$
- •3 $m_{tgt} = -2$
- •4 $y - (-1) = -2(x - (-1))$

(*b*)
- •5 $B = (0, -3)$

(*c*)
- •6 $C = (-2, 1)$

(*d*)
- •7 $r^2 = 5$
- •8 $(x+1)^2 + (y+1)^2 = 5$

ABCDEFGH is a cuboid.



K lies two thirds of the way along HG.
(i.e. HK:KG = 2:1).
L lies one quarter of the way along FG.
(i.e. FL:LG = 1:3).

$\vec{AB}$, $\vec{AD}$ and $\vec{AE}$ can be represented by the vectors

$\begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}, \begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$ respectively.

(*a*) Calculate the components of $\vec{AK}$. **2**

(*b*) Calculate the components of $\vec{AL}$. **2**

(c) Calculate the size of angle KAL. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
(*a*)	2	3.1					2		3.1.2		Source **1999 Paper 2 Qu. 3**
(*b*)	2	3.1					2		3.1.2		
(c)	5	3.1					5		3.1.11		

(*a*)

•1 obtaining for example $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$

•2 $\vec{AK} = \begin{pmatrix} -5 \\ 5 \\ 11 \end{pmatrix}$

(*b*)

•3 obtaining for example $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

•4 $\vec{AL} = \begin{pmatrix} 2 \\ 4 \\ 9 \end{pmatrix}$

(*c*)

•5 *strategy e.g.* $\cos K\hat{A}L = \frac{\vec{AK}.\vec{AL}}{|AK|\times|AL|}$

•6 109

•7 $\sqrt{171}$

•8 $\sqrt{101}$

•9 $\hat{A} = 34.0$

OR

•5 *strategy e.g.* $\cos K\hat{A}L = \frac{AK^2 + AL^2 - KL^2}{2AK \times AL}$

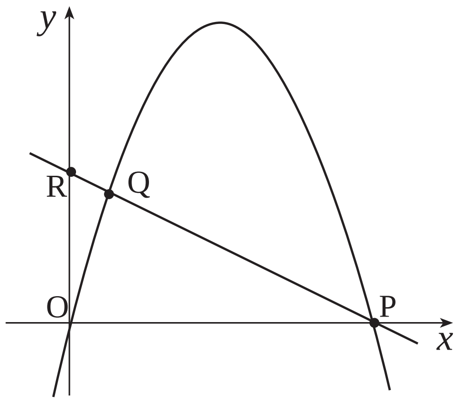
•6 $\sqrt{54}$

•7 $\sqrt{171}$

•8 $\sqrt{101}$

•9 $\hat{A} = 34.0$

The parabola shown in the diagram has equation $y = 4x - x^2$ and intersects the x-axis at the origin and P.



(*a*) Find the coordinates of the point P. **2**

(*b*) R is the point (0, 2). Find the equation of PR. **2**

(c) The line and the parabola also intersect at Q. Find the coordinates of Q. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(*a*)	2	1.2	2						1.2.9		Source
(*b*)	2	1.1	2						1.1.7		**1999 Paper 2**
(c)	4	2.1	4						2.1.8		**Qu. 4**

(*a*)

•[1] $4x - x^2 = 0$ *stated or implied by* •[2]

•[2] $(4, 0)$

(*b*)

•[3] $m = -\frac{1}{2}$

•[4] $y = -\frac{1}{2}x + 2$

or $y - 2 = -\frac{1}{2}(x - 0)$

or $y - 0 = -\frac{1}{2}(x - 4)$

(*c*)

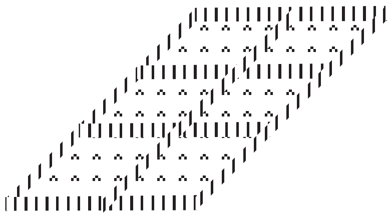
•[5] $4x - x^2 = 2 - \frac{1}{2}x$

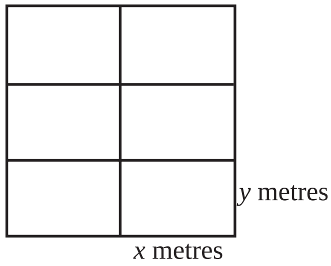
•[6] *e.g.* $2x^2 - 9x + 4 = 0$

•[7] $x = \frac{1}{2}, x = 4$

•[8] Q is $\left(\frac{1}{2}, \frac{7}{4}\right)$

A zookeeper wants to fence off six individual animal pens.





Each pen is a rectangle measuring x metres by y metres, as shown in the diagram.

(*a*) (i) Express the total length of fencing in terms of x and y.

(ii) Given that the total length of fencing is 360m, show that the total area, A m^2, of the six pens is given by $A(x) = 240x - \frac{16}{3}x^2$. **4**

(*b*) Find the values of x and y which give the maximum area and write down this maximum area. **6**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.3
(*a*)	4	0.1					2	2	0.1		Source
(*b*)	6	1.3					6		1.3.15		**1999 Paper 2 Qu. 5**

(*a*)

•1 $9y + 8x$

•2 $A = 3y \times 2x$

•3 $9y = (360 - 8x)$

•4 $2x.3.\frac{1}{9}(360 - 8x)$ and complete proof

(*b*)

•5 $A'(x) = \ldots\ldots$

•6 $240 - \frac{32}{3}x$

•7 $A'(x) = 0$ *or* $240 - \frac{32}{3}x = 0$

•8 $x = 22\frac{1}{2}$, $y = 20$

•9

x	$22\frac{1}{2}^-$	$22\frac{1}{2}$	$22\frac{1}{2}^+$
$A'(x)$	+	0	–
		maximum	

•10 2700

Functions f and g are defined on the set of real numbers by

$$f(x) = x - 1$$
$$g(x) = x^2$$

(*a*) Find formulae for (i) $f(g(x))$

(ii) $g(f(x))$. **4**

(*b*) The function h is defined by $h(x) = f(g(x)) + g(f(x))$.

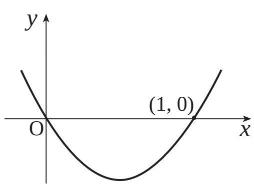
Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h. **3**

(*c*) Find the area enclosed between this graph and the x-axis. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.2
(*a*)	4	1.2	4						1.2.6		Source
(*b*)	3	1.2	3						1.2.9	0.1	**1999 Paper 2**
(*c*)	4	2.2	4						2.2.6		**Qu. 6**

(*a*)
- •1 $f(x^2)$ *stated or implied by* •2
- •2 $x^2 - 1$
- •3 $g(x-1)$ *stated or implied by* •4
- •4 $(x-1)^2$

(*b*)
- •5 $(x-1)^2 + x^2 - 1$ and complete proof
- •6 sketch as shown



- •7 minimum at $\left(\frac{1}{2}, -\frac{1}{2}\right)$ calculated or on sketch

(*c*)
- •8 $\int_0^1 (2x^2 - 2x)\ dx$
- •9 $\left[\frac{2}{3}x^3 - x^2\right]$
- •10 $-\frac{1}{3}$
- •11 dealing with $-$ve

The intensity I_t of light is reduced as it passes through a filter according to the law $I_t = I_0 e^{-kt}$ where I_0 is the initial intensity and I_t is the intensity after passing through a filter of thickness t cm. k is a constant.

(*a*) A filter of thickness 4 cm reduces the intensity from 120 candle-power to 90 candle-power. Find the value of k. **4**

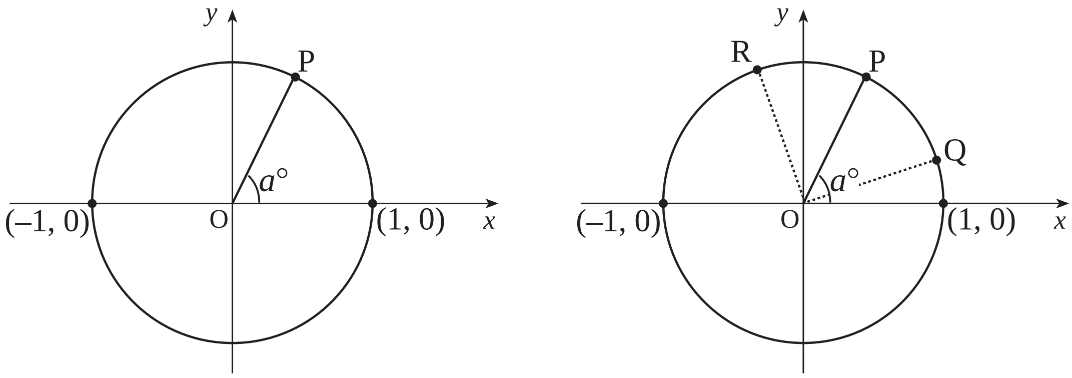
(*b*) The light is passed through a filter of thickness 10 cm. Find the percentage reduction in its intensity. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.3
(*a*)	4	3.3			2	2			3.3.7		Source **1999 Paper 2 Qu. 7**
(*b*)	3	3.3			1	2			3.3.7		

(*a*)
- •1 $90 = 120e^{-4k}$
- •2 $e^{-4k} = 0.75$ ***or*** $\ln 90 = \ln 120 + \ln e^{-4k}$
- •3 $\ln 0.75 = -4k$
- •4 $k = 0.0719$

(*b*)
- •5 $I_{10} = I_0 e^{-10\times 0.0719}$ *stated or implied by* •6
- •6 $\frac{I_{10}}{I_0} = 0.487$
- •7 51.3% reduction

The diagram shows a circle of radius 1 unit and centre the origin. The radius OP makes an angle $a°$ with the positive direction of the x-axis.



(*a*) Show that P is the point ($\cos a°$, $\sin a°$). **1**

(*b*) If $P\hat{O}Q = 45°$, deduce the coordinates of Q in terms of *a*. **1**

(*c*) If $P\hat{O}R = 45°$, deduce the coordinates of R in terms of *a*. **1**

(*d*) Hence find an expression for the gradient of QR in its simplest form. **4**

(*e*) Show that the tangent to the circle at P is parallel to QR. **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.3
(*a*)	1	0.1	1						0.1		Source
(*b*)	1	0.1	1						0.1		**1999 Paper 2**
(*c*)	1	0.1	1						0.1		**Qu. 8**
(*d*)	4	2.3		4					2.3.2	1.1.1	
(*e*)	2	1.1		2					1.1.8	1.1.9	

(*a*) •1 proof e.g. showing rt - angled triangle with "1" and a°

(*b*) •2 Q is $\left(\cos(a-45)°, \sin(a-45)°\right)$

(*c*) •3 R is $\left(\cos(a+45)°, \sin(a+45)°\right)$

(*d*) •4 $\frac{\sin(a+45)-\sin(a-45)}{\cos(a+45)-\cos(a-45)}$

•5 $\frac{\sin a\cos 45+\cos a\sin 45-\sin a\cos 45+\cos a\sin 45}{\cos a\cos 45-\sin a\sin 45-\cos a\cos 45-\sin a\sin 45}$

•6 $\frac{2\cos a\sin 45}{-2\sin a\sin 45}$

•7 $-\frac{1}{\tan a}$

(*e*) •8 $m_{OP} = \frac{\sin a}{\cos a} = \tan a$

•9 $m_{tgt\ at\ P} = -\frac{1}{\tan a}$

Solve the equation $2\sin x° - 3\cos x° = 2\cdot 5$ in the interval $0 \le x < 360$. **8**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.4
.	8	3.4				8			3.4.2		Source **1999 Paper 2 Qu. 9**

- •1 strategy: e.g. $\ldots\ldots k\sin(x-a)$ *stated or implied by* •6
- •2 $k\sin x\cos a - k\cos x\sin a$ *stated explicitly*
- •3 $k\cos a = 2$ *and* $k\sin a = 3$ *stated explicitly*
- •4 $k = \sqrt{13}$
- •5 $a = 56.3$
- •6 $\sin(x-56.3) = \frac{2.5}{\sqrt{13}}$
- •7 $x - 56.3 = 43.9,\ 136.1$ *136.1 stated or implied by the appearance of 192.4 in* •8
- •8 $100.2°$ *and* $192.4°$

OR

- •7 $x - 56.3 = 43.9,\ x = 100.2°$
- •8 $192.4°$

kcos(x − a)

$k\cos x\cos a + k\sin x\sin a$

$k\cos a = -3,\ k\sin a = 2$

$k = \sqrt{13},\quad \tan a = -\frac{2}{3}$

$a = 146.3$

$\cos(x-146.3) = 0.693$

$x - 146.3 = 46.1,\quad 313.9$

$x = 192.4,\quad 460.2$

$x = 192.4,\quad 100.2$

ksin(x + a)

$k\sin x\cos a + k\cos x\sin a$

$k\cos a = 2,\ k\sin a = -3$

$k = \sqrt{13},\quad \tan a = -\frac{3}{2}$

$a = 303.7$

$\sin(x+303.7) = 0.693$

$x + 303.7 = 43.9,\quad 136.1$

$x = -259.8,\quad -167.6$

$x = 100.2,\quad 192.4$

kcos(x + a)

$k\cos x\cos a - k\sin x\sin a$

$k\cos a = -3,\ k\sin a = -2$

$k = \sqrt{13},\quad \tan a = \frac{2}{3}$

$a = 213.7$

$\cos(x+213.7) = 0.693$

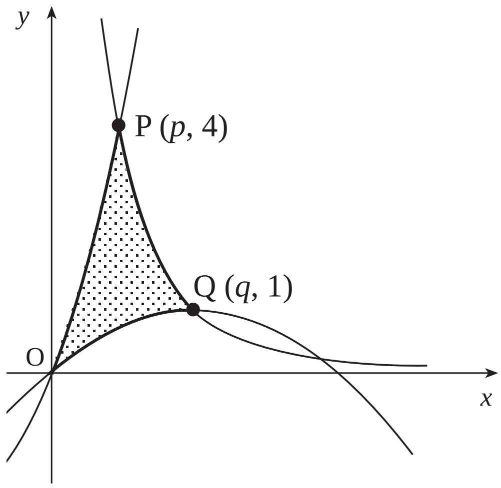
$x + 213.7 = 46.1,\quad 313.9$

$x = -167.6,\quad 100.2$

$x = 192.4,\quad 100.2$

The origin, O, and the points P and Q are the vertices of a curved 'triangle' which is shaded in the diagram.
The sides lie on curves with equations

$y = x(x+3)$, $y = x - \frac{1}{4}x^2$ and $y = \frac{4}{x^2}$.



(*a*) P and Q have coordinates $(p, 4)$ and $(q, 1)$. Find the values of p and q. **2**

(*b*) Calculate the shaded area . **7**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A / B	C	A / B	C	A / B	Main	Additional	Source
(*a*)	2	1.2	2						1.2.9		**1999 Paper 2**
(*b*)	7	2.2		7					2.2.7		**Qu. 10**

(*a*)

•[1] $p = 1$

•[2] $q = 2$

OR

(*b*)

•[3] $\int_0^1 (\text{'}OP\text{'} - \text{'}OQ\text{'})\, dx + \int_1^2 (\text{'}PQ\text{'} - \text{'}OQ\text{'})\, dx$

•[4] $\int_0^1 \left(x^2 + 3x - x + \frac{1}{4}x^2\right) dx$

•[5] $\left[\frac{5}{12}x^3 + x^2\right]$ ***or*** $\left[\frac{1}{3}x^3 + \frac{3}{2}x^2 - \frac{1}{2}x^2 + \frac{1}{12}x^3\right]$

•[6] $\frac{17}{12}$

•[7] $\int_1^2 \left(4x^{-2} - x + \frac{1}{4}x^2\right) dx$

•[8] $\left[-4x^{-1} - \frac{1}{2}x^2 + \frac{1}{12}x^3\right]$

•[9] $\frac{13}{12}$ and Area $= 2\frac{1}{2}$

•[3] $\int_0^1 \ldots\ldots\, dx + \int_1^2 \ldots\ldots\, dx - \int_0^2 \ldots\ldots\, dx$

•[4] $\int_0^1 \left(x^2 + 3x\right) dx + \int_1^2 \left(4x^{-2}\right) dx - \int_0^2 \left(x - \frac{1}{4}x^2\right) dx$

•[5] $\left[\frac{1}{3}x^3 + \frac{3}{2}x^2\right]$

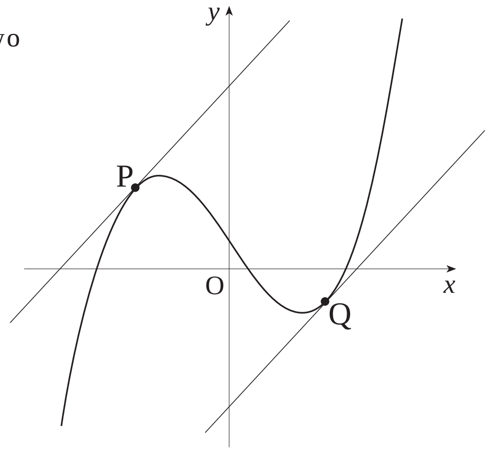
•[6] $\left[-4x^{-1}\right]$

•[7] $\left[\frac{1}{2}x^2 - \frac{1}{12}x^3\right]$

•[8] any two evaluations from $\frac{11}{6}, 2, \frac{4}{3}$

•[9] third evaluation and area $= \frac{11}{6} + 2 - \frac{4}{3} = 2\frac{1}{2}$

The diagram shows a sketch of the graph of $y = x^3 - 9x + 4$ and two parallel tangents drawn at P and Q.



(*a*) Find the equations of the tangents to the curve $y = x^3 - 9x + 4$ which have gradient 3. **6**

(*b*) Show that the shortest distance between the tangents is $\frac{16\sqrt{10}}{5}$. **6**

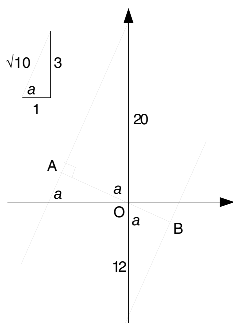
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	**1.3**
(*a*)	6	1.3					6		1.3.9	1.1.7	Source **1999 Paper 2 Qu. 11**
(*b*)	6	1.1						6	1.1.10		

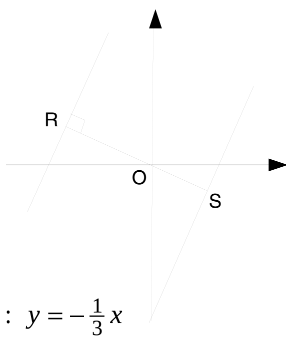
(*a*)
- •1 strategy: $\frac{dy}{dx} = \ldots\ldots = 3$
- •2 $3x^2 - 9$
- •3 $x = 2, \ -2$
- •4 $y = -6, \ 14$
- •5 $y + 6 = 3(x - 2)$
- •6 $y - 14 = 3(x + 2)$

OR

- •3 $x = 2, \quad y = -6$
- •4 $x = -2, \quad y = 14$

(*b*)
- •7 diagram with $y = -\frac{1}{3}x$
- •8 for 20 and 12
- •9 $AB = AO + OB$
- •10 $AB = 20\cos a + 12\cos a$
- •11 using $\tan a = \frac{3}{1}$
- •12 $AB = 32 \times \frac{1}{\sqrt{10}} = 32 \times \frac{\sqrt{10}}{10} = \frac{32}{5}\sqrt{10}$





- •7 $m_{RS} = -\frac{1}{3}$
- •8 *equ of RS* : $y = -\frac{1}{3}x$
- •9 $-\frac{1}{3}x = 3x - 12$ & $-\frac{1}{3}x = 3x + 20$
- •10 $R(-6, 2)$ and $S\left(\frac{18}{5}, -\frac{6}{5}\right)$
- •11 $d^2 = \left(-6 - \left(\frac{18}{5}\right)\right)^2 + \left(2 - \left(-\frac{6}{5}\right)\right)^2$
- •12 $d^2 = \frac{48^2}{25} + \frac{16^2}{25}$ and completes proof