Mathematics
Additional Question Bank
Higher

4972

Spring 1999

HIGHER STILL

Mathematics

Additional Question Bank

Higher

Support Materials

1. INTRODUCTION

1.1 Background

This Bank of Additional Questions for Higher Mathematics consists of short and extended response questions taken from the SCE Mathematics Higher Grade Examination Papers for the years 1989 to 1998. Through the question paper moderation procedures of the Scottish Examination Board(SEB) (now Scottish Qualifications Authority (SQA)), all of the past paper questions in the Bank have already undergone scrutiny for clarity of language and mathematical accuracy. In addition, the predicted difficulty levels have either been confirmed or amended as a result of actual examination performance. Both of these factors have facilitated the speedy construction of an initial Bank of valid and reliable assessment instruments for the Higher Mathematics course consisting of Mathematics 1(H), Mathematics 2(H) and Mathematics 3(H). Work is under way to produce a similar bank of questions for Statistics (H).

1.2 Structure and purpose

The structure of the Bank is such that questions from future Higher Mathematics examinations and other sources available to users can be categorised similarly and added to the Bank.

The purpose of the Bank is to prepare for course assessment and to generate evidence of attainment beyond the minimum competence necessary to pass the unit assessments for Higher Mathematics. Centres are required to submit estimates of the bands candidates are likely to attain in the external course assessment and to retain the evidence of attainment on which estimates are based for use in the event of appeals. Following the guidelines below and using questions from this bank to obtain an assessment of the candidate's own unaided work should provide quality evidence of an estimate band. Centres may, of course, devise their own assessment materials.

1.3 Quality of evidence

For assessment evidence in the form of prelim examinations or any other form of evidence to be fit for the purposes of estimates and appeals it is important that it covers as much of the course as possible. In Mathematics, evidence will normally be produced under supervision to ensure that it is the candidate's own unaided work. The following specification, which approximates to that used by SQA in the construction of the external assessment for Higher Mathematics, is offered as a guide for the construction of internal course assessments in terms of breadth, depth and variety of assessment instruments.

Breadth: Maths 1(H) : Maths 2(H) : Maths 3(H) or Stats (H): cgd.

In assessment evidence, approximately 30% of the available marks should be allocated to questions or parts of questions based on the content of each unit with the remainder, approximately 10%, allocated to questions or parts of questions which meet the course grade descriptions (cgd) for Higher Mathematics.
Approximately 20% of the available marks should be embedded in questions which integrate across the above categories.

Depth: grade C marks : grade A/B marks

In assessment evidence, approximately 60% of the available marks should be accessible to candidates capable of achieving a C grade in the external examination. The attainment of marks allocated to the A/B category is a good indication of a high level of mathematical ability. However it should be noted that the categorisation of C and A/B marks is intended to be used mainly for the construction of assessments. Judgements of ability can be arrived at on the basis of the total marks obtained by candidates on assessments of appropriate depth and duration. Evidence used for appeals purposes must be clearly supporting a grade ie A, B or C.

Variety of assessment instruments:

(a) non-calculator & calculator neutral : calculator & calculator neutral
The SQA Higher Mathematics Specimen Question Paper I, in which calculators may not be used, contains questions which assess knowledge and skills that candidates should be able to demonstrate without the aid of a calculator. In Higher Mathematics, for example, some questions of a graphical nature and some assessing basic algebraic manipulation and calculus would come into this category. Additional questions of a calculator neutral variety top up the total marks available to 50, with a time allocation of 1 hour 15 minutes. Specimen Question Paper II, in which calculators may be used, contains those questions where a calculator is required, for example, to obtain numerical values of trigonometric ratios and logarithmic/exponential expressions or to carry out computation of a complex nature. Additional questions of a calculator neutral variety top up the total marks available to 70, with a time allocation of 1 hour
45 minutes. Assessment evidence need not conform exactly to this pattern, but it is important that non-calculator skills and the ability to perform sustained work are assessed.

(b) short response : extended response
The number of marks arising from extended response questions in assessment evidence should be at least 50%.

1.4 Bank codes

In the following sections of this Additional Question Bank, codes are used for ease of reference. Mathematics 1(H), Mathematics 2(H), Mathematics 3(H) and Statistics (H) are referred to as Unit 1, Unit 2, Unit 3 and Unit 4 respectively. A 3-figure code has been applied to the items of course content as listed in the National Course Specification for Higher Mathematics. For example, 2.1.10 is the reference to the tenth item of content in the first outcome of unit 2. A code 0.1 has been used to classify content which falls into the category of course grade descriptions. In some instances there is not a content item statement which exactly describes the mathematical activity being assessed. For example, 2.2.5 evaluate definite integrals also covers using the result of a definite integration to find an unknown limit. Section 2 contains the full list of coded content for Higher Mathematics in an abbreviated form. The document, SQA Mathematics Higher:National Course Specification should be consulted for a full statement of course content and comment and course grade descriptions.

1.5 Short response analysis

Section 3 of the Bank consists of a grid which refers to the short response questions by year of past paper and provides an analysis of the questions in terms of the categories described earlier. Headings and abbreviations are explained in 1.6 below.

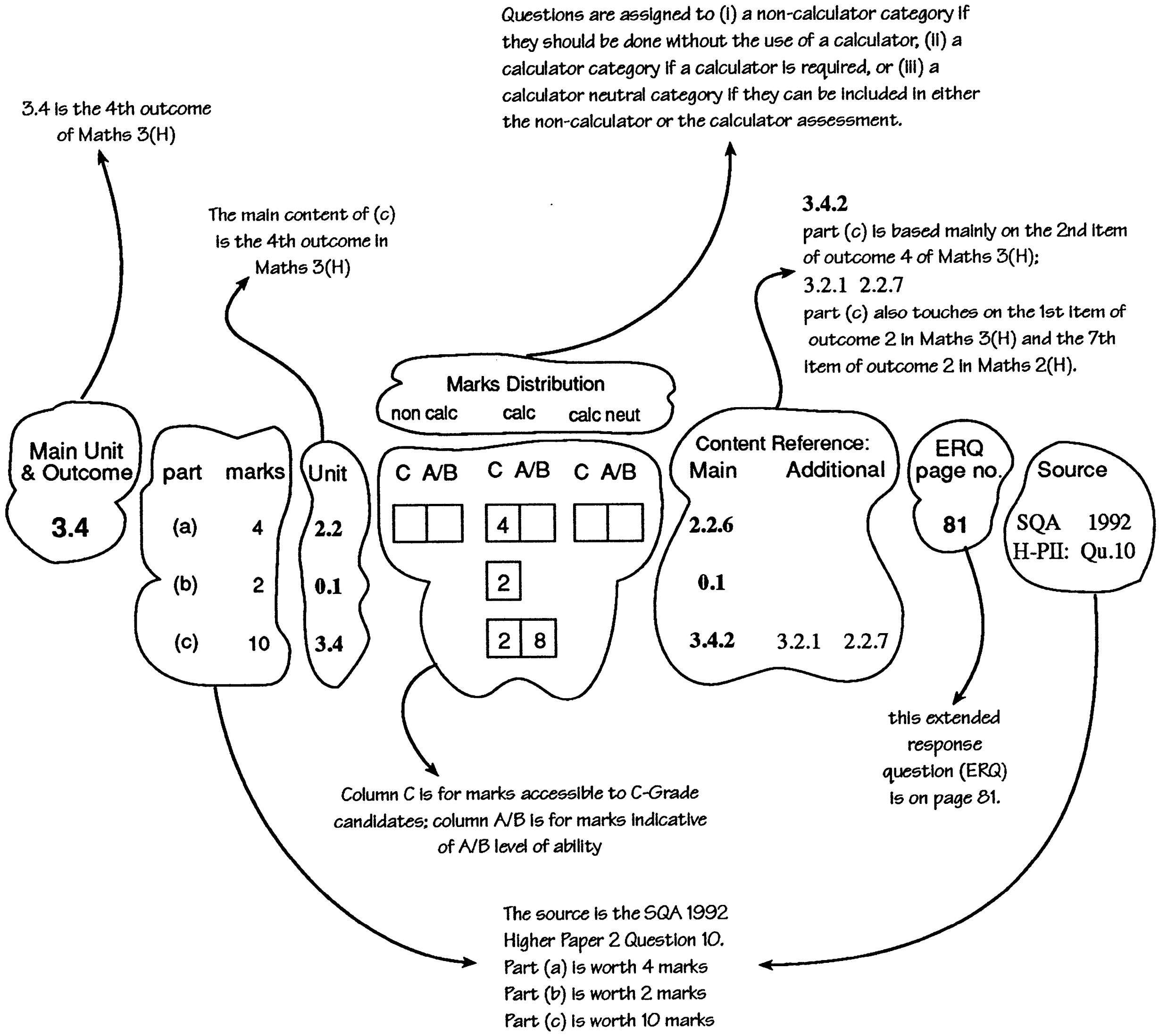
1.6 Extended response material

Section 4 of the Bank contains an analysis of the extended response questions in grid form. Headings and abbreviations are explained overleaf. Section 5 lists each of the extended response questions with, as a guide to marking, a simplified version of the actual marking instructions used in the examinations. Only one method of marking is illustrated and it should be noted that, in many instances, alternative methods are equally valid.

1.7 Important limitations on use of the initial Bank

(i)The questions in this initial Bank were not constructed to satisfy the non-calculator external assessment arrangements which now apply. Consequently some questions which have been allocated a non-calculator status may involve a level of computation higher than would normally be expected in future non-calculator questions. Users of the Bank can take account of this by minor alterations to such a question or by amending the marking scheme.

(ii)Since past examination papers for Higher Mathematics are in the public domain, it is important, for reliability, that internal course assessments are constructed with questions from a wide spread of years. For example, for an internal course assessment modelled on the Specimen Papers, a maximum of 2 extended response questions should be selected from the Paper II for any year and a maximum of 4 short response questions should be selected from the Paper I for any year.


3.4 is the 4th outcome of Maths 3(H)
The main content of (c) is the 4th outcome in Maths 3(H)
Questions are assigned to (i) a non-calculator category if they should be done without the use of a calculator, (ii) a calculator category if a calculator is required, or (iii) a calculator neutral category if they can be included in either the non-calculator or the calculator assessment.
3.4.2
part (c) is based mainly on the 2nd item of outcome 4 of Maths 3(H):
3.2.1 2.2.7
part (c) also touches on the 1st item of outcome 2 in Maths 3(H) and the 7th item of outcome 2 in Maths 2(H).
Marks Distribution
non calc
calc
calc neut
Main Unit & Outcome
3.4
part
marks
(a) 4
(b) 2
(c) 10
Unit
2.2
0.1
3.4
C A/B
C A/B
C A/B
4
2
2 8
Content Reference:
Main
Additional
2.2.6
0.1
3.4.2 3.2.1 2.2.7
ERQ page no.
81
Source
SQA 1992
H-PII: Qu.10
Column C is for marks accessible to C-Grade candidates; column A/B is for marks indicative of A/B level of ability
this extended response question (ERQ) is on page 81.
The source is the SQA 1992 Higher Paper 2 Question 10.
Part (a) is worth 4 marks
Part (b) is worth 2 marks
Part (c) is worth 10 marks

Higher Level Mathematics : Content Reference List

For a full statement of content and comments see SQA Mathematics Higher : National Course Specification

Maths 1 Outcome 1

1.1 Properties of the Straight Line

1.1.1 know the gradient formula $\frac{y_2-y_1}{x_2-x_1}$, $x_2 \neq x_1$

1.1.2 know distance formula $\sqrt{\left[(x_2-x_1)^2+(y_2-y_1)^2\right]}$

1.1.3 know gradient of st. line = tan(angle between line & positive direction of x-axis)

1.1.4 recognise the term locus

1.1.5 know equ. of line of the form $ax + by + c = 0$

1.1.6 know equ. of line of the form $y - b = m(x - a)$

1.1.7 determine equ. from 2 points or 1 point and gradient

1.1.8 know that the gradients of parallel lines are equal

1.1.9 know that lines with gradients m_1 and m_2 are perpendicular $\Leftrightarrow m_1 m_2 = -1$

1.1.10 solve problems using above properties of st. lines

1.1.11 know concurrency properties of medians, altitudes, angle bisectors and perpendicular bisectors

Maths 1 Outcome 2

1.2 Functions & Graphs

1.2.1 know domain.., range.., inverse.., composite function

1.2.2 know meaning of the terms amplitude and period

1.2.3 know general features of graphs of

$f{:}x \to \sin(ax+b),\ x \to \cos(ax+b)$

1.2.4 given graph of $f(x)$ draw graphs of related functions, $f(x)$ being simple polynomial or trig. function

$y = 3f(x)+2,\ f(3x+2),\ -3f(x),\ 3f\left(x+\frac{\pi}{2}\right)$

1.2.5 know general features of the graphs of

$f{:}x \to a^x\ (a>1 \text{ and } 0<a<1), x \in \mathbf{R}$

$f{:}x \to \log_a x\ (a>1,\ x>0)$

1.2.6 find $f(g(x))$ given $f(x)$ and $g(x)$

1.2.7 recognise probable form of function from its graph

1.2.8 complete the square for x^2+px+q

complete the square for ax^2+bx+c e.g. $2x^2-x-1$

1.2.9 interpret formulae and equations

$y=(1-\sin x)^2+2$ has min. value of 2 when $x=\frac{\pi}{2}$

$y=-2-3(2x-1)^2$ has max. of -2 when $x=\frac{1}{2}$

1.2.10 know that π radians = 180°

1.2.11 know exact values of sin/cos $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ radians and tan $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ radians.

Maths 1 Outcome 3

1.3 Basic Differentiation

1.3.1 know limit, diff. at a point, diff., derviative, diff. over an interval, derived function

1.3.2 use notation $f'(x)$ and $\frac{dy}{dx}$ for a derivative

1.3.3 know that $f'(x) = \lim\limits_{x \to 0} \frac{f(x+h)-f(x)}{h}$

1.3.4 know that

if $f(x) = x^n$, then $f'(x) = nx^{n-1}$, $n \in \mathbf{Q}$

if $f(x) = g(x)+h(x)$, then $f'(x) = g'(x)+h'(x)$

if $f(x) = kg(x)$, then $f'(x) = kg'(x)$ where k is a const.

1.3.5 know meaning of rate of change, av. gradient, strictly increasing/decreasing, stationary pt(value), max/min turning pt (value), horizontal pt of inflexion

1.3.6 know that $f'(a)$ is rate of change of f at a

1.3.7 know that $f'(a)$ is gradient of tgt at $x = a$

1.3.8 know that gradient of curve is gradient of tgt

1.3.9 find gradient of tgt to the curve $y = f(x)$ at $x = a$

1.3.10 find points on a curve where grad. has particular value

1.3.11 know and apply the fact that

if $f'(x) > 0$ ...... function is strictly increasing ......

if $f'(x) < 0$ ...... function is strictly decreasing ......

if $f'(a) = 0$ then the function has a stationary val. at $x = a$

1.3.12 find st points(values) on a curve; determine nature

1.3.13 sketch curve by finding stationary pt(s), nature, intersection with axes, behaviour of y for large +v/−ve x

1.3.14 determine greatest/least values of a f on a given int.

1.3.15 solve optimisation problems **using calculus**

Maths 1 Outcome 4

1.4 Recurrence Relations

1.4.1 know s sequence, nth term, limit as n tends to ∞

1.4.2 use u_n notation for the nth term of a sequence

1.4.3 define/interpret r.r. of form $u_{n+1} = mu_n + c$

1.4.4 know condition for limit of sequence from r.r. to exist

1.4.5 find(where possible)/interpret limit of sequence resulting from a recurrence relation in a math. model

For a full statement of content and comments see SQA Mathematics Higher : National Course Specification

Maths 2 Outcome 1

2.1 Factor/Rem Th., Quadratic Theory

2.1.1 use Rem. Th. to find remainder when ÷ by by $x-h$

2.1.2 determine roots of a polynomial equation

2.1.3 use Factor Th. to determine factors of polynomial

$f(x)=(2x-1)(3x+2)(2x-5)$

2.1.4 know roots of $ax^2+bx+c=0$, , are $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

2.1.5 know that discriminant of $ax^2+bx+c=0$ is b^2-4ac

2.1.6 use disc. to determine nature of the roots of a quad.

2.1.7 use disc. to find condition that roots of a quadratic are real, & equal or unequal

If $\frac{(x-2)^2}{x^2+2}=k,\ k\in R$, find values of k such that equation has two equal roots.

2.1.8 know condition for tangency; intersection of line and parabola (*lines and curves)*

2.1.9 solve quadratic inequalities $ax^2+bx+c\geq 0$ (or ≤ 0)

Find real values of x for $x^2+x-2\geq 0$

2.1.10 determine a quadratic equation with given roots

2.1.11 prove that an equation has root between two given values and and improve on that root

Maths 2 Outcome 2

2.2 Basic Integration

2.2.1 know meaning of terms integral, integrate, constant of integration, definite integral, limits of integration, indefinite integral, area under curve

2.2.2 know that if $f(x)=F'(x)$ then $\int_a^b f(x)dx=F(b)-F(a)$

2.2.3 know that if $f(x)=F'(x)$ then $\int f(x)dx=F(x)+C$

2.2.4 integrate $f(x)=px^n$ for all rational n, except $n=-1$ and sum or difference of such functions

2.2.5 evaluate definite integrals

2.2.6 determine area between curve $y=f(x)$, x-axis and lines $x=a$ and $x=b$

2.2.7 determine area bounded by two curves

2.2.8 solve equations of form $\frac{dy}{dx}=f(x)$ for suitable $f(x)$.

Maths 2 Outcome 3

2.3 Trigonometric formulae

2.3.1 solve trig equations in a given interval

$\cos^2 2x=1,\quad 0\leq x<2\pi$

$3\sin^2 x+7\sin x-6=0,\quad 0\leq x<2\pi$

2.3.2 know and apply the addition formulae

- $\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$
- $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

2.3.3 know and apply the double angle formulae

- $\sin 2A=2\sin A\cos A$
- $\cos 2A=\cos^2 A-\sin^2 A$
- $\cos 2A=2\cos^2 A-1=1-2\sin^2 A$

$\sin\theta=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

2.3.4 apply trigonometric formulae in solution of geometric problems

2.3.5 solve trig. equations using formulae from 2.3.2 and 2.3.3

Maths 2 Outcome 4

2.4 Equation of the Circle

2.4.1 know that equation of the circle centre (a, b) and radius r is $(x-a)^2+(y-b)^2=r^2$

2.4.2 know that $x^2+y^2+2gx+2fy+c=0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{(g^2+f^2-c)}$ provided $g^2+f^2-c>0$

2.4.3 determine the equation of circle

2.4.4 solve problems with intersection of a line and a circle, and a tangent to a circle

The line $x-3y=k$ is a tangent to $x^2+y^2-6x+8y+15=0$. Find two possible values of k.

2.4.5 determine whether two circles touch each other

For a full statement of content and comments see SQA Mathematics Higher : National Course Specification

Maths 3 Outcome 1

3.1 Vectors

3.1.1 know vector, magnitude, direction, scalar, position v., unit v., directed line seg., component, scalar product

3.1.2 know addition properties and mult. of vector by scalar

3.1.3 determine the distance between two points in 3-D

3.1.4 know and apply : $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} d \\ e \\ f \end{pmatrix} \Leftrightarrow a=d, b=e, \ c=f$

3.1.5 know and apply: for parallel vectors $\underline{v} = k\underline{u}$

3.1.6 know and apply the fact that if A, P, B are collinear such that $\frac{AP}{PB} = \frac{m}{n}$ then $\vec{AP} = \frac{m}{n}\vec{PB}$.

3.1.7 determine whether 3 points are collinear

3.1.8 know and apply the basis vectors ***i,j,k***

3.1.9 know the scalar product facts $a.b = |a| \ |b| \ \cos\theta$

$a.b = a_1b_1 + a_2b_2 + a_3b_3$

$a.(b+c) = a.b + a.c$

3.1.10 determine whether two vectors are perpendicular

3.1.11 use scalar product to find angle between two d.l.s.

Maths 3 Outcome 2

3.2 Further Differentiation and Integration

3.2.1 know and apply the facts that

$f(x) = \sin x \Rightarrow f'(x) = \cos x, \ \int \cos x \, dx = \sin x + C$

$f(x) = \cos x \Rightarrow f'(x) = -\sin x, \ \int \sin x \, dx = -\cos x + C$

3.2.2 know and apply the fact that

$f(x) = g(h(x)) \Rightarrow f'(x) = g'(h(x)).h'(x)$

differentiate $(2x+5)^3$

differentiate $\sin 3x, \ \cos^3 x$

3.2.3 integrate functions defined by $f(x) = (px+q)^n$ for all rational n except $n = -1$ and the sum or difference of such functions **integrate $(3x+1)^3$**

3.2.4 integrate functions defined by $f(x) = p\cos(qx+r)$ and $f(x) = p\sin(qx+r)$ and the sum or difference of such functions where p, q and r are constants

integrate $\sin 2x$

Maths 3 Outcome 3

3.3 Logarithmic and Exponential Functions

3.3.1 know that $a^y = x \Leftrightarrow \log_a x = y \ (a > 1, x > 0)$

3.3.2 know laws of logarithms:

$\log_a 1 = 0, \ \log_a a = 1$

$\log_a bc = \log_a b + \log_a c$

$\log_a \frac{b}{c} = \log_a b - \log_a c$

$\log_a b^n = n\log_a b$

3.3.3 simplify numerical expressions using laws of logs

3.3.4 solve simple logarithmic and exponential equations

e.g. $3.4^x = 5$

3.3.5 solve for *a* and *b* equations of the following forms, given two pairs of corresponding values of *x* and *y*:

$\log y = a\log x + b$

$y = ax^b$

$y = ab^x$

3.3.6 use a straight line graph to confirm relationships of the form

$y = ax^b$, also $y = ab^x$

3.3.7 model mathematically situations involving the logarithmic or exponential function

from experimental data draw graph of logy against logx and deduce values of a and b such that $y = ax^b$

Maths 3 Outcome 4

3.4 Further Trigonometry

3.4.1 express $a\cos\theta + b\sin\theta$ in the form $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$

3.4.2 solve, by expressing in one of the forms in 3.4.1, equations of form $a\cos\theta + b\sin\theta = c$

3.4.3 find maximum and minimum values of expressions of the form $a\cos\theta + b\sin\theta$

and find corresponding values of θ.

Main Unit & Outcome	part	marks	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
1.1		3					3		1.1.1	1.1.9 1.1.7	SQA 1997 H-PI Qu.1
1.1		4					4		1.1.1	1.1.9	SQA 1996 H-PI Qu.1
1.1		4					4		1.1.1	1.1.9 2.4.2	SQA 1996 H-PI Qu.4
1.1		2	2						1.1.1	1	SQA 1991 H-PI Qu.2
1.1	(a)	4			4				1.1.3		SQA 1993
	(b)	1			1				0.1		H-PI Qu.10
1.1		3			3				1.1.3		SQA 1992 H-PI Qu.13
1.1		3					3		1.1.6	1.1.7	SQA 1998 H-PI Qu.1
1.1		3					3		1.1.7	1.1.1	SQA 1995 H-PI Qu.5
1.1	(a)	3					3		1.1.7	1.1.9	SQA 1993
	(b)	1					1		1.2.9		H-PI Qu.2
1.1	(a)	6	6						1.1.7	1.1.9 1.1.1	SQA 1992
	(b)	2	2						1.1.10		H-PI Qu.2
1.1		2					2		1.1.7	1.1.8	SQA 1991 H-PI Qu.1
1.1		3					3		1.1.7	1.1.9	SQA 1989 H-PI Qu.1
1.1		5	5						1.1.8	1.3.7 1.3.1	SQA 1997 H-PI Qu.6
1.1		4					4		1.1.9	1.1.7	SQA 1990 H-PI Qu.3
1.1		5					5		1.1.10		SQA 1995 H-PI Qu.6
1.1		5			1	4			1.1.10	1.1.3	SQA 1990 H-PI Qu.20
1.2	(a)	1					1		1.2.1		SQA 1991
	(b)	2					2		1.2.1		H-PI Qu.14
1.2	(a)	3	3						1.2.3		SQA 1994
	(b)	3	3						2.3.1	1.2.11	H-PI Qu.12

Main Unit & Outcome	part	marks	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Additional	Source
1.2		2						2	1.2.3		SQA 1992
											H-PI Qu.12
1.2	(a)	3					3		1.2.4		SQA 1998
	(b)	2						2	1.2.4		H-PI Qu.13
	(c)	1						1	1.3.8		
1.2	(a)	2	2						1.2.4		SQA 1997
	(b)	2	2						1.2.4		H-PI Qu.16
	(c)	1		1					0.1		
1.2	(i)	2					2		1.2.4		SQA 1996
	(ii)	3						3	1.2.4		H-PI Qu.8
1.2	(a)	3					2	1	1.2.4		SQA 1995
	(b)	1						1	1.2.4		H-PI Qu.17
1.2		3	1	2					1.2.4		SQA 1993
											H-PI Qu.8
1.2		3	3						1.2.4	1.2.1	SQA 1993
											H-PI Qu.14
1.2	(a)	2					2		1.2.4		SQA 1992
	(b)	2					2		1.2.4		H-PI Qu.10
1.2	(a)	2	2						1.2.4		SQA 1991
	(b)	3	3						1.2.4		H-PI Qu.9
1.2		3	2	1					1.2.4		SQA 1990
											H-PI Qu.11
1.2		3		3					1.2.4	1.2.5	SQA 1990
											H-PI Qu.17
1.2		3		3					1.2.5		SQA 1989
											H-PI Qu.17
1.2	(a)	2					2		1.2.6		SQA 1998
	(b)	2					1	1	0.1		H-PI Qu.6
1.2		4	4						1.2.6		SQA 1997
											H-PI Qu.3
1.2	(a)	3	2	1					1.2.6		SQA 1995
	(b)	1		1					1.2.1		H-PI Qu.11
1.2		3	1	2					1.2.6		SQA 1994
											H-PI Qu.19
1.2	(a)	2	2						1.2.6		SQA 1993
	(b)	2	2						1.2.6		H-PI Qu.13
	(c)	1		1					0.1		

Main Unit & Outcome	part	marks	non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
1.2		3	3						1.2.6		SQA 1992 H-PI Qu.6
1.2	(a)	2	2						1.2.6		SQA 1991
	(b)	2		2					1.2.6		H-PI Qu.19
1.2		4					2	2	1.2.6	1.2.1	SQA 1989 H-PI Qu.19
1.2	(a)	3	3						1.2.8		SQA 1997
	(b)	1	1						1.2.9		H-PI Qu.9
1.2		3	3						1.2.8		SQA 1996 H-PI Qu.17
1.2		3	3						1.2.8	1.2.9	SQA 1994 H-PI Qu.11
1.2	(a)	2		2					1.2.8		SQA 1991
	(b)	2		2					1.2.9		H-PI Qu.15
1.2		4	4						1.2.8	1.2.9	SQA 1989 H-PI Qu.8
1.2		3	3						1.2.9		SQA 1996 H-PI Qu.3
1.3		4	4						1.3.4	1.3.7	SQA 1998 H-PI Qu.11
1.3		4	4						1.3.4		SQA 1998 H-PI Qu.14
1.3		3	3						1.3.4		SQA 1997 H-PI Qu.8
1.3		5	5						1.3.4		SQA 1996 H-PI Qu.9
1.3		4	4						1.3.4		SQA 1995 H-PI Qu.7
1.3		3	3						1.3.4	0.1	SQA 1994 H-PI Qu.2
1.3		3	1	2					1.3.4	0.1	SQA 1989 H-PI Qu.12
1.3	(a)	3					1	2	1.3.5	1.3.6	SQA 1998
	(b)	2						2	1.3.5	1.3.6	H-PI Qu.17
1.3	(a)	3	1	2					1.3.6		SQA 1995
	(b)	2		2					1.2.9		H-PI Qu.21

Main Unit & Outcome	part	marks	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
1.3		4			1	3			1.3.7	1.3.9 1.1.3	SQA 1997 H-PI Qu.20
1.3		2	2						1.3.7		SQA 1995 H-PI Qu.14
1.3		4					1	3	1.3.7	1.2.4	SQA 1992 H-PI Qu.19
1.3		6	6						1.3.9	1.1.3	SQA 1994 H-PI Qu.14
1.3		4	4						1.3.9	1.1.7	SQA 1992 H-PI Qu.1
1.3		4	4						1.3.9	1.1.7	SQA 1991 H-PI Qu.5
1.3		4	4						1.3.9	1.1.7	SQA 1990 H-PI Qu.2
1.3		4	4						1.3.9	1.1.3	SQA 1989 H-PI Qu.13
1.3	(a)	2					2		1.3.9		SQA 1989 H-PI Qu.14
	(b)	2					2		1.2.4		
1.3		4	4						1.3.10		SQA 1993 H-PI Qu.4
1.3		4	2	2					1.3.11	2.1.9	SQA 1996 H-PI Qu.16
1.3	(b)	3	3						1.3.11		SQA 1995 H-PI Qu.10
	(a)	1	1						0.1		
1.3		4	1	3					1.3.11		SQA 1993 H-PI Qu.21
1.3		5	2	3					1.3.11		SQA 1990 H-PI Qu.16
1.4	(a)	1			1				1.4.2		SQA 1994 H-PI Qu.9
	(b)	1			1				1.4.3		
	(c)	2			2				1.4.5		
1.4		4			4				1.4.3		SQA 1991 H-PI Qu.11
1.4	(a)	1			1				1.4.4		SQA 1998 H-PI Qu.8
	(b)	2			2				1.4.5		
	(c)	2				2			1.4.3		
1.4	(a)	1					1		1.4.4		SQA 1996 H-PI Qu.11
	(b)	2					2		1.4.5		

Main Unit & Outcome	part	marks	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
2.1		3					3		2.1.1		SQA 1992 H-PI Qu.3
2.1		5					5		2.1.1		SQA 1991 H-PI Qu.6
2.1		3					3		2.1.1		SQA 1990 H-PI Qu.1
2.1	(a)	2	2						2.1.2		SQA 1997 H-PI Qu.5
	(b)	3	3						2.1.7		
2.1		4	4						2.1.2		SQA 1993 H-PI Qu.7
2.1		4	4						2.1.3		SQA 1998 H-PI Qu.2
2.1		4	4						2.1.3		SQA 1996 H-PI Qu.7
2.1	(a)	2	2						2.1.3		SQA 1995 H-PI Qu.2
	(b)	2	2						2.1.3		
2.1		4	4						2.1.3		SQA 1989 H-PI Qu.2
2.1		5	1	4					2.1.6		SQA 1991 H-PI Qu.18
2.1		4					1	3	2.1.6		SQA 1990 H-PI Qu.18
2.1		2	2						2.1.7		SQA 1996 H-PI Qu.2
2.1		5					1	4	2.1.7		SQA 1995 H-PI Qu.20
2.1		2	2						2.1.7		SQA 1993 H-PI Qu.3
2.1		4	1	3					2.1.7		SQA 1992 H-PI Qu.17
2.2		5	5						2.2.4		SQA 1998 H-PI Qu.12
2.2		4	4						2.2.4	2.2.5	SQA 1996 H-PI Qu.5
2.2		3	3						2.2.4		SQA 1994 H-PI Qu.1
2.2		4	4						2.2.4	1.2.9	SQA 1993 H-PI Qu.11

Main Unit & Outcome	part	marks	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
2.2		3	3						2.2.4		SQA **1989** H-PI **Qu.5**
2.2		4	4						2.2.5		SQA **1997** H-PI **Qu.10**
2.2		5	5						2.2.5		SQA **1992** H-PI **Qu.8**
2.2	(a)	3	3						2.2.5		SQA **1991** H-PI **Qu.16**
	(b)	2	2						2.2.6		
2.2		5	5						2.2.5	2.2.6	SQA **1990** H-PI **Qu.6**
2.2		5	4	1					2.2.5		SQA **1989** H-PI **Qu.16**
2.2	(b)	4	1	3					2.2.6	3.2.1	SQA **1998** H-PI **Qu.15**
	(a)	1	1						1.2.3		
2.2	(b)	3	3						2.2.7		SQA **1990** H-PI **Qu.13**
	(a)	2	2						0.1		
2.2		3	3						2.2.8		SQA **1998** H-PI **Qu.10**
2.2		4	4						2.2.8		SQA **1992** H-PI **Qu.4**
2.2		5	5						2.2.8		SQA **1991** H-PI **Qu.10**
2.2		3	3						2.2.8		SQA **1990** H-PI **Qu.8**
2.3		4	4						2.3.1	1.2.1	SQA **1998** H-PI **Qu.9**
2.3		4	4						2.3.1	1.2.1	SQA **1996** H-PI **Qu.12**
2.3		4	4						2.3.1	1.2.1	SQA **1995** H-PI **Qu.8**
2.3		4			2	2			2.3.1		SQA **1993** H-PI **Qu.17**
2.3		5			5				2.3.1		SQA **1992** H-PI **Qu.5**
2.3	(b)	3			3				2.3.1		SQA **1990** H-PI **Qu.10**
	(a)	2			2				1.2.2	1.2.7	

Main Unit & Outcome	part	marks	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
2.3		3	3						2.3.1	1.2.1	SQA **1990** H-PI **Qu.15**
2.3		4	4						2.3.1	1.2.1	SQA **1989** H-PI **Qu.7**
2.3		4	4						2.3.1		SQA **1989** H-PI **Qu.15**
2.3		3	3						2.3.2		SQA **1997** H-PI **Qu.7**
2.3		3					3		2.3.2		SQA **1996** H-PI **Qu.15**
2.3		3	3						2.3.2		SQA **1994** H-PI **Qu.6**
2.3		3	3						2.3.2		SQA **1993** H-PI **Qu.6**
2.3		4		4					2.3.2		SQA **1989** H-PI **Qu.11**
2.3	(a)	2	2						2.3.3		SQA **1998** H-PI **Qu.7**
	(b)	1	1						2.3.3		
	(c)	2	2						2.3.2		
2.3	(a)	2					1	2	2.3.3	2.1.6	SQA **1998** H-PI **Qu.18**
	(b)	1						1	1.2.1		
2.3	(a)	2			1	1			2.3.3		SQA **1997** H-PI **Qu.18**
	(b)	4			1	3			2.3.5		
2.3		3	3						2.3.3		SQA **1995** H-PI **Qu.12**
2.3	(a)	2	2						2.3.3		SQA **1994** H-PI **Qu.13**
	(b)	3		3					2.3.3		
2.3		3	3						2.3.3		SQA **1991** H-PI **Qu.12**
2.3		4	4						2.3.3		SQA **1990** H-PI **Qu.9**
2.3		5	1	4					2.3.4		SQA **1996** H-PI **Qu.18**
2.3		5	5						2.3.5		SQA **1996** H-PI **Qu.10**
2.3		5				5			2.3.5		SQA **1995** H-PI **Qu.15**

Main Unit & Outcome	part	marks	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
2.3		5			1	4			2.3.5		SQA **1994** H-PI **Qu.15**
2.3		4	1	3					2.3.5		SQA **1991** H-PI **Qu.20**
2.4		4					4		2.4.1	1.1.9	SQA **1994** H-PI **Qu.5**
2.4	(a)	3					3		2.4.1		SQA **1993** H-PI **Qu.5**
	(b)	1					1		2.4.3		
2.4		5					5		2.4.1	1.1.2 0.1	SQA **1992** H-PI **Qu.16**
2.4		5					5		2.4.2	3.1.6	SQA **1997** H-PI **Qu.12**
2.4		3					2	1	2.4.2		SQA **1997** H-PI **Qu.14**
2.4		2					2		2.4.2		SQA **1993** H-PI **Qu.18**
2.4		5					5		2.4.2	1.1.2 0.1	SQA **1992** H-PI **Qu.9**
2.4		5					5		2.4.2	2.4.3	SQA **1990** H-PI **Qu.7**
2.4		6					1	5	2.4.3	0.1	SQA **1996** H-PI **Qu.20**
2.4		3					3		2.4.3		SQA **1995** H-PI **Qu.9**
2.4		4					4		2.4.4		SQA **1998** H-PI **Qu.4**
2.4	(a)	3					3		2.4.4		SQA **1994** H-PI **Qu.8**
	(b)	3					3		2.4.3		
2.4		5					5		2.4.4		SQA **1991** H-PI **Qu.8**
2.4		5					2	3	2.4.4		SQA **1989** H-PI **Qu.18**
3.1	(a)	2					2		3.1.1		SQA **1998** H-PI **Qu.3**
	(b)	1					1		3.1.9		
	(c)	1					1		3.1.1		
3.1	(a)	2					2		3.1.1		SQA **1998** H-PI **Qu.5**
	(b)	2					2		3.1.1		

Main Unit & Outcome	part	marks	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
3.1	(a)	1					1		3.1.1		SQA 1993 H-PI Qu.1
	(b)	2					2		3.1.3		
3.1	(a)	1					1		3.1.1		SQA 1990 H-PI Qu.5
	(b)	1					1		3.1.3		
3.1		3					3		3.1.2	3.1.1	SQA 1994 H-PI Qu.7
3.1		3						3	3.1.3		SQA 1994 H-PI Qu.18
3.1		3					3		3.1.4	3.1.1	SQA 1994 H-PI Qu.3
3.1		3					3		3.1.4		SQA 1989 H-PI Qu.4
3.1		3					3		3.1.6		SQA 1992 H-PI Qu.15
3.1		3					3		3.1.7		SQA 1997 H-PI Qu.2
3.1		4					4		3.1.7	3.1.6	SQA 1996 H-PI Qu.6
3.1		3					3		3.1.7		SQA 1994 H-PI Qu.4
3.1	(a)	4					4		3.1.7		SQA 1991 H-PI Qu.7
	(b)	1					1		3.1.6		
3.1		4					4		3.1.7	3.1.6	SQA 1990 H-PI Qu.4
3.1	(a)	2					2		3.1.8	3.1.1	SQA 1997 H-PI Qu.4
	(b)	1					1		3.1.3		
3.1		2					2		3.1.8		SQA 1995 H-PI Qu.1
3.1	(a)	3					3		3.1.8	3.1.9	SQA 1993 H-PI Qu.12
	(b)	2						2	3.1.10		
3.1		3					3		3.1.8	3.1.1	SQA 1991 H-PI Qu.3
3.1		3					3		3.1.8		SQA 1990 H-PI Qu.12
3.1	(a)	1					1		3.1.8		SQA 1989 H-PI Qu.3
	(b)	2					2		3.1.1		

Main Unit & Outcome	part	marks	non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
3.1		4					1	3	3.1.9	3.1.1	SQA 1997 H-PI Qu.13
3.1		4					1	3	3.1.9		SQA 1995 H-PI Qu.16
3.1		3					3		3.1.9		SQA 1992 H-PI Qu.18
3.1		5					5		3.1.9	3.1.1	SQA 1991 H-PI Qu.17
3.1		5	1	4					3.1.9		SQA 1989 H-PI Qu.9
3.1		3					3		3.1.10		SQA 1995 H-PI Qu.4
3.1		3					3		3.1.10		SQA 1989 H-PI Qu.6
3.2		3		3					3.2.1	3.2.2	SQA 1998 H-PI Qu.16
3.2		3		3					3.2.2	3.2.1	SQA 1996 H-PI Qu.13
3.2		4		4					3.2.2		SQA 1994 H-PI Qu.10
3.2		4	1	3					3.2.2	2.2.3	SQA 1994 H-PI Qu.17
3.2		4	2	2					3.2.2	1.3.4	SQA 1993 H-PI Qu.9
3.2		4	1	3					3.2.2		SQA 1992 H-PI Qu.11
3.2		4		4					3.2.2		SQA 1991 H-PI Qu.13
3.2		4	2	2					3.2.2	1.3.4	SQA 1990 H-PI Qu.19
3.2		4	2	2					3.2.2	1.3.1	SQA 1989 H-PI Qu.10
3.2		4		4					3.2.3		SQA 1993 H-PI Qu.16
3.2	(a)	1	1						2.3.3		SQA 1993 H-PI Qu.19
	(b)	3		3					3.2.4		
3.2		3		3					3.2.4		SQA 1997 H-PI Qu.15

Main Unit & Outcome	part	marks	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
3.2		4	4						3.2.4		SQA **1995** H-PI **Qu.3**
3.2	(a)	3		3					3.2.4		SQA **1992**
	(b)	2	1	1					1.2.3	2.2.6	H-PI **Qu.14**
3.3		4	1	3					3.3.3	3.3.1 3.3.4	SQA **1998** H-PI **Qu.19**
3.3		4				4			3.3.4		SQA **1997** H-PI **Qu.17**
3.3	(a)	2					2		3.3.4		SQA **1997**
	(b)	2						2	1.2.4		H-PI **Qu.19**
	(c)	1					1		0.1		
3.3		3						3	3.3.4		SQA **1995** H-PI **Qu.19**
3.3	(a)	3	2	1					3.3.4	1.2.4	SQA **1994**
	(b)	2		2					3.3.2	1.2.4	H-PI **Qu.16**
3.3		4				4			3.3.4		SQA **1994** H-PI **Qu.20**
3.3	(a)	1			1				3.3.4		SQA **1991**
	(b)	2			2				3.3.4		H-PI **Qu.4**
3.3	(a)	1			1				3.3.4		SQA **1989**
	(b)	4			1	3			3.3.7		H-PI **Qu.20**
3.3		3			1	2			3.3.4		SQA **1989** H-PI **Qu.21**
3.3	(a)	4			1	3			3.3.5		SQA **1993**
	(b)	1				1			1.2.7		H-PI **Qu.15**
3.3		4					1	3	3.3.5		SQA **1993** H-PI **Qu.20**
3.3		6					2	4	3.3.6	1.1.7	SQA **1990** H-PI **Qu.14**
3.3		5			1	4			3.3.7		SQA **1996** H-PI **Qu.19**
3.3	(a)	3			1	2			3.3.7		SQA **1995**
	(b)	2				2			3.3.7		H-PI **Qu.18**
3.4		4			4				3.4.1		SQA **1997** H-PI **Qu.11**
3.4		4			4				3.4.1		SQA **1995** H-PI **Qu.13**

Main Unit & Outcome	part	marks	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference Main	Content Reference Additional	Source
3.4	(a)	4			4				3.4.1		SQA **1992**
	(b)	2			1	1			3.4.3		H-PI **Qu.7**
0.1		4	4						0.1		SQA **1996** H-PI **Qu.14**

Main Unit & Outcome	part	marks	Unit	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	E.R.Q. page no.	Source
1.1	(a)	2	1.1					2		1.1.2		38	SQA 1995
	(b)	8	1.1					8		1.1.10	0.1		H-PII Qu.1
1.1	(a)	3	1.1					3		1.1.9	1.1.7	51	SQA 1994
	(b)	3	0.1					3		0.1			H-PII Qu.2
	(c)	2	1.1					2		1.1.2			
1.1	(a)	3	1.1					3		1.1.10		26	SQA 1996
	(b)i	5	1.1					5		1.1.10			H-PII Qu.2
	(b)ii	3	0.1					3		0.1			
1.1	(a)	7	1.1					7		1.1.10	0.1	94	SQA 1990
	(b)	2	0.1					2		0.1			H-PII Qu.2
1.2	(a)	4	1.2		4					1.2.3		102	SQA 1990
	(b)	3	1.2		3					1.2.4			H-PII Qu.10
	(c)	3	1.2		3					1.2.9			
1.2	(a)	4	1.2		4					1.2.4		106	SQA 1989
	(b)	2	1.2		2					1.2.1			H-PII Qu.4
	(c)	3	1.2		3					1.2.7			
1.2	(a)	3	1.2			3				1.2.5		85	SQA 1991
	(b)	3	0.1			1	2			0.1			H-PII Qu.4
1.3	(a)	3	0.1				3			0.1		11	SQA 1998
	(b)	6	1.3			3	3			1.3.15			H-PII Qu.10
1.3	(a)	4	0.1					1	3	0.1		23	SQA 1997
	(b)	6	1.3					3	3	1.3.15			H-PII Qu.10
1.3	(a)	4	0.1					1	3	0.1		37	SQA 1996
	(b)	5	1.3					2	3	1.3.15			H-PII Qu.11
1.3	(a)	4	0.1	4						0.1		57	SQA 1994
	(B)	8	1.3	3	5					1.3.15			H-PII Qu.7
1.3	(a)	4	0.1	2	2					0.1		76	SQA 1992
	(b)	6	1.3	4	2					1.3.15			H-PII Qu.5
1.3	(a)	2	0.1					2		0.1		109	SQA 1989
	(b)	6	1.3					3	3	1.3.15			H-PII Qu.7
1.3		8	1.3	2	6					1.1.3	1.3.7 0.1	45	SQA 1995 H-PII Qu.7
1.3	(a)	5	1.3					5		1.1.7	1.3.9 1.1.6	3	SQA 1998
	(b)	2	1.1					2		1.1.3	1.1.9		H-PII Qu.3
1.3	(a)	4	1.3					4		1.3.7		18	SQA 1997
	(b)	2	1.1					1	1	1.1.7			H-PII Qu.6
	(c)	4	0.1						4	0.1			

Main Unit & Outcome	part	marks	Unit	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional		E.R.Q. page no.	Source
1.3	(a)	3	1.2	3						1.2.9			93	SQA 1990
	(b)	7	1.3	7						1.3.12				H-PII Qu.1
	(c)	2	1.3	2						1.3.13				
1.3	(a)	6	1.3					6		1.3.12			2	SQA 1998
	(b)	2	1.3					2		1.3.12				H-PII Qu.2
1.3	(a)	6	1.3	6						1.3.12			25	SQA 1996
	(b)	2	1.3	2						1.3.12				H-PII Qu.1
1.3	(a)	5	1.3					5		1.3.12			82	SQA 1991
	(b)	2	1.2					2		1.2.1				H-PII Qu.1
1.4	(a)	3	1.4			3				1.4.1			8	SQA 1998
	(b)	1	1.4			1				1.4.3				H-PII Qu.8
	(c)	4	1.4			4				1.4.4	1.4.5			
1.4	(a)	4	1.4			4				1.4.1			15	SQA 1997
	(b)	2	1.4			2				1.4.1				H-PII Qu.3
	(c)	3	1.4			3				1.4.3				
1.4	(a)	3	1.4			3				1.4.1			68	SQA 1993
	(b)	3	1.4			3				1.4.1				H-PII Qu.8
	(c)	1	1.4			1				1.4.3				
	(d)	4	1.4			3	1			1.4.4	1.4.5			
1.4	(a)	4	1.4			4				1.4.1			90	SQA 1991
	(b)	6	1.4			4	2			1.4.3	1.4.5			H-PII Qu.9
1.4	(a)	5	1.4			5				1.4.1	1.4.5		108	SQA 1989
	(b)	1	0.1			1				0.1				H-PII Qu.6
1.4	(a)	3	1.4			3				1.4.2			95	SQA 1990
	(b)	5	1.4			5				1.4.2				H-PII Qu.3
1.4		7	1.4			7				1.4.3	1.4.4	1.4.5	40	SQA 1995
														H-PII Qu.3
1.4	(a)	3	1.4			3				1.4.3			74	SQA 1992
	(b)	5	1.4			5				1.4.3	1.4.5			H-PII Qu.3
2.2	(a)	3	1.2	3						1.2.9			17	SQA 1997
	(b)	5	2.2	5						2.2.7				H-PII Qu.5
	(c)	3	0.1		2					0.1				
2.1	(a)i	1	0.1	1						0.1			5	SQA 1998
	(a)ii	4	1.1	4						1.1.6	0.1			H-PII Qu.5
	(b)	7	2.1	2	5					2.1.12	2.1.2			
2.1	(a)	1	0.1					1		0.1			59	SQA 1994
	(b)	6	1.3					2	4	1.3.7	0.1			H-PII Qu.9
	(c)	2	2.1						2	2.1.6				

Main Unit & Outcome	part	marks	Unit	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	E.R.Q. page no.	Source
2.1	(a)	3	0.1	3						0.1		105	SQA 1989
	(b)	1	0.1	1						0.1			H-PII Qu.3
	(c)	4	2.1	1	3					2.1.11			
2.1	(a)	3	2.1	3						2.1.3		50	SQA 1994
	(b0)	4	2.1	4						2.1.3			H-PII Qu.1
2.1		8	2.1					8		2.1.3	1.3.12	61	SQA 1993
													H-PII Qu.1
2.1	(a)	3	2.1	3						2.1.3		72	SQA 1992
	(b)	2	1.2	2						1.2.9			H-PII Qu.1
	(c)	6	1.3	6						1.3.12			
2.1	(a)	4	1.2	4						1.2.6		28	SQA 1996
	(b)	7	2.1	7						2.1.6	2.1.7 0.1		H-PII Qu.4
2.1	(a)	4	1.2	4						1.2.6		98	SQA 1990
	(b)	3	2.1	3						2.1.3			H-PII Qu.6
	(c)	2	1.2	2						1.2.1			
2.1	(a)	3	1.2					3		1.2.7		30	SQA 1996
	(b)	4	0.1					4		0.1			H-PII Qu.6
	(c)	4	2.1					4		2.1.8	0.1		
2.1	(a)	4	1.2					1	3	1.2.7		91	SQA 1991
	(b)	7	2.1					3	4	2.1.8	1.1.1 1.1.7		H-PII Qu.10
2.1	(a)	7	2.1	7						2.1.8	2.1.3	67	SQA 1993
	(b)	1	2.1		1					2.1.8			H-PII Qu.7
2.1	(a)	4	1.3	4						1.3.9	1.1.7	39	SQA 1995
	(b)	5	2.1	5						2.1.2	2.1.8		H-PII Qu.2
2.2	(a)	2	0.1	2						0.1		62	SQA 1993
	(b)	4	2.2	4						2.2.6			H-PII Qu.2
2.2	(a)	2	2.1	2						2.1.2		16	SQA 1997
	(b)	7	2.2	6	1					2.2.6			H-PII Qu.4
2.2	(a)	5	1.1	3	2					1.1.3	1.1.10	112	SQA 1989
	(b)	4	1.1	1	3					1.1.7	0.1		H-PII Qu.10
	(c)	6	2.2		6					2.2.6			
2.2	(a)	2	1.2	2						1.2.7		4	SQA 1998
	(b)	4	2.2	4						2.2.6			H-PII Qu.4
	(c)i	2	2.1	2						2.1.8			
	(c)ii	3			3					2.2.7			
2.2	(a)	5	2.2	5						2.2.7		48	SQA 1995
	(b)	3	2.2		3					2.2.5			H-PII Qu.10
	(c)	5	2.1	2	3					2.1.3	0.1		

Main Unit & Outcome	part	marks	Unit	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional		E.R.Q. page no.	Source
2.2		9	2.2					3	6	2.2.7	0.1		60	SQA 1994 H-PII Qu.10
2.2	(a)	6	2.2	3	3					2.2.7			99	SQA 1990
	(b)	4	2.2	2	2					2.2.5	0.1			H-PII Qu.7
	(c)	4	0.1		4					0.1				
2.2	(a)	8	2.1					5	3	2.1.8	1.1.7	1.3.9	33	SQA 1996
	(b)	3	2.2						3	2.2.7				H-PII Qu.8
2.2	(a)	5	1.3	5						1.3.9	1.1.7		92	SQA 1991
	(b)	2	0.1	2						0.1				H-PII Qu.11
	(c)	3	2.2	3						2.2.6				
	(d)	3	0.1	1	2					0.1				
2.2	(a)	2	1.2	2						1.2.9			103	SQA 1989
	(b)	6	1.3	6						1.3.12				H-PII Qu.1
	(c)	2	1.3	2						1.3.13				
	(d0	4	2.2	4						2.2.6				
2.3	(a)	2	0.1			1	1			0.1			79	SQA 1992
	(b)	3	2.3				3			2.3.4				H-PII Qu.8
	(c)	3	0.1			3				0.1				
2.3	(a)	3	0.1					2	1	0.1			84	SQA 1991
	(b)	5	2.3					5		2.3.5	0.1			H-PII Qu.33
2.3	(a)	5	2.3	3	2					2.3.1			66	SQA 1993
	(b)	1	0.1	1						0.1				H-PII Qu.6
	(c)	3	0.1		3					0.1				
2.3	(a)	4	1.2			2	2			1.2.3			22	SQA 1997
	(b)	4	2.3				4			2.3.1				H-PII Qu.9
2.3	(a)	4	2.3			4				2.3.5			78	SQA 1992
	(b)	1	1.2			1				1.2.7				H-PII Qu.7
	(c)	2	1.2			2				1.2.9				
	(d)	3	1.2			2	1			1.2.10				
2.4	(a)	6	2.4					3	3	2.4.1	1.1.9	1.3.9	35	SQA 1996
	(b)	4	2.4						4	2.4.4				H-PII Qu.10
2.4	(a)	3	1.1					3		1.1.1	1.1.7		80	SQA 1992
	(b)	1	2.4					1		2.4.3				H-PII Qu.9
	(c)	5	2.4					2	3	2.4.4				
2.4	(a)	8	2.4			8				2.4.2	1.1.2		53	SQA 1994
	(b)	8	1.1			8				1.1.1	1.1.9	2.4.4		H-PII Qu.4
2.4		6	2.4			6				2.4.2	2.4.4		107	SQA 1989 H-PII Qu.5

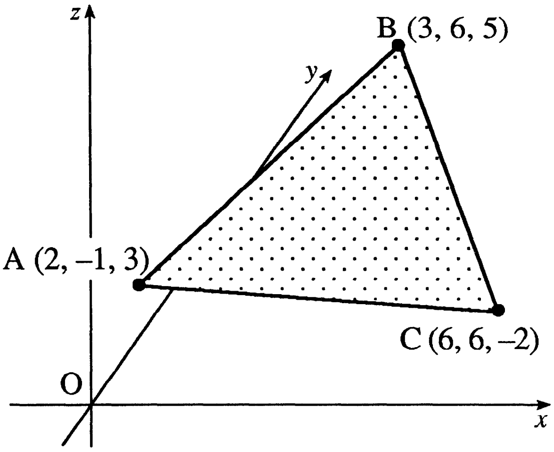
Main Unit & Outcome	part	marks	Unit	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	E.R.Q. page no.	Source
2.4	(a)	3	2.4					3		2.4.3		6	SQA **1998**
	(b)	3	2.4					3		2.4.2	3.1.6		H-PII **Qu.6**
	(c)	3	0.1						3	0.1			
2.4	(a)	4	2.4					4		2.4.3		100	SQA **1990**
	(b)	7	2.4					3	4	2.4.4			H-PII **Qu.8**
2.4	(a)	5	2.4					5		2.4.4		13	SQA **1997**
	(b)i	1	2.4					1		2.4.2			H-PII **Qu.1**
	(b)ii	3	1.1					3		1.1.10	1.1.7		
2.4	(a)	3	1.1					3		1.1.9	1.1.7	63	SQA **1993**
	(b)	5	2.4					5		2.4.3			H-PII **Qu.3**
2.4	(a)	4	1.1					4		1.1.9	1.1.7	83	SQA **1991**
	(b)	6	2.4					6		2.4.3	1.1.2		H-PII **Qu.2**
3.1	(a)	3	0.1					3		0.1		77	SQA **1992**
	(b)	4	3.1					4		3.1.9	3.1.10		H-PII **Qu.6**
3.1	(a)	1	0.1					1		0.1		104	SQA **1989**
	(b)	3	3.1					3		3.1.6			H-PII **Qu.2**
	(c)	4	3.1					4		3.1.7	3.1.6		
3.1	(a)	2	3.1			2				3.1.1		1	SQA **1998**
	(b)	5	3.1			5				3.1.11			H-PII **Qu.1**
	(c)	2	0.1			2				0.1			
3.1	(a)	2	3.1			2				3.1.1		27	SQA **1996**
	(b)	7	3.1			7				3.1.11			H-PII **Qu.3**
3.1		8	3.1					8		2.4.1	2.4.3 3.1.6	46	SQA **1995**
													H-PII **Qu.8**
3.1	(a)	3	3.1			3				3.1.1		52	SQA **1994**
	(b)	1	3.1			1				3.1.6			H-PII **Qu.3**
	(c)	4	3.1			4				3.1.11			
	(d)	2	0.1			2				0.1			
3.1	(a)	3	3.1			3				3.1.3		73	SQA **1992**
	(b)	2	3.1			2				3.1.3			H-PII **Qu.2**
	(c)	3	3.1			3				3.1.10			
	(d)	5	3.1			5				3.1.11			
3.1	(a)	3	3.1			3				3.1.6		14	SQA **1997**
	(b)	7	3.1			7				3.1.11			H-PII **Qu.2**
3.1	(a)	3	3.1			3				3.1.6		65	SQA **1993**
	(b)	4	3.1			4				3.1.3	0.1		H-PII **Qu.5**
	(c)	5	0.1			5				0.1			

Main Unit & Outcome	part	marks	Unit	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	E.R.Q. page no.	Source
3.1	(a)	3	3.1			3				3.1.6		96	SQA **1990**
	(b)	5	3.1			5				3.1.11			H-PII **Qu.4**
3.1	(a)	4	3.1					4		3.1.11		42	SQA **1995**
	(b)	6	3.1					6		3.1.6	3.1.3		H-PII **Qu.5**
	(c)	2	3.1					1	1	3.1.3	0.1		
3.1	(a)	7	3.1			7				3.1.11	3.3.1	86	SQA **1991**
	(b)	3	0.1			3				0.1			H-PII **Qu.5**
3.2	(a)	1	0.1						1	0.1		49	SQA **1995**
	(b)	5	3.2					1	4	3.2.2			H-PII **Qu.11**
	(c)	3	1.3					1	2	1.3.15			
3.2	(a)	3	0.1	1	2					0.1		71	SQA **1993**
	(b)	7	1.3	1	6					1.3.15	3.2.2		H-PII **Qu.11**
3.2	(a)	4	2.3	2	2					2.3.2	2.3.3	47	SQA **1995**
	(b)	4	3.2		4					3.2.4			H-PII **Qu.9**
3.2		6	3.2	2	4					3.2.4	2.2.6	29	SQA **1996**
													H-PII **Qu.5**
3.3	(a)	3	1.1			3				1.1.1	1.1.7	70	SQA **1993**
	(b)	4	3.3				4			3.3.6			H-PII **Qu.10**
3.3	(a)	2	3.3			2				3.3.4		75	SQA **1992**
	(b)	3	3.3			1	2			3.3.4			H-PII **Qu.4**
	(c)	3	1.2			1	2			1.2.5			
3.3	(a)	5	3.3			2	3			3.3.4		88	SQA **1991**
	(b)	4	3.3			1	3			3.3.4			H-PII **Qu.7**
3.3	(a)	3	3.3				3			3.3.7		12	SQA **1998**
	(b)	6	3.3				6			3.3.5			H-PII **Qu.11**
3.3	(a)	3	3.3			1	2			3.3.7		21	SQA **1997**
	(b)	3	3.3				3			3.3.7			H-PII **Qu.8**
3.3	(a)	3	1.1			3				1.1.7		34	SQA **1996**
	(b)	4	3.3				4			3.3.6			H-PII **Qu.9**
3.4	(a)	3	0.1	2	1					0.1		24	SQA **1997**
	(b)	6	3.4	4	2					3.4.1	3.4.3		H-PII **Qu.11**
	(c)	4	0.1	1	3					0.1			
3.4	(a)	5	0.1			1	4			0.1	2.3.3	69	SQA **1993**
	(b)	4	3.4			4				3.4.1			H-PII **Qu.9**
	(c)	4	3.4			1	3			3.4.2			

Main Unit & Outcome	part	marks	Unit	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	E.R.Q. page no.	Source
3.4	(a)	4	3.4			4				3.4.1		7	SQA **1998**
	(b)	4	3.4			1	3			3.4.3			H-PII **Qu.7**
	(c)	1	0.1				1			0.1			
3.4	(a)	4	3.4			4				3.4.1		32	SQA **1996**
	(b)	3	3.4			3				3.4.2			H-PII **Qu.7**
3.4	(a)	4	3.4			4				3.4.1		54	SQA **1994**
	(b)	4	3.4			4				3.4.2			H-PII **Qu.5**
	(c)	2	3.4				2			3.4.2			
3.4	(a)	4	3.4			4				3.4.1		89	SQA **1991**
	(b)	4	1.2			2	2			1.2.3			H-PII **Qu.8**
	(c)	3	2.3			1	2			2.3.1			
3.4	(a)	4	3.4			4				3.4.1		111	SQA **1989**
	(b)	6	1.2			2	4			1.2.3	1.2.4		H-PII **Qu.9**
	(c)	1	0.1				1			0.1			
	(d)	2	0.1				2			0.1			
3.4	(a)	4	2.3			4				2.3.2		41	SQA **1995**
	(b)	5	3.4			5				3.4.1			H-PII **Qu.4**
3.4	(a)	3	2.3			3				2.3.2	1.2.11	97	SQA **1990**
	(b)	4	3.4			4				3.4.1			H-PII **Qu.5**
	(c)	3	3.4				3			3.4.2			
3.4	(a)	4	2.2			4				2.2.6		81	SQA **1992**
	(b)	2	0.1			2				0.1			H-PII **Qu.10**
	(c)	10	3.4			2	8			3.4.2	3.2.1 2.2.7		
0.1		6	0.1					6		0.1		19	SQA **1997**
													H-PII **Qu.7**
0.1		7	0.1					7		0.1		43	SQA **1995**
													H-PII **Qu.6**
0.1	(a)	2	0.1			2				0.1		55	SQA **1994**
	(b)	6	0.1			6				0.1			H-PII **Qu.6**
0.1	(a)	5	0.1					5		0.1		58	SQA **1994**
	(b)	4	0.1					4		0.1			H-PII **Qu.8**
	(c)	2	0.1					2		0.1			
0.1	(a)	3	0.1					3		0.1		64	SQA **1993**
	(b)	5	2.1					5		2.1.7	0.1		H-PII **Qu.4**
0.1		5	0.1					5		0.1		87	SQA **1991**
													H-PII **Qu.6**

Main Unit & Outcome	part	marks	Unit	marks distribution: non calc C	non calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	E.R.Q. page no.	Source
0.1	(a)	4	0.1	4						0.1		110	SQA 1989
	(b)	5	3.2	2	3					3.2.1	3.2.4		H-PII Qu.8
	(c)	2	1.2	2						1.2.3			
	(d)	2	0.1		2					0.1			
0.1	(a)	4	2.2					4		2.2.5		9	SQA 1998
	(b)	4	2.2					4		2.2.5			H-PII Qu.9
	(c)	1	0.1						1	0.1			
0.1	(a)	3	2.1			3				2.1.11		101	SQA 1990
	(b)	6	0.1			3	3			0.1			H-PII Qu.9

A triangle ABC has vertices A (2, –1, 3), B(3, 6, 5) and C (6, 6, –2).



(*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Calculate the size of angle BAC. **(5)**

(*c*) Hence find the area of the triangle. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	2	3.1			2				3.1.1		Source
(*b*)	5	3.1			5				3.1.11		**1998 Paper 2**
(*c*)	2	0.1			2				0.1		**Qu. 1**

(*a*) •1 $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

•2 $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$

(*b*) •3 $\cos B\hat{A}C = \dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}|\,|\overrightarrow{AC}|}$ *stated or implied by responses to •4 to •7*

•4 $\overrightarrow{AB}.\overrightarrow{AC} = 4 + 49 - 10$

•5 $\overrightarrow{AB} = \sqrt{54}$

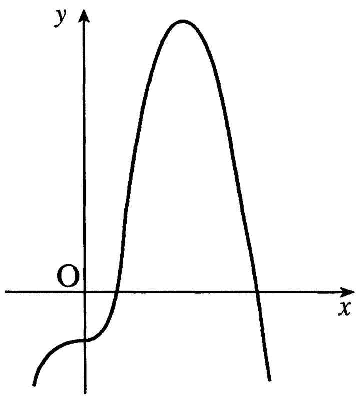
•6 $\overrightarrow{AC} = \sqrt{90}$

•7 $B\hat{A}C = 51 \cdot 9°$

(*c*) •8 **identify** 2 sides and included angle *e.g.* $\sqrt{54}, \sqrt{90}, B\hat{A}C$

•9 $27 \cdot 4$

A curve has equation $y = -x^4 + 4x^3 - 2$. An incomplete sketch of the graph is shown in the diagram.



(*a*) Find the coordinates of the stationary points. **(6)**

(*b*) Determine the nature of the stationary points. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	1.3
(*a*)	6	1.3					6		1.3.12	Source **1998 Paper 2 Qu. 2**
(*b*)	2	1.3					2		1.3.12	

(*a*)

•[1] $\frac{dy}{dx} = \ldots\ldots$ *stated or implied* by •[2]

•[2] $-4x^3 + 12x^2$

•[3] $-4x^3 + 12x^2 = 0$ *or* $\frac{dy}{dx} = 0$ *explicitly stated*

•[4] $-4x^2(x-3)$ (*accept* $x^2(-4x+12)$)

•[5] $x = 0$ *and* 3

•[6] $y = -2$ *and* 25

(*b*)

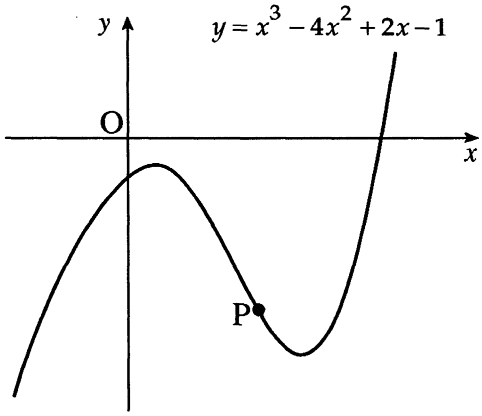
•[7]

x	0^-	0	0^+	3	3^+
$\frac{dy}{dx}$					

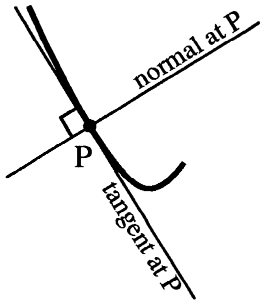
•[8] + 0 + 0 −

PI at $x = 0$, max *at* $x = 3$

(*a*) The diagram shows an incomplete sketch of the curve with equation $y = x^3 - 4x^2 + 2x - 1$. Find the equation of the tangent to the curve at the point P where $x = 2$. **(5)**



(*b*) The normal to the curve at P is defined as the straight line through P which is perpendicular to the tangent to the curve at P.
Find the angle which the normal at P makes with the positive direction of the *x*-axis. **(2)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	1.3
(*a*)	5	1.3					5		1.1.7, 1.3.9, 1.1.6	Source **1998 Paper 2 Qu. 3**
(*b*)	2	1.1					2		1.1.3, 1.1.9	

(*a*)
- $\bullet^1$ $\frac{dy}{dx} = \ldots$
- $\bullet^2$ $3x^2 - 8x + 2$
- $\bullet^3$ gradient $= -2$ (calculated from $\frac{dy}{dx}$)
- $\bullet^4$ $y_A = -5$
- $\bullet^5$ $y + 5 = -2(x - 2)$

(*b*)
- $\bullet^6$ $m_{\text{normal}} = \frac{1}{2}$
- $\bullet^7$ angle $= \tan^{-1}\frac{1}{2}$

A parabola passes through the points (0, 0), (6, 0) and (3, 9) as shown in Diagram 1.

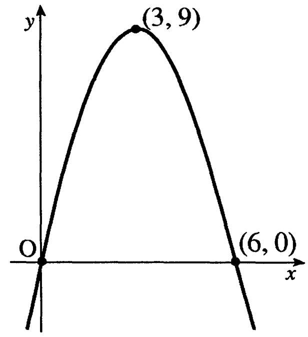


Diagram 1

(*a*) The parabola has equation of the form $y = ax(b - x)$. Determine the values of a and b. **(2)**

(*b*) Find the area enclosed by the parabola and the x-axis. **(4)**

(*c*) (i) Diagram 2 shows the parabola from (*a*) and the straight line with equation $y = x$. Find the coordinates of P, the point of intersection of the parabola and the line.

(ii) Calculate the area enclosed between the parabola and the line. **(5)**

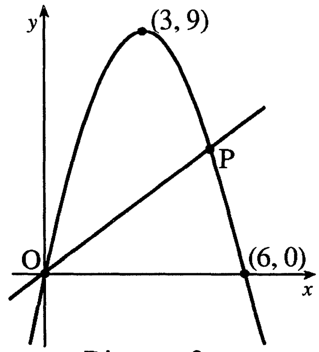


Diagram 2

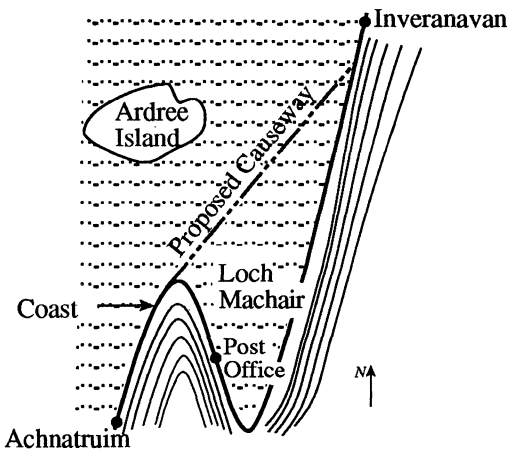
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	2	1.2	2						1.2.7		Source
(*b*)	4	2.2	4						2.2.6		**1998 Paper 2**
(c)i	2	2.1	2						2.1.8		**Qu. 4**
(c)ii	3	2.2		3					2.2.7		

(*a*)
- $\bullet^1$ $a = 1$
- $\bullet^2$ $b = 6$

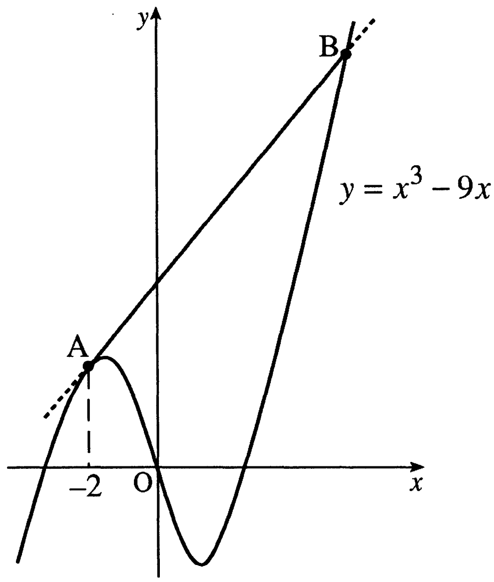
(*b*)
- $\bullet^3$ $\int_0^6 x(6-x)\,dx$
- $\bullet^4$ $\int \left(6x - x^2\right) dx$
- $\bullet^5$ $3x^2 - \frac{1}{3}x^3$
- $\bullet^6$ 36

(*c*)
- $\bullet^7$ $x = 6x - x^2$
- $\bullet^8$ $x_P = 5$
- $\bullet^9$ $\int_0^5 \left(6x - x^2 - x\right) dx$ or equiv.
- $\bullet^{10}$ $\left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^5$ or equiv.
- $\bullet^{11}$ $\frac{125}{6}$ or equiv.

The map shows part of the coast road from Achnatruim to Inveranavan. In order to avoid the hairpin bends, it is proposed to build a straight causeway, as shown, with the southern end tangential to the existing road.



With the origin taken at the Post Office the part of the coast road shown lies along the curve with equation $y = x^3 - 9x$. The causeway is represented by the line AB. The southern end of the proposed causeway is at the point A where $x = -2$, and the line AB is a tangent to the curve at A.



(*a*) (i) Write down the coordinates of A. **(5)**

(ii) Find the equation of the line AB.

(*b*) Determine the coordinates of the point B which represents the northern end of the causeway. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.1
(*a*)i	1	0.1	1						0.1	Source
(*a*)ii	4	1.1	4						1.1.6, 4	**1998 Paper 2**
(*b*)	7	2.1	2	5					2.1.12 & 2.1.2	**Qu. 5**

(*a*)

- •1 $y_{x=-2} = 10$
- •2 $\frac{dy}{dx} = \ldots\ldots$
- •3 $3x^2 - 9$
- •4 $m_{x=-2} = 3$
- •5 $y - 10 = 3(x+2)$

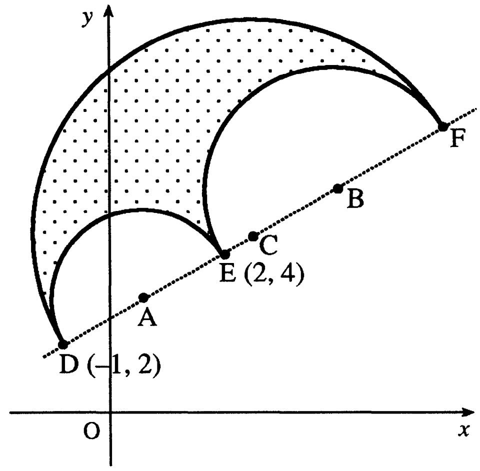
(*b*)

- •6 $y = 3x + 16$
- •7 $3x + 16 = x^3 - 9x$
- •8 $x^3 - 12x - 16 = 0$
- •9 *e.g.*

–2	1	0	–12	–16
		–2	4	16
	1	–2	–8	0

- •10 *e.g.* $x^2 - 2x - 8$
- •11 *e.g.* $(x+2)(x-4)$
- •12 B is $(4, 28)$

The shape shown in the diagram is composed of 3 semicircles with centres A, B and C which lie on a straight line.



DE is a diameter of one of the semicircles. The coordinates of D and E are (–1, 2) and (2, 4).

(*a*) Find the equation of the circle with centre A and diameter DE. **(3)**

The circle with centre B and diameter EF has equation $x^2 + y^2 - 16x - 16y + 76 = 0$.

(*b*) (i) Write down the coordinates of B.

(ii) Determine the coordinates of F and C. **(3)**

(*c*) In the diagram the perimeter of the shape is represented by the thick black line. Show that the perimeter is $5\pi\sqrt{13}$ units. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.4
(*a*)	3	2.4					3		2.4.3	Source
(*b*)	3	2.4					3		2.4.2 & 3.1.6	**1998 Paper 2**
(*c*)	3	0.1						3	0.1	**Qu. 6**

(*a*) •[1] $A = \left(\frac{1}{2}, 3\right)$

•[2] $r^2 = \frac{9}{4} + 1$ *or* $d^2 = 13$

•[3] $\left(x - \frac{1}{2}\right)^2 + (y - 3)^2 = \frac{13}{4}$

or $x^2 + y^2 - x - 6y + 6 = 0$

(*b*) •[4] $B\ (8,8)$

•[5] $F\ (14,12)$

•[6] $C\ \left(\frac{13}{2}, 7\right)$

(*c*) •[7] $\frac{1}{2}\pi DF + \frac{1}{2}\pi DE + \frac{1}{2}\pi EF$

•[8] $\frac{1}{2}\pi DF = \frac{5}{2}\pi\sqrt{13}$ ***OR*** $\frac{1}{2}\pi EF = 2\pi\sqrt{13}$

•[9] $\frac{5}{2}\pi\sqrt{13} + \frac{1}{2}\pi\sqrt{13} + 2\pi\sqrt{13}$

The function f is defined by $f(x) = 2\cos x° - 3\sin x°$.

(a) Show that $f(x)$ can be expressed in the form $f(x) = k\cos(x+\alpha)°$ where $k > 0$ and $0 \le \alpha < 360$, and determine the values of k and α. **(4)**

(b) Hence find the maximum and minimum values of $f(x)$ and the values of x at which they occur, where x lies in the interval $0 \le x < 360$. **(4)**

(c) Write down the minimum value of $(f(x))^2$. **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.4
(a)	4	3.4			4				3.4.1		Source
(b)	4	3.4			1	3			3.4.3		**1998 Paper 2**
(c)	1	0.1				1			0.1		**Qu. 7**

(a)
- •[1] $k\cos x\cos\alpha - k\sin x\sin\alpha$ **stated explicitly**
- •[2] $k\sin\alpha = 3$ *and* $k\cos\alpha = 2$ **stated explicitly**
- •[3] $k = \sqrt{13}$
- •[4] $\alpha = 56.3$

(b)
- •[5] $\sqrt{13}\cos(x + 56.3)$
- •[6] $\text{Max} = \sqrt{13}$ ***and*** $\text{min} = -\sqrt{13}$
- •[7] $x = 303.7$ and no further answers
- •[8] $x = 123.7$ and no further answers

(c)
- •[9] Min Value $= 0$

A gardener feeds her trees weekly with "Bioforce, the wonder plant food". It is known that in a week the amount of plant food in the tree falls by about 25%.

(*a*) The trees contain no Bioforce initially and the gardener applies 1g of Bioforce to each tree every Saturday. Bioforce is only effective when there is continuously more than 2g of it in the tree. Calculate how many weekly feeds will be necessary before the Bioforce becomes effective. **(3)**

(*b*) (i) Write down a recurrence relation for the amount of plant food in the tree immediately after feeding. **(1)**

(ii) If the level of Bioforce in the tree exceeds 5g, it will cause leaf burn. Is it safe to continue feeding the trees at this rate indefinitely? **(4)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	1.4
			C	A/B	C	A/B	C	A/B	Main Additional	
(*a*)	3	1.4			3				1.4.1	Source
(*b*)	1	1.4			1				1.4.3	**1998 Paper 2**
(*c*)	4	1.4			4				1.4.4, 1.4.5	**Qu. 8**

(*a*) •1 75% or equivalent

•2 0.75, 1.31 **and** 1.73

•3 2.05 **and** "after fourth feed"

(*b*) •4 $u_{n+1} = 0.75u_n + 1$

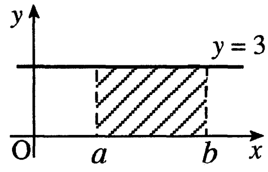
(*c*) •5 $-1 < 0.75 < 1$ so sequence has a limit

•6 *e.g.* $L = 0.75L + 1$

•7 $L = 4$

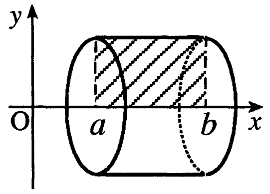
•8 Safe to continue

Diagram 1 shows the area between the line $y = 3$ and the x-axis from $x = a$ to $x = b$. If this area is rotated through 360° about the x-axis, it forms a solid shape (a cylinder) as shown in Diagram 2.



Diag.1

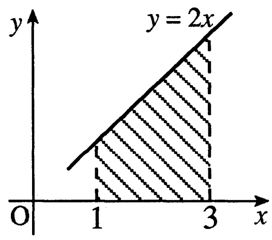
The volume of this solid may be obtained by evaluating the integral $\pi\int_a^b y^2\ dx$.



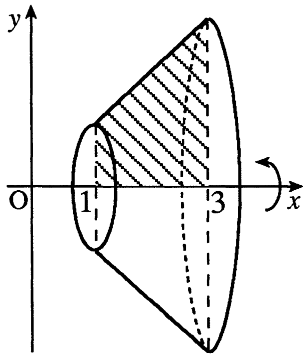
Diag.2

Worked Example

The area between $y = 2x$ and the x-axis from $x = 1$ to $x = 3$ is rotated about the x-axis. The volume of the solid is calculated as follows:

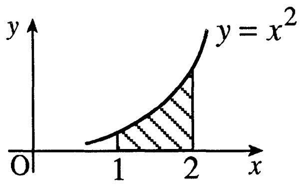


$$y = 2x$$
$$y^2 = (2x)^2 = 4x^2$$

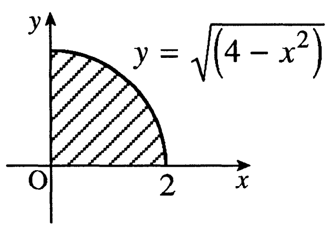


$$\pi\int_1^3 y^2\ dx$$
$$= \pi\int_1^3 4x^2\ dx$$
$$= \pi\left[\tfrac{4}{3}x^3\right]_1^3$$
$$= \pi\left[36 - \tfrac{4}{3}\right]$$
$$\text{Volume} = \tfrac{104}{3}\pi \text{ units}^3$$

(*a*) Use this method to find the volume of the solid formed when the area between $y = x^2$ and the x-axis from $x = 1$ to $x = 2$ is rotated about the x-axis. **(4)**



(*b*) (i) Use this method to find the volume of the solid formed when the area between $y = \sqrt{(4 - x^2)}$ and the x-axis from $x = 0$ to $x = 2$ is rotated about the x-axis. **(4)**



(ii) **Hence** write down the volume of a sphere of radius 2. **(1)**

1998 Paper 2 Qu. 9

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	4
(*a*)	4	2.2					4		2.2.5		Source
(*b*)	4	2.2					4		2.2.5		**1998 Paper 2**
(*c*)	1	0.1						1	0.1		**Qu. 9**

(*a*) •1 $y^2 = x^4$

•2 $\pi\int_1^2 x^4 \; dx$

•3 $\pi\left[\frac{1}{5}x^5\right]_1^2$

•4 $\frac{31}{5}\pi$ (*accept* 19.5)

(*b*) •5 $y^2 = 4 - x^2$

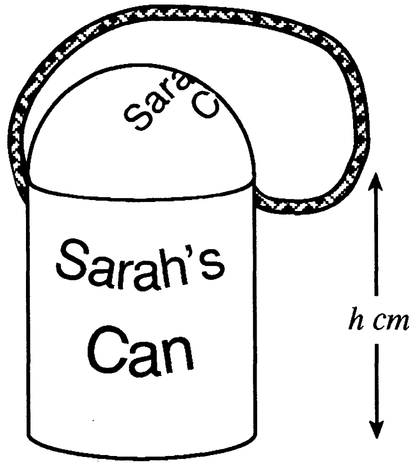
•6 $\pi\int_0^2 \left(4 - x^2\right) \; dx$

•7 $\pi\left[4x - \frac{1}{3}x^3\right]_0^2$

•8 $\frac{16}{3}\pi$

(*c*) •9 $\frac{32}{3}\pi$ **or** $2 \times \frac{16}{3}\pi$

A child's drinking beaker is in the shape of a cylinder with a hemispherical lid and a circular flat base. The radius of the cylinder is r cm and the height is h cm. The volume of the cylinder is 400 cm^3.



(*a*) Show that the surface area of plastic, $A(r)$, needed to make the beaker is given by $A(r) = 3\pi r^2 + \frac{800}{r}$. **(3)**

Note: The curved surface area of a hemisphere of radius r is $2\pi r^2$.

(*b*) Find the value of r which ensures that the surface area of plastic is minimised. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.3
(*a*)	3	0.1				3			0.1		Source
(*b*)	6	1.3			3	3			1.3.15		**1998 Paper 2 Qu. 10**

(*a*)
- •1 $\pi r^2 + 2\pi rh + 2\pi r^2$
- •2 $h = \frac{400}{\pi r^2}$ or equivalent (*e.g.* $\pi rh = \frac{400}{r}$)
- •3 $2\pi r \frac{400}{\pi r^2} + 3\pi r^2$ and completes proof

(*b*)
- •4 $\frac{dA}{dr} = \ldots$
- •5 $800r^{-1}$
- •6 $6\pi r - 800r^{-2}$
- •7 *e.g.* $6\pi r - \frac{800}{r^2} = 0$
- •8 3.5
- •9

r	3.5^-	3.5	3.5^+
$\frac{dA}{dr}$	$-ve$	0	$+ve$

(*a*) The variables x and y are connected by a relationship of the form $y = ae^{bx}$ where a and b are constants. Show that there is a linear relationship between $\log_e y$ and x. **(3)**

(*b*) From an experiment some data was obtained. The table shows the data which lies on the line of best fit.

x	3.1	3.5	4.1	5.2
y	21 876	72 631	439 392	11 913 076

The variables x and y in the above table are connected by a relationship of the form $y = ae^{bx}$. Determine the values of a and b. **(6)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		**3.3**
			C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*)	3	3.3				3			3.3.7		Source
(*b*)	6	3.3				6			3.3.5		**1998 Paper 2 Qu. 11**

(*a*)
- $\bullet^1$ $\log_e y = \log_e ae^{bx}$
- $\bullet^2$ $\log_e y = \log_e a + \log_e e^{bx}$
- $\bullet^3$ $\log_e y = \log_e a + bx$

(*b*)
- $\bullet^4$ evidence for strategy being carried out will be appearance of two equations at $\bullet^5$ stage
- $\bullet^5$ *e.g.* $3.1b + \log a = 9.99,\ 5.2b + \log a = 16.29$
- $\bullet^6$ strategy: know to subtract
- $\bullet^7$ $b = 3$
- $\bullet^8$ $a = e^{0.69}$
- $\bullet^9$ $a = 2$

Diagram 1 shows a circle with equation $x^2 + y^2 + 10x - 2y - 14 = 0$ and a straight line, l_1, with equation $y = 2x + 1$.
The line intersects the circle at A and B.

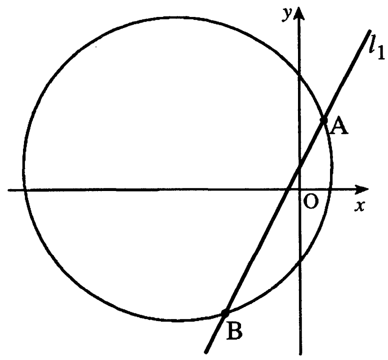


Diagram 1

(*a*) Find the coordinates of the points A and B. **(5)**

(*b*) Diagram 2 shows a second line, l_2, which passes through the centre of the circle, C, and is at right angles to line l_1.

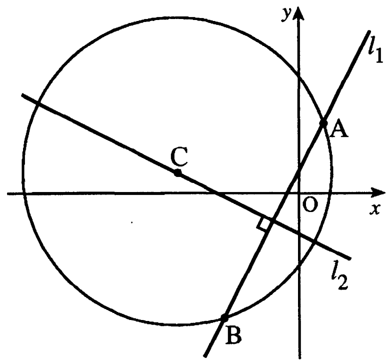


Diagram 2

(i) Write down the coordinates of C. **(1)**

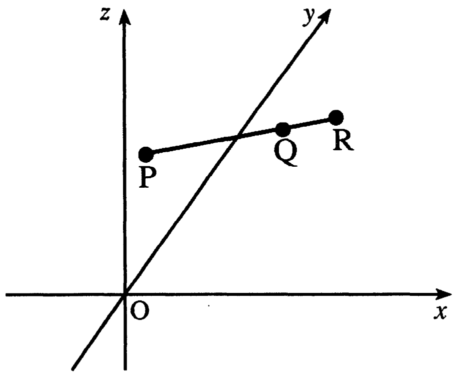
(ii) Find the equation of the line l_2. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.4
(*a*)	5	2.4					5		2.4.4		Source
(*b*)i	1	2.4					1		2.4.2		**1997 Paper 2**
(*b*)ii	3	1.1					3		1.1.10	1.1.7	**Qu.1**

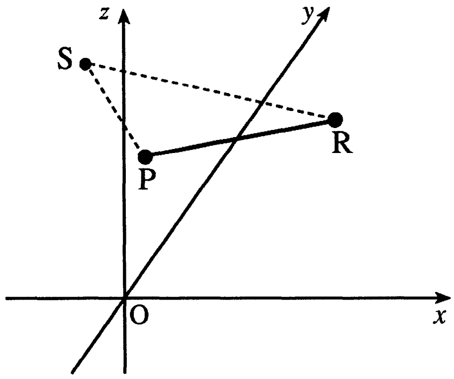
(*a*)
- •[1] know to substitute
- •[2] correct substitution
- •[3] a "quadratic" = 0
- •[4] $x = -3, \ 1$
- •[5] $y = -5, \ 3$

(*b*)
- •[6] $m_{diameter} = 2$
- •[7] $m_{perpendicular} = -\frac{1}{2}$
- •[8] centre = $(-1, -1)$
- •[9] equation: $y + 1 = -\frac{1}{2}(x + 1)$

Relative to the axes shown and with an appropriate scale, P(–1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.



(*a*) Find the coordinates of R. **(3)**



(*b*) Roads from P and R are built to meet at the point S (–2, 2, 5). Calculate the size of angle PSR. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	3.1
(*a*)	3	3.1			3				3.1.6	Source 1997 Paper 2 Qu.2
(*b*)	7	3.1			7				3.1.11	

(*a*)

•[1] $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$ •[2] $\begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix}$

•[3] $R = (7, -1, 6)$

(*b*)

•[4] $\overrightarrow{SP}.\overrightarrow{SR} = |SP||SR|\cos P\hat{S}R$

•[5] $\overrightarrow{SP} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$ •[6] $\overrightarrow{SR} = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$

•[7] $|SP| = \sqrt{11}$ •[8] $|SR| = \sqrt{91}$

•[9] $\overrightarrow{SP}.\overrightarrow{SR} = 3$

•[10] $P\hat{S}R = 84 \cdot 6°$

The sum of £1000 is placed in an investment account on January 1st and, thereafter, £100 is placed in the account on the first day of each month.

- Interest at the rate of 0·5% per month is credited to the account on the last day of each month.
- This interest is calculated on the amount in the account on the first day of the month.

(*a*) How much is in the account on June 30th ? **(4)**

(*b*) On what date does the account first exceed £2000? **(2)**

(*c*) Find a recurrence relation which describes the amount in the account, explaining your notation carefully. **(3)**

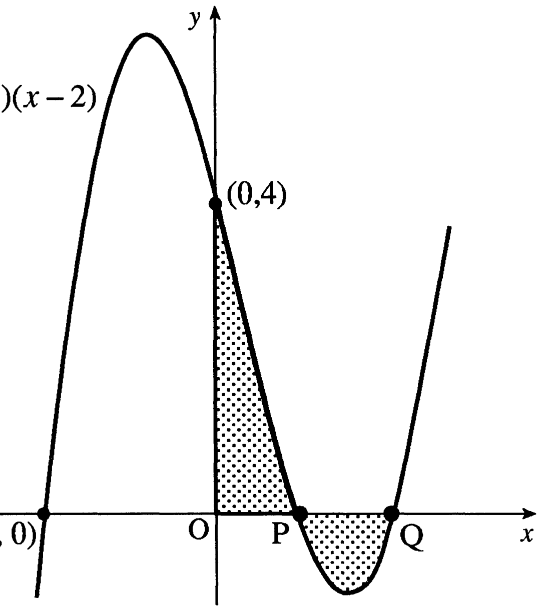
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.4
(*a*)	4	1.4			4				1.4.1		Source
(*b*)	2	1.4			2				1.4.1		**1997 Paper 2**
(*c*)	3	1.4			3				1.4.3		**Qu.3**

(*a*)
- •1 1.005
- •2 £1000 + interest = £1005
- •3 £1005 + £100 + interest = £1110.525
- •4 £1537.93

(*b*)
- •5 complete another month
- •6 £2073.94 on Nov.1st

(*c*)
- •7 $u_{n+1} = 1.005u_n + 100$
- •8 u_n = amount on 1st day of each month
- •9 $u_0 = 1000$ (on 1st January)

The diagram shows a sketch of the graph of $y=(x+2)(x-1)(x-2)$. The graph cuts the axes at (–2, 0), (0, 4) and the points P and Q.



$y=(x+2)(x-1)(x-2)$

(*a*) Write down the coordinates of P and Q. **(2)**

(*b*) Find the total shaded area. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	2	2.1	2						2.1.2		Source
(*b*)	7	2.2	6	1					2.2.6		**1997 Paper 2 Qu.4**

(*a*) •1 (1,0)

•2 (2,0)

(*b*) •3 $\int f(x)dx$

•4 $\int_0^1 - \int_1^2$

•5 $(x+2)(x^2-3x+2)$ or equiv.

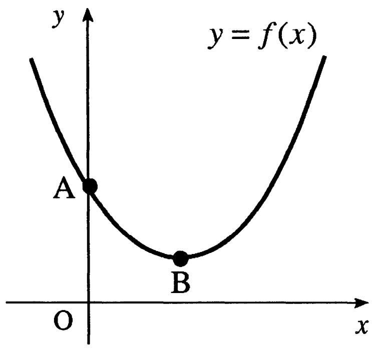
•6 x^3-x^2-4x+4

•7 $\frac{1}{4}x^4-\frac{1}{3}x^3-2x^2+4x$

•8 $1\frac{11}{12}$ or $-\frac{7}{12}$

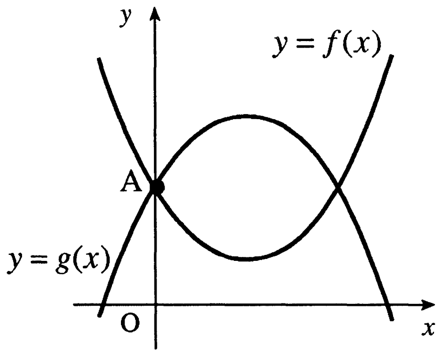
•9 $2\frac{1}{2}$

The first diagram shows a sketch of part of the graph of $y = f(x)$ where $f(x) = (x-2)^2 + 1$. The graph cuts the y-axis at A and has a minimum turning point at B.



(*a*) Write down the coordinates of A and B. **(3)**

(*b*) The second diagram shows the graphs of $y = f(x)$ and $y = g(x)$ where $g(x) = 5 + 4x - x^2$.
Find the area enclosed by the two curves. **(5)**



(*c*) $g(x)$ can be written in the form $m + n \times f(x)$ where m and n are constants.
Write down the values of m and n. **(2)**

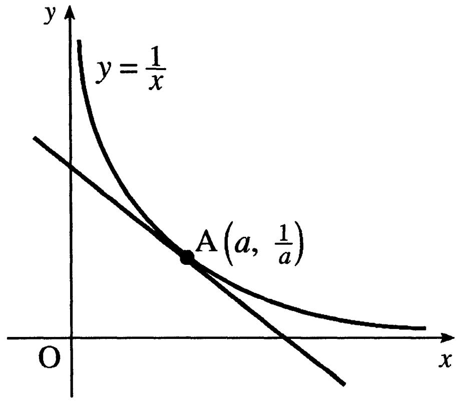
part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*)	3	1.2	3						1.2.9		Source
(*b*)	5	2.2	5						2.2.7		**1997 Paper 2**
(*c*)	3	0.1		2					0.1		**Qu.5**

(*a*)
- •1 $A = (0,5)$
- •2 $x_B = 2$
- •3 $y_B = 1$

(*b*)
- •4 $\int_0^4$
- •5 $\int \left(\left(5 + 4x - x^2\right) - \left(x^2 - 4x + 5\right) \right) dx$
- •6 $8x - 2x^2$ *or equiv.*
- •7 $4x^2 - \frac{2}{3}x^3$ *or equiv.*
- •8 $\frac{64}{3}$

(*c*)
- •9 $n = -1$
- •10 $m = 10$

(*a*) A sketch of part of the graph of $y = \frac{1}{x}$ is shown in the diagram. The tangent at A $\left(a, \frac{1}{a}\right)$ has been drawn.
Find the gradient of this tangent. **(4)**



(*b*) Hence show that the equation of this tangent is $x + a^2 y = 2a$. **(2)**

(*c*) This tangent cuts the y-axis at B and the x-axis at C.

(i) Calculate the area of triangle OBC **(3)**

(ii) Comment on your answer to *c*(i). **(1)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(*a*)	4	1.3					4		1.3.7		**1997 Paper 2**
(*b*)	2	1.1					1	1	1.1.7		**Qu.6**
(*c*)	4	0.1						4	0.1		

(*a*)
- •1 $\frac{1}{x} = x^{-1}$
- •2 $\frac{dy}{dx} = \ldots\ldots$
- •3 $\frac{dy}{dx} = -x^{-2}$
- •4 gradient = $-a^{-2}$

(*b*)
- •5 use $y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$
- •6 $a^2 y - a = -(x - a)$ and completes proof

(*c*)
- •7 $y_B = \frac{2a}{a^2}$
- •8 $x_A = 2a$
- •9 2
- •10 independent of a

In certain topics in Mathematics, such as calculus, we often require to write an expression such as $\dfrac{8x+1}{(2x+1)(x-1)}$ in the form $\dfrac{2}{2x+1}+\dfrac{3}{x-1}$.

$\dfrac{2}{2x+1}+\dfrac{3}{x-1}$ are called **Partial Fractions** for $\dfrac{8x+1}{(2x+1)(x-1)}$.

The worked example shows you how to find partial fractions for the expression $\dfrac{6x+2}{(x+2)(x-3)}$.

Worked Example

Find partial fractions for $\dfrac{6x+2}{(x+2)(x-3)}$.

Let $\dfrac{6x+2}{(x+2)(x-3)}=\dfrac{A}{x+2}+\dfrac{B}{x-3}$ where A and B are constants

$$=\frac{A(x-3)}{(x+2)(x-3)}+\frac{B(x+2)}{(x-3)(x+2)}$$

i.e. $\dfrac{6x+2}{(x+2)(x-3)}=\dfrac{A(x-3)+B(x+2)}{(x+2)(x-3)}$

Hence $6x+2=A(x-3)+B(x+2)$ for all values of x.

A and B can be found as follows:

Select a value of x that makes the first bracket zero

Let $x = 3$ (this eliminates A)

$18+2=A\times 0 \ + \ B\times 5$

$20=5B$

$\underline{B=4}$

Select a value of x that makes the second bracket zero

Let $x = -2$ (this eliminates B)

$-12+2=A\times(-5) \ + \ B\times 0$

$-10=-5A$

$\underline{A=2}$

Therefore $\dfrac{6x+2}{(x+2)(x-3)}=\dfrac{2}{x+2}+\dfrac{4}{x-3}$.

Find partial fractions for $\dfrac{5x+1}{(x-4)(x+3)}$. **(6)**

1997 Paper 2 Qu.7

part marks	Unit	non-calc		calc		calc neut		Content Reference :	4
		C	A/B	C	A/B	C	A/B	Main Additional	Source
6	0.1					6		0.1	**1997 Paper 2 Qu.7**

- •[1] $\frac{A}{x-4}+\frac{B}{x+3}$
- •[2] $\frac{A(x+3)+B(x-4)}{(2x-1)(x+3)}$
- •[3] $5x+1=A(x+3)+B(x-4)$
- •[4] choose to let $x=-3$ and 4 in turn
- •[5] $A=3$
- •[6] $B=2$

The radioactive element carbon-14 is sometimes used to estimate the age of organic remains such as bones, charcoal, and seeds.

Carbon-14 decays according to a law of the form $y = y_0 e^{kt}$ where y is the amount of radioactive nuclei present at time t years and y_0 is the initial amount of radioactive nuclei.

(*a*) The half-life of carbon-14, i.e. the time taken for half the radioactive nuclei to decay, is 5700 years. Find the value of the constant k, correct to 3 significant figures. **(3)**

(*b*) What percentage of the carbon-14 in a sample of charcoal will remain after 1000 years? **(3)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	3.3
			C	A/B	C	A/B	C	A/B	Main Additional	Source
(*a*)	3	3.3			1	2			3.3.7	**1997 Paper 2 Qu.8**
(*b*)	3	3.3				3			3.3.7	

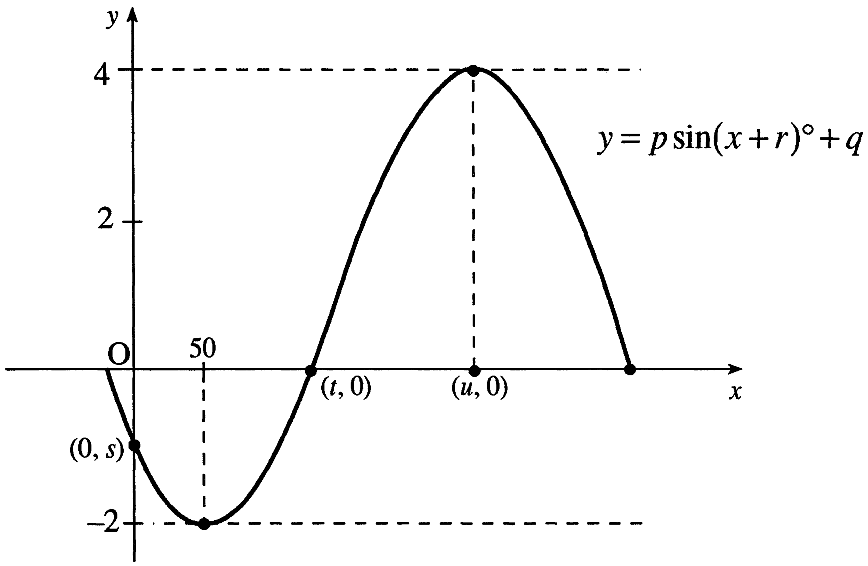
(*a*)
- •[1] $\frac{1}{2}y_0 = y_0 e^{5700k}$
- •[2] $\ln\frac{1}{2} = 5700k$
- •[3] $k = -0.000122$

(*b*)
- •[4] $y = y_0 e^{-0.000122\times1000}$
- •[5] $\frac{y}{y_0} = \ldots$
- •[6] 88.5%

The sketch represents part of the graph of a trigonometric function of the form $y = p\sin(x+r)° + q$. It crosses the axes at (0, s) and (t,0), and has turning points at (50, –2) and (u, 4).

(i) Write down values for p, q, r and u. **(4)**

(ii) Find the values for s and t. **(4)**

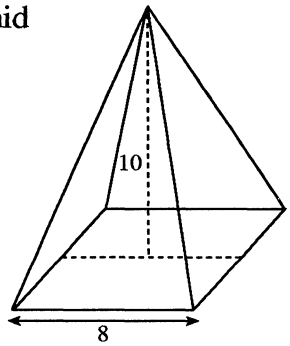


part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(a)	4	1.2			2	2			1.2.3		Source **1997 Paper 2 Qu.9**
(b)	4	2.3				4			2.3.1		

(a)
- •[1] $p = -3$
- •[2] $q = 1$
- •[3] $r = 40$ *or* -320
- •[4] $u = 230$

(b)
- •[5] replace x by 0
- •[6] –0.928
- •[7] replace y by 0
- •[8] 120.5

A cuboid is to be cut out of a right square-based pyramid. The pyramid has a square base of side 8 cm. and a vertical height of 10cm.

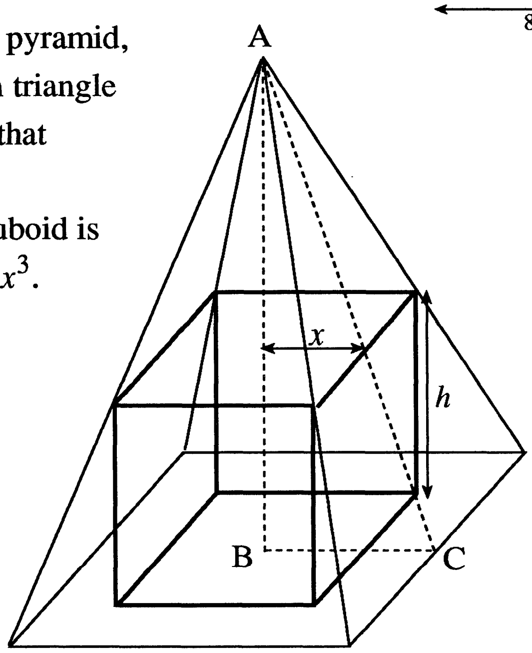


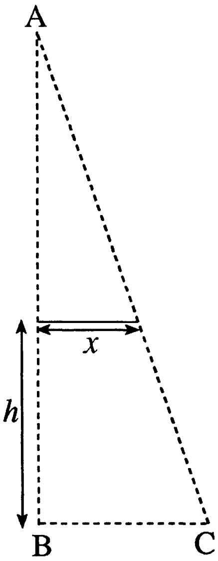
(*a*) The cuboid has a square base of side $2x$ cm and a height of h cm.

If the cuboid is to fit into the pyramid, use the information shown in triangle ABC, or otherwise, to show that

(i) $h = 10 - \frac{5}{2}x$. **(3)**

(ii) the volume, V, of the cuboid is given by $V = 40x^2 - 10x^3$. **(1)**





(*b*) Hence find the dimensions of the square-based cuboid with the greatest volume which can be cut from the pyramid. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.3
(*a*)	4	0.1					1	3	0.1		Source
(*b*)	6	1.3					3	3	1.3.15		**1997 Paper 2 Qu.10**

(*a*)

$\bullet^1$ strategy: e.g. equate ratios from similar triangles

$\bullet^2$ $\frac{10}{4} = \frac{10-h}{x}$ or equivalent

$\bullet^3$ complete proof

$\bullet^4$ $V = 40x^2 - 10x^3$

(*b*)

$\bullet^5$ $\frac{dV}{dx} =$

$\bullet^6$ $80x - 30x^2$

$\bullet^7$ $\frac{dV}{dx} = 0$ for stationary points

$\bullet^8$ $0, \frac{8}{3}$

$\bullet^9$

x	...	$\frac{8}{3}$	...
$\frac{dV}{dx}$	+	0	−
		max	

$\bullet^{10}$ $\frac{16}{3}$ and $\frac{10}{3}$

Two identical coins, radius 1 unit, are supported by horizontal and vertical plates at B and C. Diagram 1 shows the coins touching each other and the line of centres is inclined at p radians to the vertical.

Let d be the length of BC.

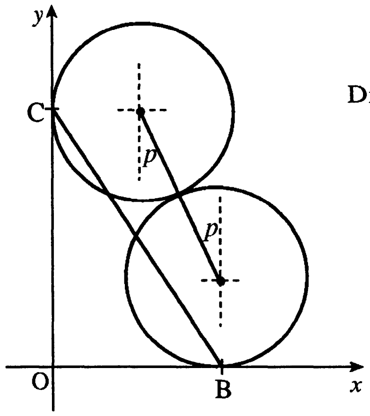


Diagram 1

(*a*) (i) Show that $OB = 1 + 2\sin p$ **(1)**

(ii) Write down a similar expression for OC and hence show that $d^2 = 6 + 4\cos p + 4\sin p$. **(2)**

(*b*) (i) Express d^2 in the form $6 + k\cos(p - \alpha)$ **(4)**

(ii) Hence write down the exact maximum value of d^2 and the value of p for which this occurs. **(2)**

(*c*) Diagram 2 shows the special case where $p = \frac{\pi}{4}$.

(i) Show that $OB = 1 + \sqrt{2}$ and find the exact length of BD. **(2)**

(ii) Using your answer to (*b*)(ii) find the exact value of $\sqrt{6 + 4\sqrt{2}}$. **(2)**

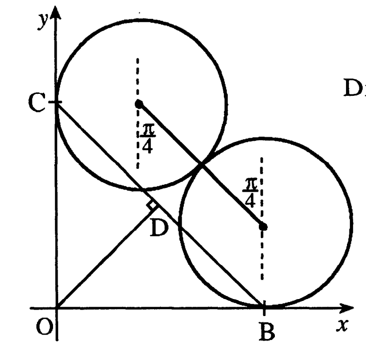


Diagram 2

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.4
(*a*)	3	0.1	2	1					0.1		Source
(*b*)	6	3.4	4	2					3.4.1	3.4.3	**1997 Paper 2**
(*c*)	4	0.1	1	3					0.1		**Qu.11**

(*a*)

•1 $\sin p = \frac{\text{"hor"}}{2}$ *and* $OB = 1 + \text{"hor"}$

•2 $OC = 1 + 2\cos p$

•3 $d^2 = (1 + 2\cos p)^2 + (1 + 2\sin p)^2$ and completes proof

(*b*)

•4 $k\cos(p - \alpha) = k\cos p\cos\alpha + k\sin p\sin\alpha$

•5 $k\cos\alpha = 4$ *and* $k\sin\alpha = 4$

•6 $k = 4\sqrt{2}$

•7 $\alpha = \frac{\pi}{4}$

•8 maximum value $= 6 + 4\sqrt{2}$

•9 occurs when $p = \frac{\pi}{4}$

(*c*)

•10 $OB = 1 + 2\sin\frac{\pi}{4}$ and completes proof

•11 $BD = (1 + \sqrt{2}) \times \frac{1}{\sqrt{2}}$

•12 $BC = 2 + \sqrt{2}$

•13 $6 + 4\sqrt{2} = (2 + \sqrt{2})^2$ *so* $\sqrt{6 + 4\sqrt{2}} = 2 + \sqrt{2}$

A curve has equation $y = x^4 - 4x^3 + 3$.

(a) Find algebraically the coordinates of the stationary points. **(6)**

(b) Determine the nature of the stationary points. **(2)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	1.3
			C	A/B	C	A/B	C	A/B	Main Additional	Source
(a)	6	1.3	6						1.3.12	**1996 Paper 2 Qu.1**
(b)	2	1.3	2						1.3.12	

(a)

- •[1] $\frac{dy}{dx} =$
- •[2] $4x^3 - 12x^2$
- •[3] $= 0$ stated explicitly
- •[4] *e.g.* $4x^2(x-3)$
- •[5] $x = 0,\ 3$
- •[6] $y = 3,\ -24$

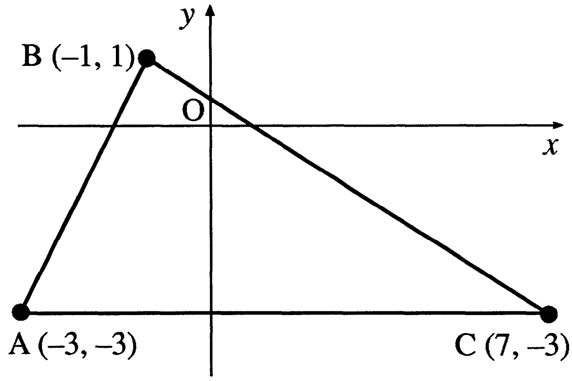
(b)

- •[7]

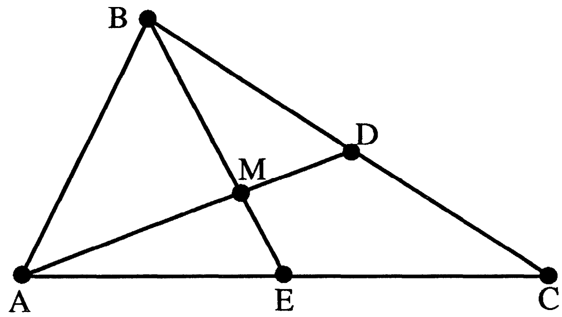
x	0^-	0	0^+	3	3^+
$\frac{dy}{dx}$	$-$	0	$-$	0	$+$

- •[8] pt of inflection at $x = 0$
  minimum at $x = 3$

A triangle ABC has vertices A(–3, –3), B(–1, 1) and C(7, –3).



(*a*) Show that the triangle ABC is right-angled at B. **(3)**



(*b*) The medians AD and BE intersect at M.

(i) Find the equations of AD and BE. **(5)**

(ii) Hence find the coordinates of M. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	1.1
(*a*)	3	1.1					3		1.1.10	Source
(*b*)i	5	1.1					5		1.1.10	**1996 Paper 2**
(b)ii	3	0.1					3		0.1	**Qu.2**

(*a*) •1 $m_{AB} = 2$

•2 $m_{BC} = -\frac{1}{2}$

•3 $m_{AB} \times m_{BC} = -1 \Rightarrow m_{AB} \perp m_{BC}$

(*b*) •4 $D = (3,-1)$ ***and*** $E = (2,-3)$

•5 $m_{AD} = \frac{1}{3}$

•6 AD: $y+1 = \frac{1}{3}(x-3)$ *or equiv.*

•7 $m_{BE} = -\frac{4}{3}$

•8 BE: $y-1 = -\frac{4}{3}(x+1)$ *or equiv.*

•9 *eg* clear fractions

•10 *eg* substitute

•11 $x = 1,\ y = -\frac{5}{3}$

The first four levels of a stepped pyramid with a square base are shown in Diagram 1.

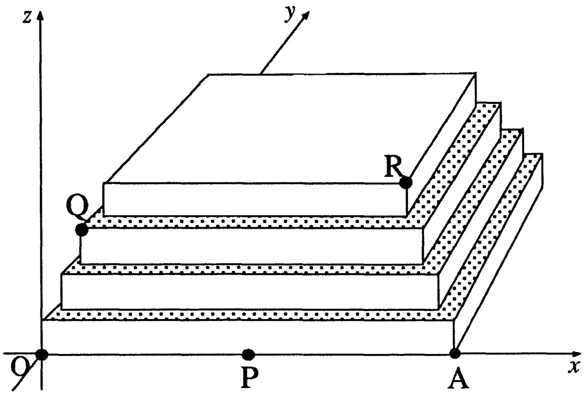


Diagram 1

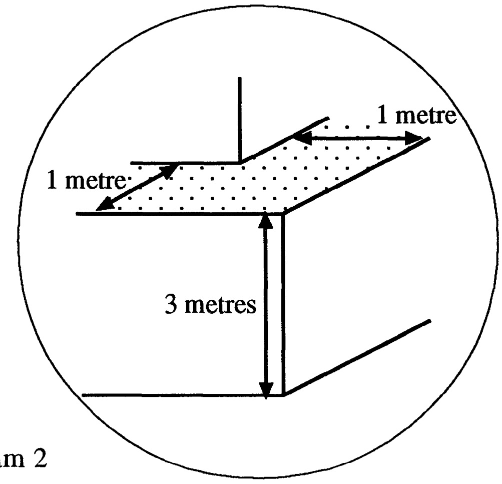


Diagram 2

Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m.
The height and "width" of a step at a corner are shown in the enlargement in Diagram 2.
With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0).

(*a*) Find the coordinates of Q and R. **(2)**

(*b*) Find the size of angle QPR. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	2	3.1			2				3.1.1		Source **1996 Paper 2 Qu.3**
(*b*)	7	3.1			7				3.1.11		

(*a*) •[1] $Q=(2,2,9)$

•[2] $R=(21,3,12)$

(*b*) •[3] $\cos\theta=\frac{a.b}{|a||b|}$ *with some subsequent use*

eg $\cos Q\hat{P}R=\frac{\vec{PQ}.\vec{PR}}{|\vec{PQ}||\vec{PR}|}$

•[4] $\vec{PQ}=\begin{pmatrix}-10\\2\\9\end{pmatrix}$ •[5] $\vec{PR}=\begin{pmatrix}9\\3\\12\end{pmatrix}$

•[6] $|\vec{PQ}|=\sqrt{185}$

•[7] $|\vec{PR}|=\sqrt{234}$

•[8] $\vec{PQ}.\vec{PR}=24$

•[9] $Q\hat{P}R=83\cdot4^\circ$

(*a*) $f(x) = 2x + 1$, $g(x) = x^2 + k$, where k is a constant.

(i) Find $g(f(x))$. **(2)**

(ii) Find $f(g(x))$. **(2)**

(*b*) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$. **(2)**

(ii) Determine the nature of the roots of this equation when $k = 6$. **(2)**

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. **(3)**

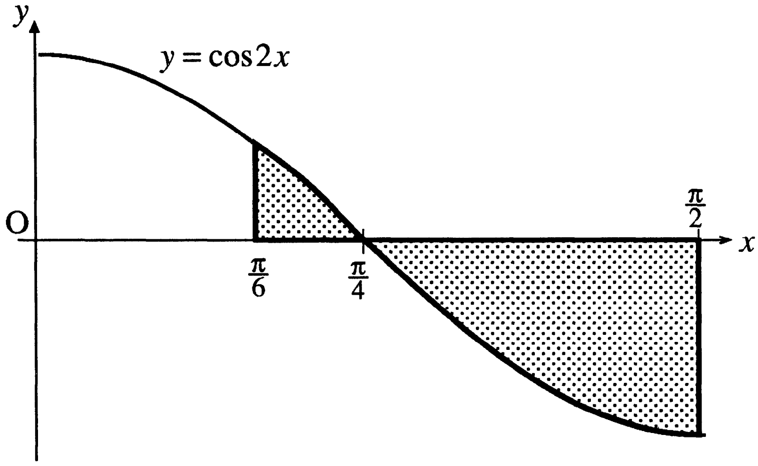
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	4	1.2	4						1.2.6		Source
(*b*)	7	2.1	7						2.1.6,	2.1.7, 0.1	1996 Paper 2 Qu.4

(*a*)
- •1 $g(2x+1)$
- •2 $(2x+1)^2 + k$
- •3 $f(x^2+k)$
- •4 $2(x^2+k)+1$

(*b*)
- •5 $4x^2+4x+k+1$ *AND* $2x^2+2k+1$
- •6 $4x^2+4x+k+1-(2x^2+2k+1)=0$

  so $2x^2+4x-k=0$
- •7 $b^2-4ac = 16-4\times2\times(-6) = 64$
- •8 so roots real & distinct
- •9 $b^2-4ac = 16-4\times2\times(-k)$
- •10 $b^2-4ac=0$ for equal roots
- •11 $k=-2$

An artist has designed a 'bow' shape which he finds can be modelled by the shaded area below. Calculate the area of this shape. **(6)**



part	marks	Unit	non-calc		calc		calc neut		Content Reference :	3.2
			C	A/B	C	A/B	C	A/B	Main Additional	
-	6	3.2	2	4					3.2.4, 2.2.6	Source **1996 Paper 2 Qu.5**

(-) •1 evidence of two integrals

•2 $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos 2x \ dx$ *and* $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \ dx$

•3 $\frac{1}{2}\sin 2x$

•4 $\frac{1}{2}\sin\frac{\pi}{2} - \frac{1}{2}\sin\frac{\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{4}$

•5 $\frac{1}{2}\sin\pi - \frac{1}{2}\sin\frac{\pi}{2} = -\frac{1}{2}$

•6 $1 - \frac{\sqrt{3}}{4}$

Diagram 1 shows :

- the point A(1, 2),
- the straight line $\boldsymbol{l}$ passing through the origin O and the point A.
- the parabola $\boldsymbol{p}$ with a minimum turning point at O and passing through A.
- and the circle $\boldsymbol{c}$, centre O, passing through A.

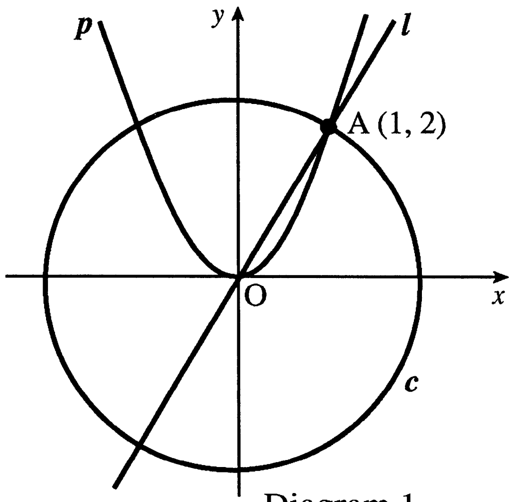


Diagram 1

(a) Write down the equations of the line, the parabola and the circle. **(3)**

The following transformations are carried out:

- the line is given a translation of 4 units down (i.e. – 4 units in the direction of the y-axis).
  Diagram 2 shows the line $\boldsymbol{l'}$, the image of line $\boldsymbol{l}$, after this translation.
- the parabola is reflected in the x-axis.
- the circle is given a translation of 2 units to the right (i.e. +2 units in the direction of the x-axis).

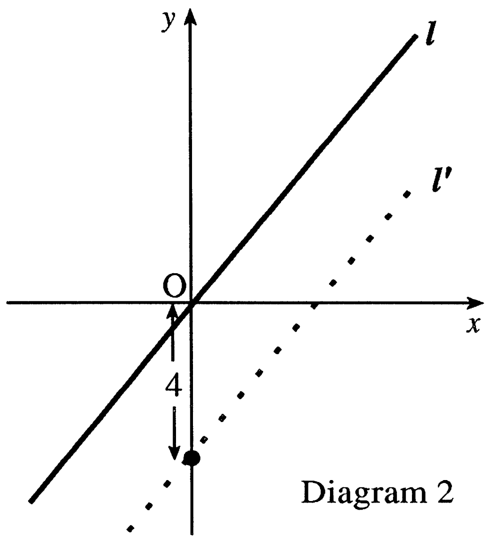


Diagram 2

(b) Write down the equations of $\boldsymbol{l'}$, $\boldsymbol{p'}$ (the image of the parabola p) and $\boldsymbol{c'}$ (the image of the circle c). **(4)**

(c) (i) Show that the line $\boldsymbol{l'}$ passes through the centre of the circle $\boldsymbol{c'}$. **(1)**

(ii) Find the coordinates of the points where the line $\boldsymbol{l'}$ intersects the parabola $\boldsymbol{p'}$. **(3)**

1996 Paper 2 Qu.6

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.1
(*a*)	3	1.2					3		1.2.7	Source
(*b*)	4	0.1					4		0.1	**1996 Paper 2**
(*c*)	4	2.1					4		2.1.8, 0.1	**Qu.6**

(*a*) •[1] $y = 2x$

•[2] $y = 2x^2$

•[3] $x^2 + y^2 = 5$

(*b*) •[4] $y = 2x - 4$

•[5] $y = -2x^2$

•[6] $centre = (2,0)$

•[7] $(x-2)^2 + y^2 = 5$

(*c*) •[8] *show* $(2,0)$ *satisfies* $y = 2x - 4$

•[9] $2x - 4 = -2x^2$

•[10] $(x+2)(x-1) = 0$

•[11] $(-2,-8),\ (1,-2)$

$f(x) = 2\cos x° + 3\sin x°$.

(a) Express $f(x)$ in the form $k\cos(x - \alpha)°$ where $k > 0$ and $0 \le \alpha < 360$. **(4)**

(b) Hence solve algebraically $f(x) = 0.5$ for $0 \le x < 360$. **(3)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(a)	4	3.4			4				3.4.1		1996 Paper 2 Qu.7
(b)	3	3.4			3				3.4.2		

(a)

$\bullet^1$ $k\cos x\cos\alpha + k\sin x\sin\alpha$

$\bullet^2$ $k\cos\alpha = 2$ ***and*** $k\sin\alpha = 3$

$\bullet^3$ $k = \sqrt{13}$

$\bullet^4$ $\alpha = 56.3$

(b)

$\bullet^5$ $\cos(x - 56.3)° = \frac{0.5}{\sqrt{13}}$

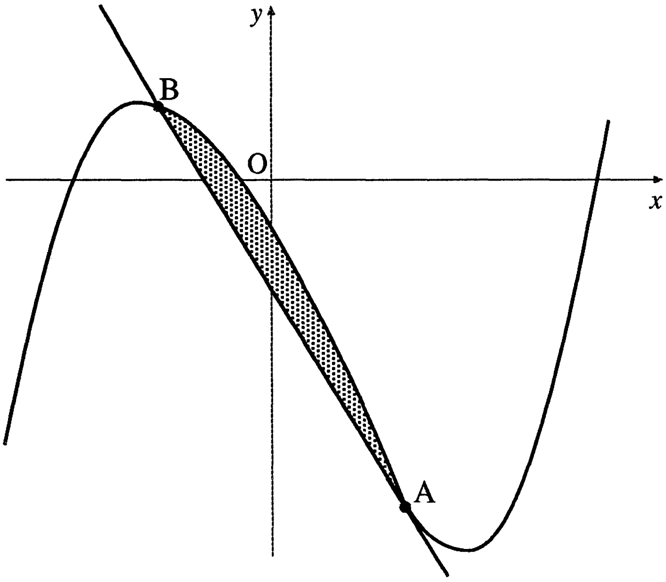
$\bullet^6$ $x - 56.3 = 82.0,\ 278.0$

$\bullet^7$ $x = 138.3,\ 334.3$

In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, – 8).

(*a*) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(*b*) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.2
(*a*)	8	2.1					5	3	2.1.8, 1.1.7, 1.3.9	Source **1996 Paper 2 Qu.8**
(*b*)	3	2.2						3	2.2.7	

(*a*)

•1 *strat:* $\frac{dy}{dx} = \ldots$

•2 $\frac{dy}{dx} = 3x^2 - 2x - 6$

•3 $m_{tgt} = -5$

•4 $y + 8 = -5(x - 1)$

•5 *strat:* attempt to simplify and equate *y*'s

•6 $x^3 - x^2 - x + 1 = 0$

•7 *strat:* *e.g.* try to factorise

•8 $B = (-1, 2)$

(*b*)

•9 $\int \left(x^3 - x^2 - 6x - 2\right) - (-5x - 3)\ dx$

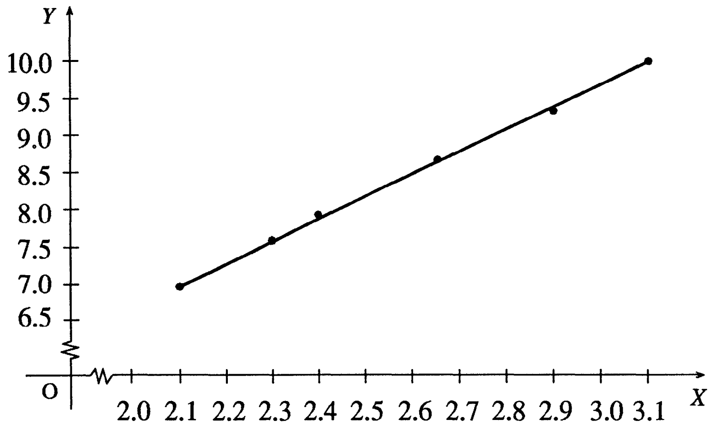
•10 $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x\right]$

•11 $1\frac{1}{3}$

Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

X (=$\log_e x$)	Y (=$\log_e y$)
2.10	7.00
2.31	7.60
2.40	7.92
2.65	8.70
2.90	9.38
3.10	10.00



A graph was drawn and is shown above.

(a) Find the equation of the line in the form $Y = mX + c$. **(3)**

(b) Hence find the values of the constants a and b in the relationship $y = ax^b$. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.3
(a)	3	1.1			3				1.1.7		Source **1996 Paper 2 Qu.9**
(b)	4	3.3				4			3.3.6		

(a) •[1] *e.g.* $m = 3$

•[2] *e.g.* $8.70 = 3 \times 2.65 + c$ *or equiv.*

•[3] *e.g.* $Y = 3X + 0.75$

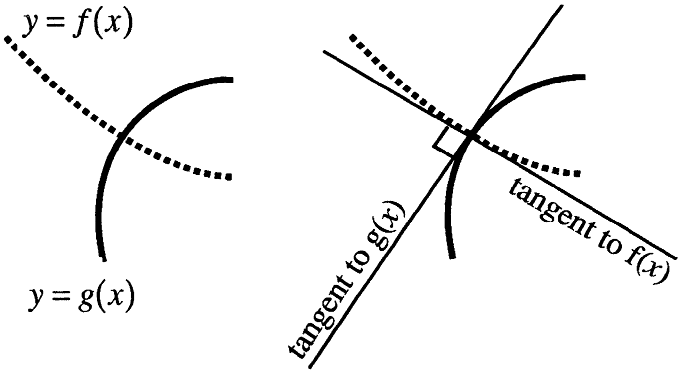
(b) •[4] $\ln y = 3\ln x + 0{\cdot}7$

•[5] $\ln y = \ln 2.01x^3$

•[6] $b = 3$

•[7] $a = 2.01$

Two curves, $y = f(x)$ and $y = g(x)$, are called orthogonal if, at each point of intersection, their tangents are at right angles to each other.



(*a*) Diagram 1 shows the parabola with equation $y = 6 + \frac{1}{9}x^2$ and the circle M with equation $x^2 + (y-5)^2 = 13$.
These two curves intersect at (3, 7) and (–3, 7).
Prove that these curves are orthogonal. **(6)**

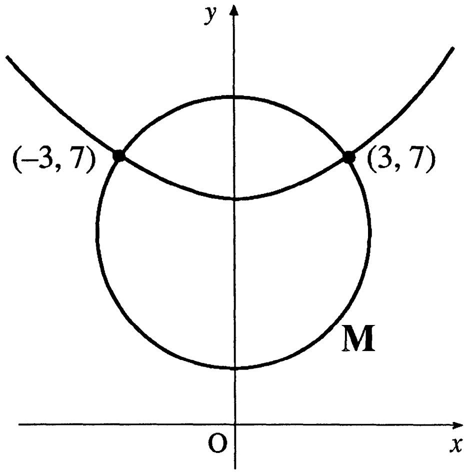


Diagram 1

(*b*) Diagram 2 shows the circle M, from (*a*) above, which is orthogonal to the circle N. The circles intersect at (3, 7) and (–3, 7).

(i) Write down the equation of the tangent to circle M at the point (–3, 7). **(1)**

(ii) Hence find the equation of circle N. **(3)**

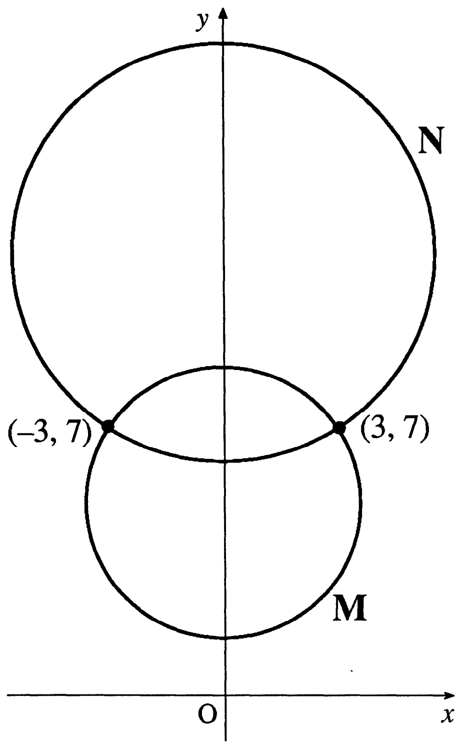


Diagram 2

1996 Paper 2 Qu.10

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.4
(*a*)	6	2.4					3	3	2.4.1, 1.1.9, 1.3.9	Source 1996 Paper 2 Qu.10
(*b*)	4	2.4						4	2.4.4	

(a)

•1 $\frac{dy}{dx} = \frac{2}{9}x$

choose 1 point e.g. (3,7)

•2 *parabola*: $m_{tgt\ x=3} = \frac{2}{3}$

•3 circle centre $= (0,5)$

•4 *circle*: $m_{rad\ x=3} = \frac{2}{3}$, $m_{tgt\ x=3} = -\frac{3}{2}$

•5 $\left(m_{tgt(P)\ x=3}\right) \times \left(m_{tgt(C)\ x=3}\right) = -1$

so "curves orthogonal"

•6 deal totally with other point

(b)

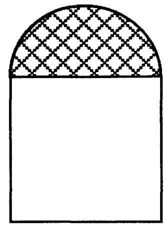
•7 $y - 7 = \frac{3}{2}(x+3)$

•8 circle centre $= \left(0, 11\frac{1}{2}\right)$

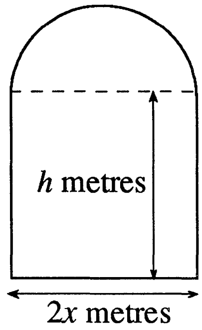
•9 $r^2 = \frac{117}{4}$

•10 $x^2 + \left(y - 11\frac{1}{2}\right)^2 = \frac{117}{4}$

A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.
The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.



The rectangle measures $2x$ metres by h metres.



(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **(2)**

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**

(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(*a*)	4	0.1					1	3	0.1		Source
(*b*)	5	1.3					2	3	1.3.15		1996 Paper 2 Qu.11

(*a*)
- •[1] eg $2h + 2x + \text{semicircle} = 10$
- •[2] $h = \frac{1}{2}(10 - \pi x - 2x)$
- •[3] $L = 2 \times 2xh + \frac{1}{2}\pi x^2$
- •[4] $L = 4x \times \frac{1}{2}(10 - \pi x - 2x) + \frac{1}{2}\pi x^2$

$L = 20x - 4x^2 - \frac{3}{2}\pi x^2$

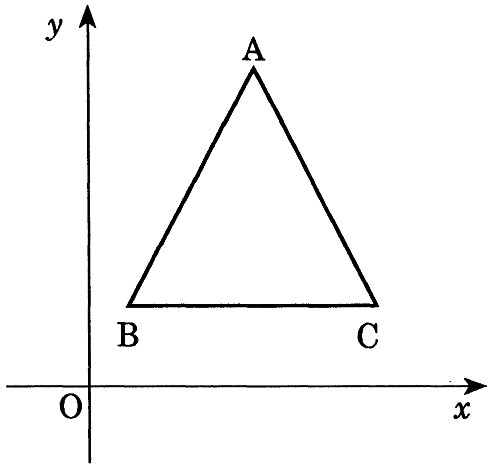
(*b*)
- •[5] $L' = 20 - 8x - 3\pi x$
- •[6] $L' = 0$
- •[7] $x = \dfrac{20}{3\pi + 8} = x_0 \ (= 1.148)$
- •[8]

x	x_0^-	x_0	x_0^+
L'	+	0	−

maximum at x_0

- •[9] $h = \dfrac{5\pi + 20}{3\pi + 8} \ (= 2.049)$

A triangle ABC has vertices A(4, 8), B(1, 2) and C(7, 2).



(*a*) Show that the triangle is isosceles. **(2)**

(*b*) (i) The altitudes AD and BE intersect at H, where D and E lie on BC and CA respectively. Find the coordinates of H. **(7)**

(ii) Hence show that H lies one quarter of the way up DA. **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	1.1
(*a*)	2	1.1					2		1.1.2	Source **1995 Paper 2 Qu.1**
(*b*)	8	1.1					8		1.1.10, 0.1	

(*a*) •1 Calculate the length of the sides

•2 $AB = AC = \sqrt{3^2 + 6^2}$

(*b*) •3 knows to find equ. of an altitude

•4 $m_{AC} = -2$

•5 $m_{BE} = \frac{1}{2}$

•6 $y - 2 = \frac{1}{2}(x - 1)$

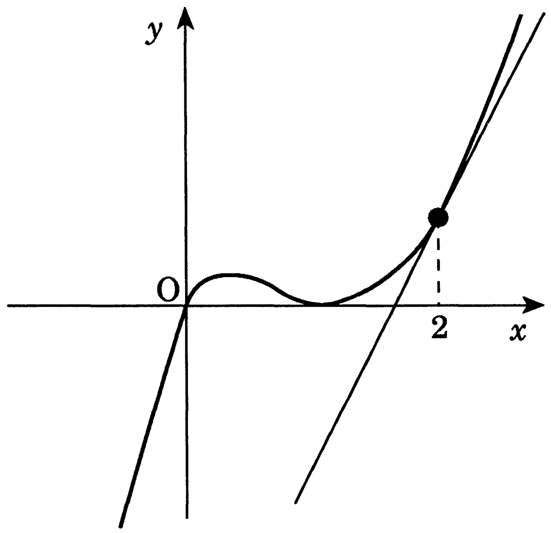
•7 $x = 4$ stated or implied

•8 knows how to find intersection

•9 $H = \left(4, \frac{7}{2}\right)$

•10 $DA = 6$ and $DH = 1\frac{1}{2}$

The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.



(*a*) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

(*b*) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	4	1.3	4						1.3.9,	1.1.7	Source
(*b*)	5	2.1	5						2.1.2,	2.1.8	**1995 Paper 2 Qu.2**

(*a*)
- •1 $\frac{dy}{dx} = \ldots\ldots\ldots$
- •2 $3x^2 - 4x + 1$
- •3 $m_{x=2} = 5$
- •4 $y - 2 = 5(x - 2)$

(*b*)
- •5 equate 'y's
- •6 $x^3 - 2x^2 - 4x + 8 = 0$
- •7 e.g. synthetic division
- •8 the appearance of:
  - $x^2 - 4$
  - **or** $x^2 - 4x + 4$
  - **or** ± 2
  - **or** $-2, 2, 2$
- •9 $x = -2,\ y = -18$

Trees are sprayed weekly with the pesticide, KILLPEST, whose manufacturers claim it will destroy 65% of all pests. Between the weekly sprayings it is estimated that 500 new pests invade the trees.

A new pesticide, PESTKILL, comes onto the market. The manufacturers claim that it will destroy 85% of existing pests but it is estimated that 650 new pests per week will invade the trees.

Which pesticide will be more effective in the long term ? **(7)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	1.4
			C	A/B	C	A/B	C	A/B	Main Additional	
-	7	1.4			7				1.4.3, 1.4.4, 1.4.5	Source **1995 Paper 2 Qu.3**

(-) •[1] $0\cdot35$ stated or implied

•[2] $0\cdot35u_n + 500$

•[3] $0\cdot15$ stated or implied

•[4] $0\cdot15u_n + 650$

•[5] $l = al + b$......... *or* limit $= \frac{b}{1-a}$.........

•[6] limits $= 769$ and 765

•[7] Limits are valid since $|a| < 1$ in both cases
and Pestkill is more effective

(*a*) (i) Diagram 1 shows part of the graph of the function f defined by $f(x) = b\sin ax^\circ$, where a and b are constants.
Write down the values of a and b.

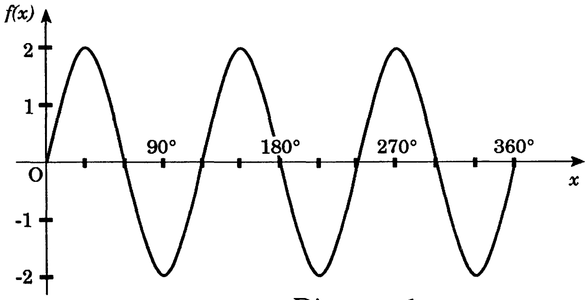


Diagram 1

(ii) Diagram 2 shows part of the graph of the function g defined by $g(x) = d\cos cx^\circ$, where c and d are constants.
Write down the values of c and d.

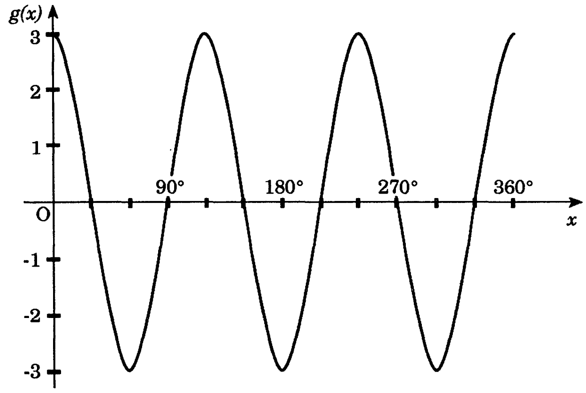


Diagram 2

(4)

(*b*) The function h is defined by $h(x) = f(x) + g(x)$.
Show that $h(x)$ can be expressed in terms of a single trigonometric function of the form $q\sin(px + r)^\circ$ and find the values of p, q and r. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.4
(*a*)	4	2.3			4				2.3.2		Source **1995 Paper 2 Qu.4**
(*b*)	5	3.4			5				3.4.1		

(*a*) •[1] $a = 3$

•[2] $b = 2$

•[3] $c = 3$

•[4] $d = 3$

(*b*) •[5] $p = 3$

•[6] $q\sin(px + r)^\circ$

$= q\sin px^\circ \cos r^\circ + q\cos px^\circ \sin r^\circ$

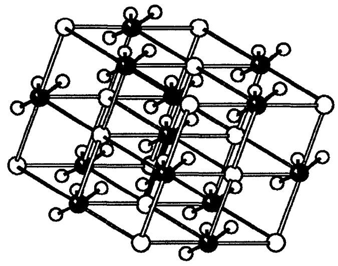
•[7] $q = \sqrt{13}$

•[8] $q\cos r^\circ = 2,\ q\sin r^\circ = 3$

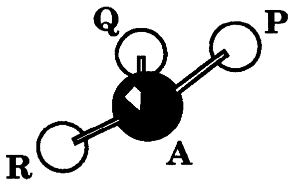
or $\tan r^\circ = \frac{3}{2}$

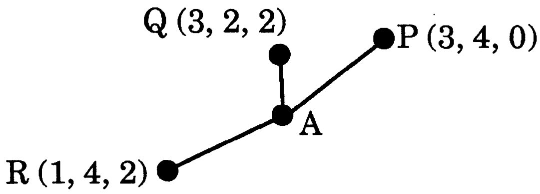
•[9] $r = 56 \cdot 3$

The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown below.





(*a*) Calculate the size of angle PQR. **(4)**

(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. **(6)**

(*c*) The coordinates of A are (2, 3, 1).

(i) Show that P, Q and R are also equidistant from A

(ii) Explain why T, and not A, is the centre of the circle through P, Q and R. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	4	3.1					4		3.1.11		Source
(*b*)	6	3.1					6		3.1.6,	3.1.3	**1995 Paper 2**
(*c*)	2	3.1					1	1	3.1.3,	0.1	**Qu.5**

(*a*)

•1 $PQ = \sqrt{8},\ RQ = \sqrt{8},$

•2 Use s.p.: $\overrightarrow{PQ}.\overrightarrow{RQ} = |\overrightarrow{PQ}|.|\overrightarrow{RQ}|\cos\theta$

•3 $\begin{pmatrix}0\\-2\\2\end{pmatrix}.\begin{pmatrix}2\\-2\\0\end{pmatrix} = 4$

•4 60°

(*b*)

•5 $M = (2,3,2)$

•6 $\overrightarrow{PT} = \frac{2}{3}\overrightarrow{PM}$ or equivalent

•7 $\overrightarrow{PT} = \frac{2}{3}\begin{pmatrix}-1\\-1\\2\end{pmatrix}$ or equiv.

•8 $T = \left(\frac{7}{3}, \frac{10}{3}, \frac{4}{3}\right)$

•9 $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ stated or implied

•10 $PT = 2\sqrt{\frac{2}{3}}, QT = 2\sqrt{\frac{2}{3}}, RT = 2\sqrt{\frac{2}{3}}$ or equivalent

(*c*)

•11 $PA = QA = RA = \sqrt{3}$

•12 A is in a different plane

A system of 3 equations in 3 unknowns can be solved by a method known as Gaussian Elimination as shown below.

Example

Solve the system of equations by Gaussian Elimination.

$$\begin{aligned} x + 2y - 3z &= 11 \\ 2x + 2y - z &= 11 \\ 3x - 2y + 4z &= -4 \end{aligned}$$

A Write out the coefficients in an array:

• Row 1	1	2	–3	11
• Row 2	2	2	–1	11
• Row 3	3	–2	4	–4

B Keep Row 1 the same. Make Row 2 and Row 3 each begin with a zero by subtracting multiples of Row 1 from them.

• Row 1 is kept the same	1	2	–3	11
• Row 2 becomes 'Row 2 – 2 x Row 1'	**0**	**–2**	**5**	**–11**
• Row 3 becomes 'Row 3 – 3 x Row 1'	**0**	**–8**	**13**	**–37**

C Keep Row 1 and Row 2 the same. Make Row 3 begin with two zeros, by subtracting a multiple of Row 2 from it.

• Row 1 is kept the same	1	2	–3	11	.........(1)
• Row 2 is kept the same	0	–2	5	–11	.........(2)
• Row 3 becomes 'Row 3 – 4 x Row 2'	**0**	**0**	**–7**	**7**	.........(3)

D

- Line (3) gives $-7z = 7, \quad z = -1$
- Line (2) gives $-2y + 5z = -11$
  $-2y + (-5) = -11, \quad y = 3$
- Line (1) gives $x + 2y - 3z = 11$
  $x + 6 + 3 = 11, \quad x = 2$

So the solution is $x = 2, y = 3, z = -1$

Solve the following system of equations by Gaussian Elimination **as shown above.**

$$\begin{aligned} x - 2y + z &= 6 \\ 3x + y - z &= 7 \\ 4x - y + 2z &= 15 \end{aligned}$$

(7)

1995 Paper 2 Qu.6

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	4
-	7	0.1					7		0.1		Source **1995 Paper 2 Qu.6**

(-)

•1
$$\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 3 & 1 & -1 & 7 \\ 4 & -1 & 2 & 15 \end{array}$$

•2
$$\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 7 & -4 & -11 \end{array}$$

•3
$$\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 7 & -4 & -11 \\ 0 & 7 & -2 & -9 \end{array}$$

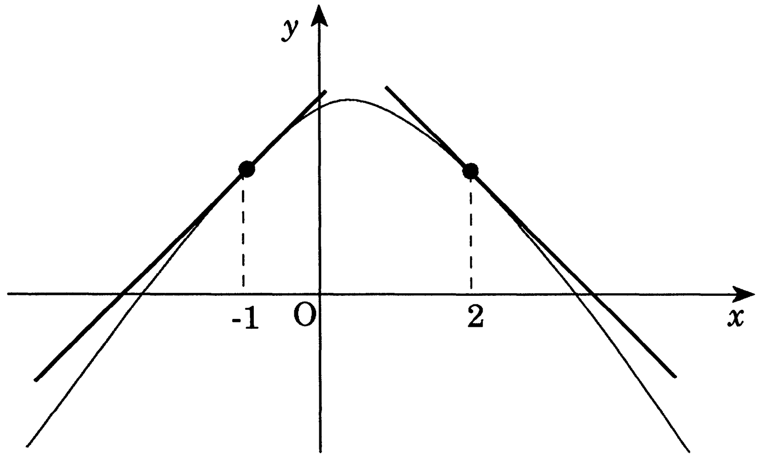
•4
$$\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 7 & -4 & -11 \\ 0 & 0 & 2 & 2 \end{array}$$

•5 $2z = 2,\ z = 1$

•6 $7y - 4z = -11,\ y = -1$

•7 $x - 2y + z = 6,\quad x = 3$

The parabola $y = ax^2 + bx + c$ crosses the y-axis at (0, 3) and has two tangents drawn, as shown in the diagram.



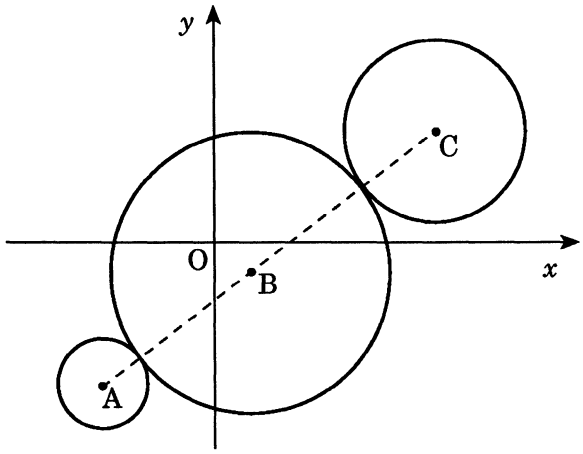
The tangent at $x = -1$ makes an angle of 45° with the positive direction of the x-axis and the tangent at $x = 2$ makes an angle of 135° with the positive direction of the x-axis.
Find the values of a, b and c. **(8)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Content Reference : Additional	1.3
-	8	1.3	2	6					1.1.3,	1.3.7, 0.1	Source **1995 Paper 2 Qu.7**

(-)

- •[1] $c = 3$
- •[2] $2ax + b$
- •[3] $m = \tan 45° = 1$
- •[4] $-2a + b = 1$
- •[5] $m = \tan 135° = -1$
- •[6] $4a + b = -1$
- •[7] method for solving pr. of equ
- •[8] $a = -\frac{1}{3}$, $b = \frac{1}{3}$

When newspapers were printed by lithograph, the newsprint had to run over three rollers, illustrated in the diagram by three circles. The centres A, B and C of the three circles are collinear.



The equations of the circumferences of the outer circles are

$(x+12)^2+(y+15)^2=25$ and $(x-24)^2+(y-12)^2=100$.

Find the equation of the central circle. **(8)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
-	8	3.1					8		2.4.1,	2.4.3, 3.1.6	Source **1995 Paper 2 Qu.8**

(-)

- $\bullet^1$ $(-12,-15)$ and $(24,12)$
- $\bullet^2$ radii are 5 and 10
- $\bullet^3$ AC = 45
- $\bullet^4$ radius = 15
- $\bullet^5$ B divides AC in ratio 4:5
- $\bullet^6$ $\vec{OB}=\frac{1}{9}\left[4\vec{OC}+5\vec{OA}\right]$ stated or implied
- $\bullet^7$ $\vec{OB}=\frac{1}{9}\left[4\begin{pmatrix}24\\12\end{pmatrix}+5\begin{pmatrix}-12\\-15\end{pmatrix}\right]$
- $\bullet^8$ $(x-4)^2+(y+3)^2=15^2$

(*a*) By writing $\sin 3x$ as $\sin(2x + x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$. **(4)**

(*b*) Hence find $\int \sin^3 x \, dx$. **(4)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	3.2
			C	A/B	C	A/B	C	A/B	Main Additional	Source
(*a*)	4	2.3	2	2					2.3.2, 2.3.3	**1995 Paper 2 Qu.9**
(*b*)	4	3.2		4					3.2.4	

(*a*)

- •[1] $\sin 2x \cos x + \cos 2x \sin x$
- •[2] $2\sin x \cos x \cos x + \ldots\ldots\ldots$
- •[3] $\ldots\ldots\ldots + \left(1 - 2\sin^2 x\right)\sin x$
- •[4] $2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

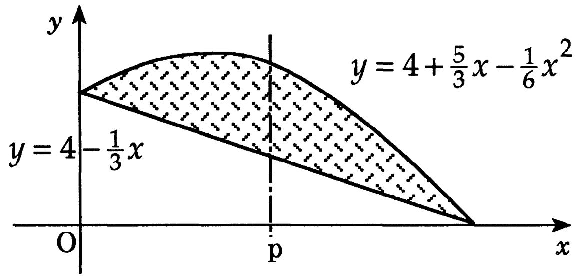
(*b*)

- •[5] $\int \frac{1}{4}(3\sin x - \sin 3x) \; dx$
- •[6] $-3\cos x$
- •[7] $+\cos 3x$
- •[8] $\div 3$

When building a road beside a vertical rockface, engineers often use wire mesh to cover the rockface. This helps to prevent rocks and debris from falling onto the road. The shaded region of the diagram below represents a part of such a rockface.

This shaded region is bounded by a parabola and a straight line.

The equation of the parabola is $y = 4 + \frac{5}{3}x - \frac{1}{6}x^2$ and the equation of the line is $y = 4 - \frac{1}{3}x$.



(*a*) Find algebraically the area of wire mesh required for this part of the rockface. **(5)**

(*b*) To help secure the wire mesh, weights are attached to the mesh along the line $x = p$ so that the area of mesh is bisected.
By using your answer to part (*a*), or otherwise, show that

$$p^3 - 18p^2 + 432 = 0.$$ **(3)**

(*c*) (i) Verify that $p = 6$ is a solution of this equation.
(ii) Find algebraically the other two solutions of this equation.
(iii) Explain why $p = 6$ is the only valid solution to this problem. **(5)**

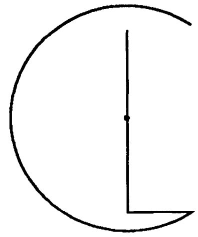
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.2
(*a*)	5	2.2	5						2.2.7		Source
(*b*)	3	2.2		3					2.2.5		**1995 Paper 2**
(*c*)	5	2.1	2	3					2.1.3		**Qu.10**

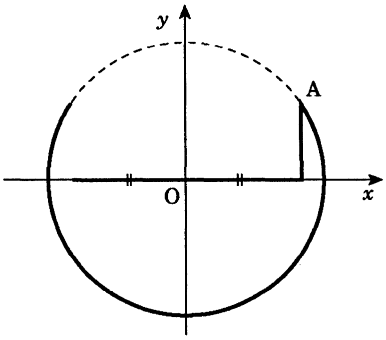
(*a*)
- •1 Area under curve - area under line
- •2 abscissae at intersection are 0 **and** 12
- •3 $\int_0^{12} \left(4 + \frac{5}{3}x - \frac{1}{6}x^2 - \left(4 - \frac{1}{3}x\right)\right) dx$
- •4 $\left[x^2 - \frac{1}{18}x^3\right]_0^{12}$ or equivalent
- •5 48

(*b*)
- •6 $\int \ldots\ldots\ldots \; dx = 24$
- •7 $\int_0^p \left(2x - \frac{1}{6}x^2\right) dx = 24$ or equivalent statement
- •8 $p^2 - \frac{1}{18}p^3 = 24$

(*c*)
- •9 "$f(6) = 0$" or equivalent
- •10 divide by $(p - 6)$
- •11 $p^2 - 12p - 72$
- •12 $p = 6 \pm \sqrt{108}$ or equivalent
- •13 outside range 0 – 12

Linktown Church is considering designs for a logo for their Parish magazine. The 'C' is part of a circle and the centre of the circle is the mid-point of the vertical arm of the 'L'. Since the 'L' is clearly smaller than the 'C', the designer wishes to ensure that the total length of the arms of the 'L' is as long as possible.





The designer decides to call the point where the 'L' and 'C' meet A and chooses to draw co-ordinate axes so that A is in the first quadrant. With axes as shown, the equation of the circle is $x^2 + y^2 = 20$.

(a) If A has co-ordinates (x,y), show that the total length T of the arms of the 'L' is given by $T = 2x + \sqrt{20 - x^2}$. **(1)**

(*b*) Show that for a stationary value of T, x satisfies the equation

$$x = 2\sqrt{20 - x^2}.$$ **(5)**

(*c*) By squaring both sides, solve this equation. Hence find the greatest length of the arms of the 'L'. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.2
(*a*)	1	0.1						1	0.1		Source
(*b*)	5	3.2					1	4	3.2.2		**1995 Paper 2**
(*c*)	3	1.3					1	2	1.3.15		**Qu.11**

(*a*) •1 $T = x + x + y$ **and** $y^2 = 20 - x^2$

(*b*) •2 appearance of $\frac{dT}{dx} = 2 + \ldots\ldots$

•3 $\frac{1}{2}\left(20 - x^2\right)^{-\frac{1}{2}}$

•4 $\times -2x$

•5 $\frac{dT}{dx} = 0$ stated or implied

•6 completing proof

(*c*) •7 $x^2 = 4\left(20 - x^2\right)$

•8 $x = 4$ (accept $x = \pm 4$)

•9 justifying $x = 4$ gives $T_{max} = 10$

The graph of the curve with equation $y = 2x^3 + x^2 - 13x + a$ crosses the x-axis at the point (2,0).

(*a*) Find the value of a and hence write down the coordinates of the point at which this curve crosses the y-axis. **(3)**

(*b*) Find algebraically the coordinates of the other points at which the curve crosses the x-axis. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
											Source
(*a*)	3	2.1	3						2.1.3		1994 Paper 2
(*b*)	4	2.1	4						2.1.3		Qu.1

(*a*) •[1] strategy

eg

$$\begin{array}{c|cccc} 2 & 2 & 1 & -13 & a \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

or $f(2) = 0 = 16 + 4 - 26 + a$

•[2] $a = 6$

•[3] $(0,6)$

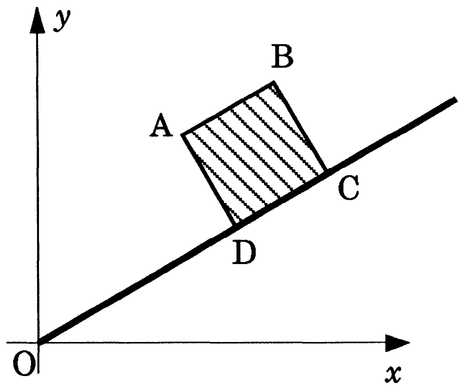
(*b*) •[4] $2x^2 + 5x - 3$

•[5] $(x+3)(2x-1)$

•[6] $x = -3,\ \frac{1}{2}$

•[7] $(-3,0),\ \left(\frac{1}{2},0\right)$

ABCD is a square. A is the point with coordinates (3,4) and ODC has equation $y = \frac{1}{2}x$.



(a) Find the equation of the line AD. **(3)**

(b) Find the coordinates of D. **(3)**

(c) Find the area of the square ABCD. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.1
(a)	3	1.1					3		1.1.9,	1.1.7	Source
(b)	3	0.1					3		0.1		**1994 Paper 2**
(c)	2	1.1					2		1.1.2		**Qu.2**

(a) $\bullet^1$ using $m_1 m_2 = -1$

$\bullet^2$ $m_{AD} = -2$

$\bullet^3$ $y - 4 = -2(x - 3)$

(b) $\bullet^4$ strategy for sim. equations

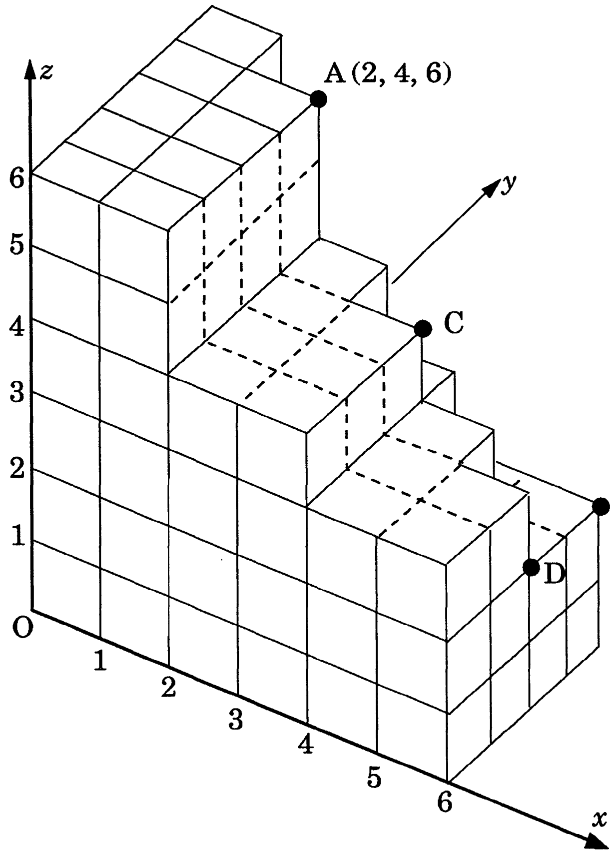
$\bullet^5$ $2x + y = 10$ or equiv

$\bullet^6$ (4,2)

(c) $\bullet^7$ strategy : find length of AD

$\bullet^8$ 5

With coordinate axes as shown, the point A is (2,4,6).



(*a*) Write down the coordinates of B,C and D. **(3)**

(*b*) Show that C is the midpoint of AD. **(1)**

(*c*) By using the components of the vectors $\vec{OA}$ and $\vec{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

(*d*) Hence calculate the size of angle OAB. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
(*a*)	3	3.1			3				3.1.1		Source
(*b*)	1	3.1			1				3.1.6		**1994 Paper 2**
(*c*)	4	3.1			4				3.1.11		**Qu.3**
(*d*)	2	0.1			2				0.1		

(*a*) •1 One of B,C or D

•2 Remaining two of B, C and D

•3 B (6,4,2), C (4,3,4), D (6,2,2)

(*b*) •4 $\left(\frac{2+6}{2}, \frac{4+2}{2}, \frac{6+2}{2}\right)$

(*c*) •5 $\cos A\hat{O}B = \frac{\vec{OA}.\vec{OB}}{|\vec{OA}||\vec{OB}|}$ *or* $\frac{OA^2 + OB^2 - AB^2}{2 \times OA \times OB}$ or equivalents

•6 $\vec{OA}.\vec{OB} = 40$ *or* $AB^2 = 32$

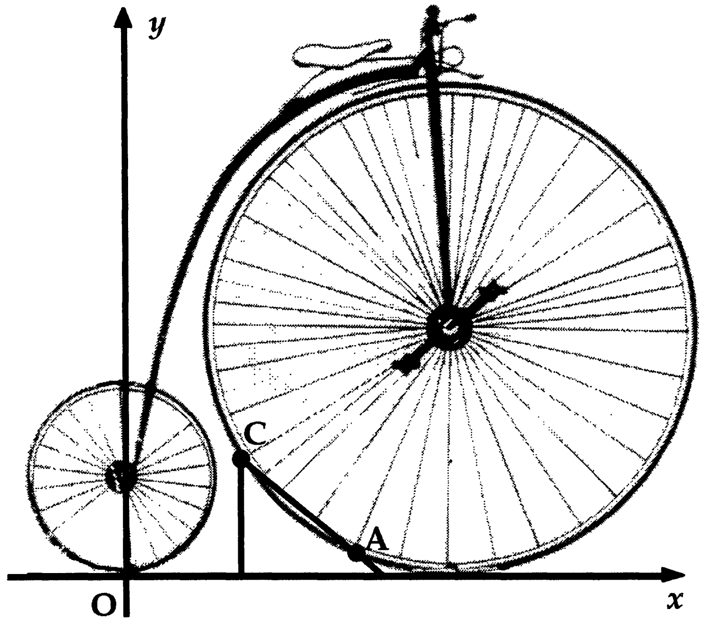
•7 $OA = \sqrt{56} = OB$

•8 44°

(*d*) •9 strategy: e.g. use isosceles Δ

•10 68°

A penny-farthing bicycle on display in a museum is supported by a stand at points A and C. A and C lie on the front wheel.
With coordinate axes as shown and 1 unit = 5cm, the equation of the rear wheel (the small wheel) is
$x^2 + y^2 - 6y = 0$ and
the equation of the front wheel is
$x^2 + y^2 - 28x - 20y + 196 = 0$.



(*a*) (i) Find the distance between the centres of the two wheels.
(ii) Hence calculate the clearance, i.e. the smallest gap, between the front and rear wheels. Give your answer to the nearest millimetre. **(8)**

(*b*) B(7,3) is half-way between A and C, and P is the centre of the front wheel.
(i) Find the gradient of PB.
(ii) Hence find the equation of AC and the coordinates of A and C. **(8)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.4
											Source
(*a*)	8	2.4			8				2.4.2,	1.1.2	**1994 Paper 2**
(*b*)	8	1.1			8				1.1.1,	1.1.9, 2.4.4	**Qu.4**

(*a*)
- •[1] centre (0, 3)
- •[2] centre (14, 10)
- •[3] distance between centres = √245
- •[4] radius = 3
- •[5] radius = 10
- •[6] strategy (clearance = distance between centres minus sum of radii)
- •[7] √245 – 13
- •[8] 133 mm or equivalent

(*b*)
- •[9] $m_{PB} = 1$
- •[10] $m_{AC} = -1$
- •[11] $y - 3 = -(x - 7)$ for AC
- •[12] strategy: substitute
- •[13] substituting correctly
- •[14] *eg* $2x^2 - 28x + 96 = 0$
- •[15] $x = 6,\ 8$ (*or* $y = 2, 4$)
- •[16] $(6,4)$ **and** $(8,2)$

(*a*) Express $3\sin x^\circ - \cos x^\circ$ in the form $k\sin(x-\alpha)^\circ$, where $k > 0$ and $0 \le \alpha \le 90\cdot$ **(4)**

(*b*) Hence find algebraically the values of x between 0 and 180 for which $3\sin x^\circ - \cos x^\circ = \sqrt{5}$. **(4)**

(*c*) Find the range of values of x between 0 and 180 for which $3\sin x^\circ - \cos x^\circ \le \sqrt{5}$. **(2)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	3.4
			C	A/B	C	A/B	C	A/B	Main Additional	Source
(*a*)	4	3.4			4				3.4.1	
(*b*)	4	3.4			4				3.4.2	1994 Paper 2
(*c*)	2	3.4				2			3.4.2	Qu.5

(*a*)

•1 $k(\sin x\cos\alpha - \cos x\sin\alpha)$ or equivalent

•2 $k\cos\alpha = 3$ **and** $k\sin\alpha = 1$

•3 $k = \sqrt{10}$

•4 $\alpha = 18\cdot 4$

(*b*)

•5 $\sqrt{10}\sin(x - 18\cdot 4)^\circ = \sqrt{5}$

•6 $\sin(x - 18\cdot 4)^\circ = \frac{1}{\sqrt{2}}$ or equivalent

•7 $63\cdot 4$

•8 $153\cdot 4$

(*c*)

•9 strategy stated or implied

•10 $x \le 63.4$ **and** $x \ge 153.4$

EXAMPLE

(i) Let $f(x) = x^3 + 5x - 1$.
Since $f(0) = -1$ and $f(1) = 5$
the equation $f(x) = 0$ has a root in the interval $0 < x < 1$ because $f(0) < 0$ and $f(1) > 0$.

(ii) To find this root, the equation $x^3 + 5x - 1 = 0$ can be rearranged as follows :

$$x^3 + 5x - 1 = 0$$
$$x^3 + 5x = 1$$
$$x\left(x^2 + 5\right) = 1$$
$$x = \frac{1}{x^2 + 5}$$

We can write this result as a recurrence relation

$$x_{n+1} = \frac{1}{x_n^2 + 5}$$

and use it to find this root. In this example we will work to 3 decimal places and can therefore give the final answer to 2 decimal places.

(iii) For our first estimate, x_1, we use the mid-point of the interval $0 < x < 1$ [from part (i)].

$$x_1 = 0 \cdot 5, \quad x_2 = \frac{1}{0 \cdot 5^2 + 5} = 0 \cdot 190$$
$$x_2 = 0 \cdot 190 \quad x_3 = \frac{1}{0 \cdot 190^2 + 5} = 0 \cdot 199$$
$$x_3 = 0 \cdot 199 \quad x_4 = \frac{1}{0 \cdot 199^2 + 5} = 0 \cdot 198$$
$$x_4 = 0 \cdot 198 \quad x_5 = \frac{1}{0 \cdot 198^2 + 5} = 0 \cdot 198$$

Hence, correct to 2 decimal places, the root is $x = 0{\cdot}20$.

(*a*) Show that the equation $2x^3 + 3x - 1 = 0$ has a root in the interval $0 < x < 0{\cdot}5$. **(2)**

(*b*) By using the technique described above find this root correct to 2 decimal places. **(6)**

1994 Paper 2 Qu.6

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	4
											Source
(a)	2	0.1			2				0.1		**1994 Paper 2**
(b)	6	0.1			6				0.1		**Qu.6**

(a) •1 $f(0) = -1$ **and** $f(0\cdot5) = 0\cdot75$

•2 "$f(0) < 0$ and $f(0.5) > 0$" or equiv. **explicitly stated**

(b) •3 $x = \frac{1}{2x^2+3}$

•4 $x_1 = 0.25$

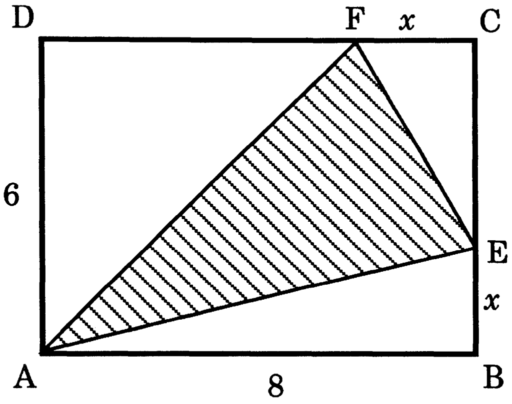
•5 $x_2 = 0.32$

•6 $x_3 = 0.312$ rounded to 3dp

•7 $x_4 = 0.313$ *and* $x_5 = 0.313$

•8 0.31 correct to 2dp

An yacht club is designing its new flag. The flag is to consist of a red triangle on a yellow rectangular background. In the yellow rectangle ABCD, AB measures 8 units and AD is 6 units. E and F lie on BC and CD, x units from B and C as shown in the diagram.



(a) Show that the area, H square units, of the red triangle AEF is given by $H(x) = 24 - 4x + \frac{1}{2}x^2$. **(4)**

(b) Hence find the greatest and least possible values of the area of triangle AEF. **(8)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	1.3
			C	A/B	C	A/B	C	A/B	Main Additional	Source
(a)	4	0.1	4						0.1	**1994 Paper 2**
(b)	8	1.3	3	5					1.3.15	**Qu.7**

(a)
- $\bullet^1$ rectangle minus 3 triangles
- $\bullet^2$ area of Δ's ADF and ABE
- $\bullet^3$ area of Δ FCE
- $\bullet^4$ 3 triangles : $24 + 4x - \frac{1}{2}x^2$ **or** $48 - 4x - 3x + \frac{1}{2}x^2 - 24 + 3x$

(b)
- $\bullet^5$ $H'(x) = \ldots\ldots$
- $\bullet^6$ $x - 4$
- $\bullet^7$ put $H'(x) = 0$ **stated explicitly**
- $\bullet^8$ $x = 4$ and $H = 16$
- $\bullet^9$ justify minimum
- $\bullet^{10}$ consider $x = 0$ **and** $x = 6$
- $\bullet^{11}$ $H(0) = 24$, **and** $H(6) = 18$
- $\bullet^{12}$ communication re greatest and least.

(*a*) $f(x) = 4x^2 - 3x + 5$.

Show that $f(x + 1)$ simplifies to $4x^2 + 5x + 6$ and find a similar expression for $f(x - 1)$.

Hence show that $\dfrac{f(x+1) - f(x-1)}{2}$ simplifies to $8x - 3$. **(5)**

(*b*) $g(x) = 2x^2 + 7x - 8$.

Find a similar expression for $\dfrac{g(x+1) - g(x-1)}{2}$. **(4)**

(*c*) By examining your answers for (*a*) and (*b*), **write down** the simplified expression for $\dfrac{h(x+1) - h(x-1)}{2}$, where $h(x) = 3x^2 + 5x - 1$. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	4
(*a*)	5	0.1					5		0.1		Source
(*b*)	4	0.1					4		0.1		**1994 Paper 2**
(*c*)	2	0.1					2		0.1		**Qu.8**

(*a*) •1 $f(x+1) = 4(x+1)^2 - 3(x+1) + 5$

•2 $4x^2 + 8x + 4 - 3x - 3 + 5$

•3 $f(x-1) = 4(x-1)^2 - 3(x-1) + 5$

•4 $f(x-1) = 4x^2 - 11x + 12$

•5 $\frac{16x-6}{2}$

(*b*) •6 $g(x+1) = 2(x+1)^2 + 7(x+1)$ *and* $g(x-1) = 2(x-1)^2 + 7(x-1) - 8$

•7 $g(x+1) = 2x^2 + 11x + 1$

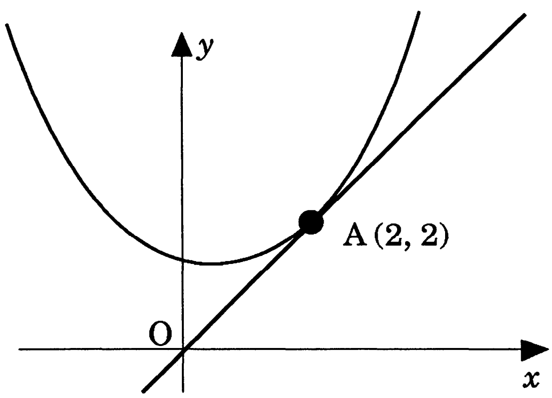
•8 $g(x-1) = 2x^2 + 3x - 13$

•9 $4x + 7$

(*c*) •10 strategy stated or implied

•11 $6x + 5$

(*a*) The point A(2, 2) lies on the parabola $y = x^2 + px + q$. Find a relationship between p and q. **(1)**



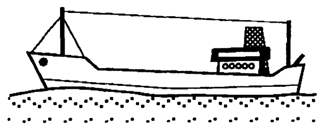
(*b*) The tangent to the parabola at A is the line $y = x$. Find the value of p. Hence find the equation of the parabola. **(6)**

(*c*) Using your answers for p and q, find the value of the discriminant of $x^2 + px + q = 0$. What feature of the above sketch is confirmed by this value ? **(2)**

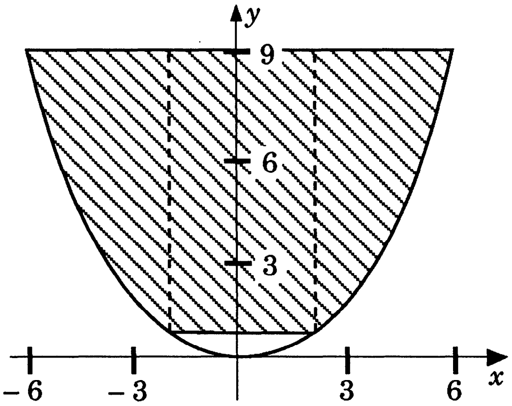
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(*a*)	1	0.1					1		0.1		Source
(*b*)	6	1.3					2	4	1.3.7,	0.1	**1994 Paper 2**
(*c*)	2	2.1						2	2.1.6		**Qu.9**

(*a*)	•1	$2p + q = -2$
(*b*)	•2	strategy
	•3	$2x + p$
	•4	gradient = 1, or equivalent
	•5	$4 + p$
	•6	$p = -3$
	•7	$q = 4$
(*c*)	•8	$\Delta = -7$
	•9	$\sqrt{-7}$ means no roots

The cargo space of a small bulk carrier is 60m long.



The shaded part of the diagram represents the uniform cross-section of this space. It is shaped like the parabola with equation $y = \frac{1}{4}x^2, \ -6 \le x \le 6,$ between the lines $y = 1$ and $y = 9$. Find the area of this cross-section and hence find the volume of cargo that this ship can carry.



(9)

part marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.2
- 9	2.2					3	6	2.2.7, 0.1	Source **1994 Paper 2 Qu.10**

(-) $\bullet^1$ strategy: split into approp. parts

$\bullet^2$ $y = 1 \Rightarrow x = \pm 2$

$\bullet^3$ first rectangular area

$\bullet^4$ $9 - \frac{1}{4}x^2$ for integrand of shaped area

$\bullet^5$ $\int_2^5 dx$ for limits of shaped area

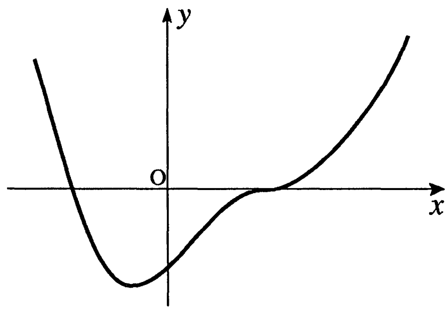
$\bullet^6$ for integrating......$\left(9x - \frac{1}{12}x^3\right)$

$\bullet^7$ for evaluating......$\left(\frac{56}{3}\right)$

$\bullet^8$ total cross – sectional area $= \frac{208}{3}(m^2)$

$\bullet^9$ volume $= 4160 \ (m^3)$

The function f, whose incomplete graph is shown in the diagram, is defined by $f(x) = x^4 - 2x^3 + 2x - 1$.
Find the coordinates of the stationary points and justify their nature. **(8)**



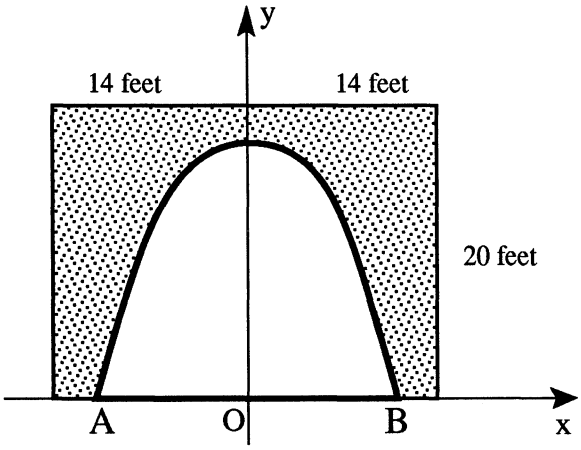
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
-	8	2.1					8		2.1.3,	1.3.12	Source 1993 Paper 2 Qu.1

- •1 for knowing to differentiate
- •2 $f'(x) = 4x^3 - 6x^2 + 2$
- •3 for putting $f'(x) = 0$
- •4 for factorising **or** checking zeros
- •5 $x = -\frac{1}{2},\ x = 1$
- •6 $y = -\frac{27}{16},\ y = 0$
- •7 completed nature table

x	$<-\frac{1}{2}$	$-\frac{1}{2}$	$>-\frac{1}{2}$	<1	1	>1
$f'(x)$	$-ve$	0	$+ve$	$+ve$	0	$+ve$
	╲	—	╱	╱	—	╱

- •8 (1,0) is pt. of inflexion, $\left(-\frac{1}{2}, -1\frac{11}{16}\right)$ is min t.p.

The concrete on the 20 feet by 28 feet rectangular facing of the entrance to an underground cavern is to be repainted.



Coordinate axes are chosen as shown in the diagram with a scale of 1 unit equal to 1 foot.The roof is in the form of a parabola with equation $y = 18 - \frac{1}{8}x^2$.

(*a*) Find the coordinates of the points A and B. **(2)**

(*b*) Calculate the total cost of repainting the facing at £3 per square foot. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	2	0.1	2						0.1		Source
(*b*)	4	2.2	4						2.2.6		**1993 Paper 2 Qu.2**

(*a*) •[1] $18 - \frac{1}{8}x^2 = 0$

•[2] $x = \pm 12$

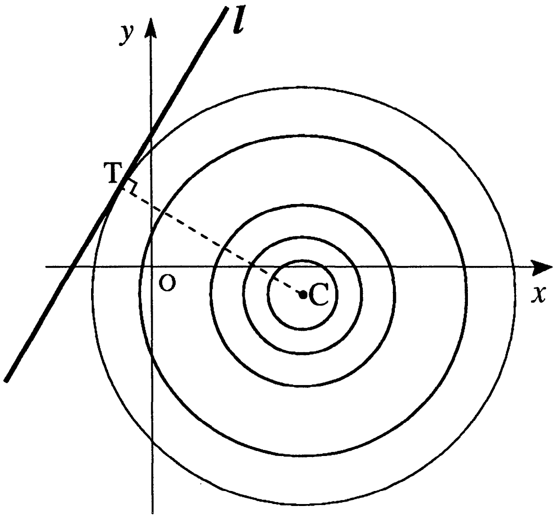
(*b*) •[3] $Area = 2\int_0^{12} y\ dx$

•[4] integrating

•[5] 288

•[6] for knowing to subtract area of parabola from area of rectangle and multiply by 3.

In an experiment with a ripple tank, a series of concentric circles with centre C(4,–1) is formed as shown in the diagram.
The line l with equation $y = 2x + 6$ represents a barrier placed in the tank.
The largest complete circle touches the barrier at the point T.



(*a*) Find the equation of the radius CT. **(3)**

(*b*) Find the equation of the largest complete circle. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.4
(*a*)	3	1.1					3		1.1.9,	1.1.7	Source
(*b*)	5	2.4					5		2.4.3		**1993 Paper 2 Qu.3**

(*a*)
- •[1] $m_l = 2$
- •[2] $m_r = -\frac{1}{2}$
- •[3] $y + 1 = -\frac{1}{2}(x - 4)$

(*b*)
- •[4] $(x-4)^2 + (y+1)^2 = r^2$
- •[5] $(x-4)^2 + (2x+7)^2 = r^2$
- •[6] $5x^2 + 20x + (65 - r^2) = 0$
- •[7] $\Delta = 400 - 4 \times 5(65 - r^2) = 0$
- •[8] $r^2 = 45$

An array of numbers such as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is called a matrix. The eigenvalues of the matrix

$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ are defined to be the roots of the equation $(a-x)(d-x)-bc=0$.

EXAMPLE

In order to find the eigenvalues of the matrix $\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$

solve $(1-x)(2-x)-4x3=0$

solution:

$$2-3x+x^2-12=0$$
$$x^2-3x-10=0$$
$$(x+2)(x-5)=0$$
$$x=-2 \text{ or } x=5$$

so the eigenvalues of **B** are –2 and 5

(*a*) Find the eigenvalues of $\mathbf{C} = \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$. **(3)**

(*b*) Find the value of *t* for which the eigenvalues of the matrix $\mathbf{D} = \begin{pmatrix} 3 & -1 \\ t & 1 \end{pmatrix}$ are equal. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	0.1
(*a*)	3	0.1					3		0.1		Source
(*b*)	5	2.1					5		2.1.7,	0.1	**1993 Paper 2 Qu.4**

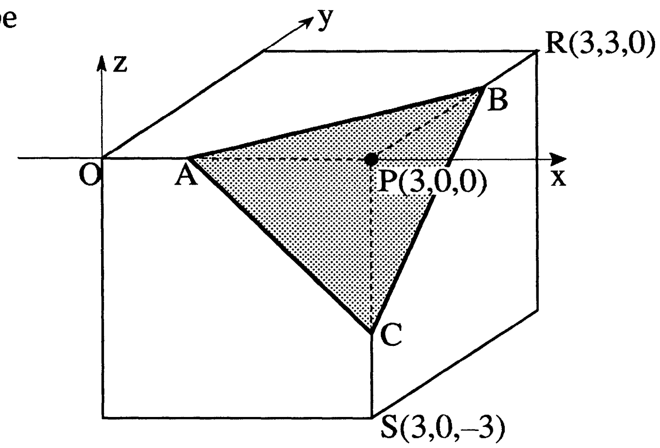
(*a*)

- •[1] $(3-x)(5-x)-2\times 4=0$
- •[2] $x^2-8x+7=0$
- •[3] eigenvalues are 1, 7

(*b*)

- •[4] $(3-x)(1-x)+t=0$
- •[5] $x^2-4x+(3+t)=0$
- •[6] $\Delta=0$ for equal roots or equiv.
- •[7] $\Delta=16-4\times 1\times(3+t)$ or equiv.
- •[8] $t=1$

A model of a crystal was made from a cube of side 3 units by slicing off the corner at P to leave a triangular face ABC. Coordinate axes have been introduced as shown in the diagram.
The point A divides OP in the ratio 1:2. Points B and C similarly divide RP and SP respectively in the ratio 1:2.



(*a*) Find the coordinates of A, B and C. **(3)**

(*b*) Calculate the area of triangle ABC. **(4)**

(*c*) Calculate the percentage increase or decrease in the surface area of the crystal compared with the cube. **(5)**

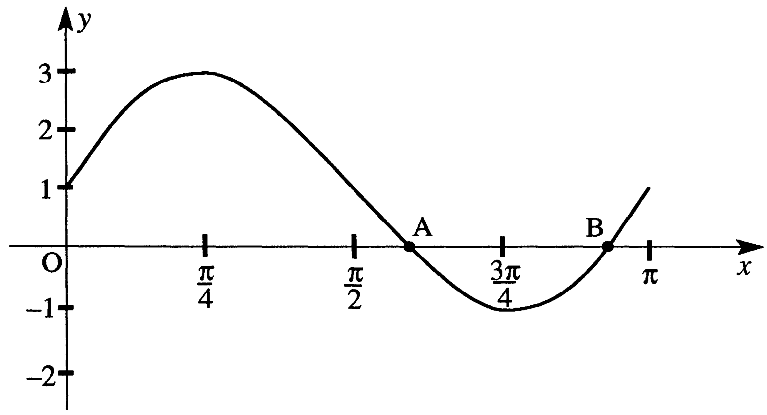
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	3	3.1			3				3.1.6		Source
(*b*)	4	3.1			4				3.1.3,	0.1	**1993 Paper 2**
(*c*)	5	0.1			5				0.1		**Qu.5**

(*a*)
- •1 $A(1,0,0)$
- •2 $B(3,2,0)$
- •3 $C(3,0,-2)$

(*b*)
- •4 strategy for area of triangle and attempt to calculate parts
- •5 60° or altitude $= \sqrt{6}$
- •6 side $= 2\sqrt{2}$
- •7 using chosen formula correctly

(*c*)
- •8 54 unit^2 for cube
- •9 know how to calculate s.a of crystal
- •10 area of 1 pentagonal face = 7 unit^2
- •11 51.5 unit^2 for crystal $(48 + 2\sqrt{3})$
- •12 strategy for finding % decrease

The diagram below shows the graph of $y = 2\sin 2x + 1$ for $0 \le x \le \pi$.



(*a*) Find the coordinates of A and B (as shown in the diagram) by solving an appropriate equation algebraically. **(5)**

(*b*) The points $(0, 2)$ and $(\pi, 0)$ are joined by a straight line l. In how many points does l intersect the given graph ? **(1)**

(*c*) C is the point on the given graph with an x-coordinate of $\frac{\pi}{2}$. Explain whether C is above, below or on the line l. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(*a*)	5	2.3	3	2					2.3.1		Source
(*b*)	1	0.1	1						0.1		1993 Paper 2
(*c*)	3	0.1		3					0.1		Qu.6

(*a*) $\bullet^1$ $2\sin 2x + 1 = 0$

$\bullet^2$ $\sin 2x = -\frac{1}{2}$

$\bullet^3$ for any valid sol of equ. in form $\sin ax = -\frac{b}{c}$

$\bullet^4$ $\left(\frac{7\pi}{12}, 0\right)$

$\bullet^5$ $\left(\frac{11\pi}{12}, 0\right)$

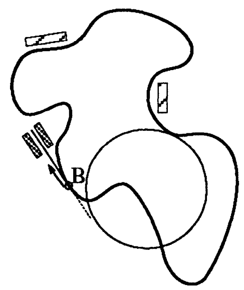
(*b*) $\bullet^6$ 3

(*c*) $\bullet^7$ $y_C = 1$

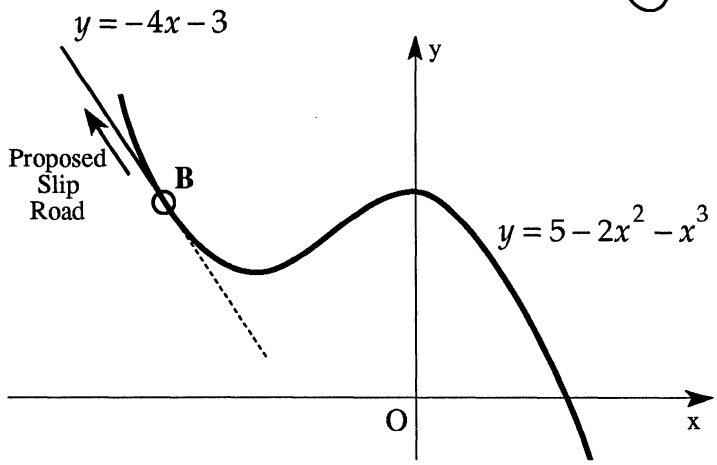
$\bullet^8$ for a strategy to make a decision about C

$\bullet^9$ for making a consistent decision about C

The diagram shows the plans for a proposed new racing circuit. The designer wishes to introduce a slip road at B for cars wishing to exit from the circuit to go into the pits. The designer needs to ensure that the two sections of road touch at B in order that drivers may drive straight on when they leave the circuit.



Relative to appropriate axes, the part of the circuit circled above is shown below. This part of the circuit is represented by a curve with equation $y = 5 - 2x^2 - x^3$ and the proposed slip road is represented by a straight line with equation $y = -4x - 3$.



(*a*) Calculate the coordinates of B. **(7)**

(*b*) Justify the designer's decision that this direction for the slip road does allow drivers to go straight on. **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(*a*)	7	2.1	7						2.1.8,	2.1.3	Source
(*b*)	1	2.1		1					2.1.8		**1993 Paper 2 Qu.7**

(*a*) •1 equating expressions for y

•2 re-arranging cubic...... "..."= 0

•3 strategy for solving cubic

•4 first linear factor

•5 quadratic factor

•6 $x = -2, 2$

•7 intersection at (– 2, 5)

(*b*) •8 double root $\Rightarrow$ tangency **or** $y'(-2) = -4$ = gradient of line

Secret Agent 004 has been captured and his captors are giving him a 25 milligram dose of a truth serum every 4 hours. 15% of the truth serum present in his body is lost every hour.

(*a*) Calculate how many milligrams of serum remain in his body after 4 hours (that is immediately **before** the second dose is given). **(3)**

(*b*) It is known that the level of serum in the body has to be continuously above 20 milligrams before the victim starts to confess. Find how many doses are needed before the captors should begin their interrogation. **(3)**

(*c*) Let u_n be the amount of serum (in milligrams) in his body just **after** his n^{th} dose. Show that $u_{n+1} = 0 \cdot 522u_n + 25$. **(1)**

(*d*) It is also known that 55 milligrams of this serum in the body will prove fatal, and the captors wish to keep Agent 004 alive. Is there any maximum length of time for which they can continue to administer this serum and still keep him alive ? **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.4
(*a*)	3	1.4			3				1.4.1		Source
(*b*)	3	1.4			3				1.4.1		**1993 Paper 2**
(*c*)	1	1.4			1				1.4.3		**Qu.8**
(*d*)	4	1.4			3	1			1.4.4,	1.4.5	

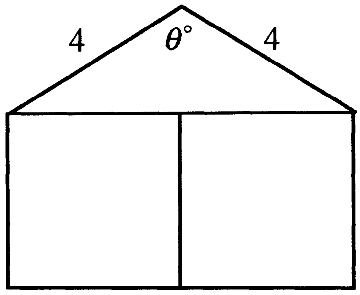
(*a*)
- •1 strategy for each hour (e.g. using 0·85)
- •2 using strategy 4 times (e.g. $(0{\cdot}85^4)$)
- •3 13·05

(*b*)
- •4 apply a correct dose strategy
- •5 a relevant sequence e.g. 13·05, 19·86, 23·4,
  or 25, 38·05, 44·9, 48·4
- •6 3 doses

(*c*)
- •7 valid explanation i.e. $(0 \cdot 85)^4 = 0 \cdot 522$ explicitly stated

(*d*)
- •8 statement that limit exists because $(0 \cdot 85)^4 < 1$
- •9 $\therefore l = 0 \cdot 522l + 25$ or using $l = \frac{b}{1-a}$
- •10 $l = 52{\cdot}3$
- •11 52·3 < 55 so no maximum length of time

A builder has obtained a large supply of 4 metre rafters. He wishes to use them to build some holiday chalets. The planning department insists that the gable end of each chalet should be in the form of an isosceles triangle surmounting two squares, as shown in the diagram.



(*a*) If θ ° is the angle shown in the diagram and A is the area (in square metres) of the gable end, show that $A = 8\left(2 + \sin\theta^\circ - 2\cos\theta^\circ\right)$. **(5)**

(*b*) Express $8\sin\theta^\circ - 16\cos\theta^\circ$ in the form $k\sin(\theta - \alpha)^\circ$. **(4)**

(*c*) Find algebraically the value of θ for which the area of the gable end is 30 square metres. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.4
(*a*)	5	0.1			1	4			0.1,	2.3.3	Source
(*b*)	4	3.4			4				3.4.1		**1993 Paper 2**
(*c*)	4	3.4			1	3			3.4.2		**Qu.9**

(*a*)
- •[1] area of triangle = $\frac{1}{2} \times 4 \times 4\sin\theta$ *or* $2 \times \frac{1}{2} \times 4\sin\frac{\theta}{2} \times 4\cos\frac{\theta}{2}$
- •[2] strategy for finding length of side of square or rectangle
- •[3] for length of side or (length of side)^2 of square/rectangle
- •[4] area of rectangle
- •[5] simplifying

Note : For •[3] various forms of the length are

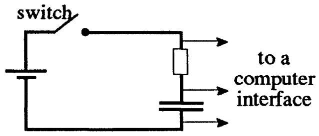
square: $4\sin\frac{\theta}{2}, \frac{2\sin\theta}{\sin\left(90-\frac{\theta}{2}\right)}, \sqrt{16 - 16\cos^2\frac{\theta}{2}}$

rect: $\frac{4\sin\theta}{\sin\left(90-\frac{\theta}{2}\right)}, \sqrt{32 - 32\cos\theta}$

(*b*)
- •[6] strategy including expansion of $k\sin(\theta - \alpha)$
- •[7] $k\cos\alpha = 8$ & $k\sin\alpha = 16$
- •[8] $k = 8\sqrt{5}$ or equiv.
- •[9] $\tan\alpha = 2 \Rightarrow \alpha = 63 \cdot 4$

(*c*)
- •[10] $8(2 + \sin\theta - 2\cos\theta) = 30$
- •[11] $8\sqrt{5}\sin(\theta - 63\cdot4)^\circ = 14$
- •[12] $\sin(\theta - 63\cdot4)^\circ = 0\cdot783$
- •[13] $\theta = 51\cdot5 + 63\cdot4 = 114\cdot9$

When the switch in this circuit was closed, the computer printed out a graph of the current flowing (I microamps) against the time (t seconds).This graph is shown in fig. 1.



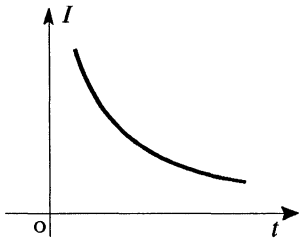


figure 1

In order to determine the equation of the graph shown in figure 1, values of $\log_e I$ were plotted against $\log_e t$ and the best fitting straight line was drawn as shown in figure 2.

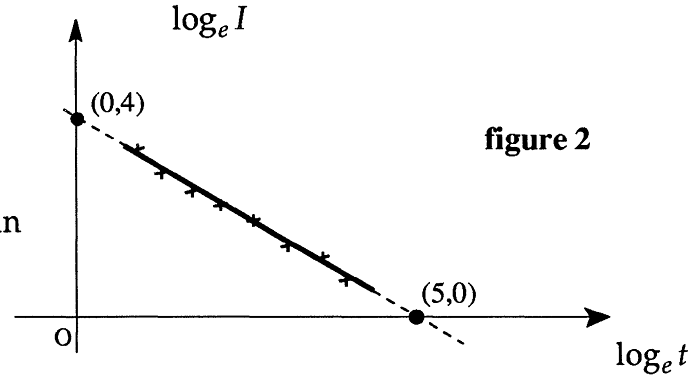


figure 2

(*a*) Find the equation of the line shown in figure 2 in terms of $\log_e I$ and $\log_e t$. **(3)**

(*b*) Hence or otherwise show that I and t satisfy a relationship of the form $I = kt^r$ stating the values of k and r. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.3
(*a*)	3	1.1			3				1.1.1,	1.1.7	Source
(*b*)	4	3.3				4			3.3.6		**1993 Paper 2 Qu.10**

(*a*) $\bullet^1$ $m = -\frac{4}{5}$ stated or implied

$\bullet^2$ $y = mx + 4$ stated or implied

$\bullet^3$ $\log_e I = -\frac{4}{5}\log_e t + 4$

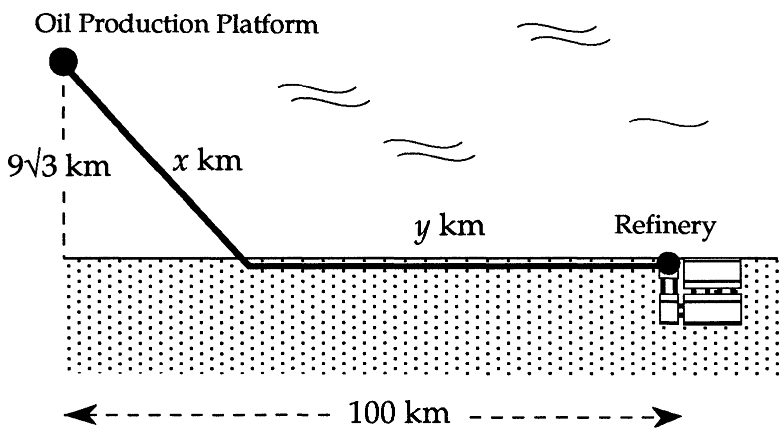
(*b*) $\bullet^4$ $\log_e t^{-\frac{4}{5}}$

$\bullet^5$ $\log_e 54.6$

$\bullet^6$ $\log_e 54.6t^{-\frac{4}{5}}$

$\bullet^7$ $I = 54.6t^{-0.8}$

An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.



The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

(*a*) Show that the total cost of this pipeline is £$C(x)$ million where

$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}.$$ **(3)**

(*b*) Show that $x = 18$ gives a minimum cost for this pipeline.
Find this minimum cost and the corresponding total length of the pipeline. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.2
(*a*)	3	0.1	1	2					0.1		Source
(*b*)	7	1.3	1	6					1.3.15,	3.2.2	**1993 Paper 2 Qu.11**

(*a*)

•[1] $C = 2x + y$

•[2] $\sqrt{x^2 - \left(9\sqrt{3}\right)^2}$

•[3] for completing proof

(*b*)

•[4] knowing to differentiate

•[5] $\frac{1}{2}\left(x^2 - 243\right)^{-\frac{1}{2}}$

•[6] $\times\ 2x$

•[7] $C'(18) = 0$

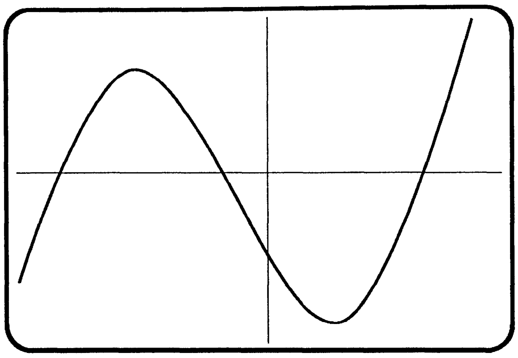
•[8] justification of minimum e.g. nature table

•[9] $C = 127$

•[10] $x + y = 109$

	18^-	18	18^+
$C'(x)$	–	0	+
	\	—	/
		minimum	

The diagram shows part of the graph of the curve with equation $f(x) = x^3 + x^2 - 16x - 16$.



(*a*) Factorise $f(x)$. **(3)**

(*b*) Write down the co-ordinates of the four points where the curve crosses the x and y axes. **(2)**

(*c*) Find the turning points and justify their nature. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	3	2.1	3						2.1.3		Source
(*b*)	2	1.2	2						1.2.9		**1992 Paper 2**
(*c*)	6	1.3	6						1.3.12		**Qu.1**

(*a*) •1 any linear factor

•2 corresponding quadratic factor

•3 $f(x) = (x+1)(x-4)(x+4)$

(*b*) •4 For all 3 points on x-axis

•5 (0,–16)

(*c*) •6 $f'(x) = 3x^2 + 2x - 16$

•7 use $f'(x) = 0$

•8 $x = 2$, **and** $x = -\frac{8}{3}$

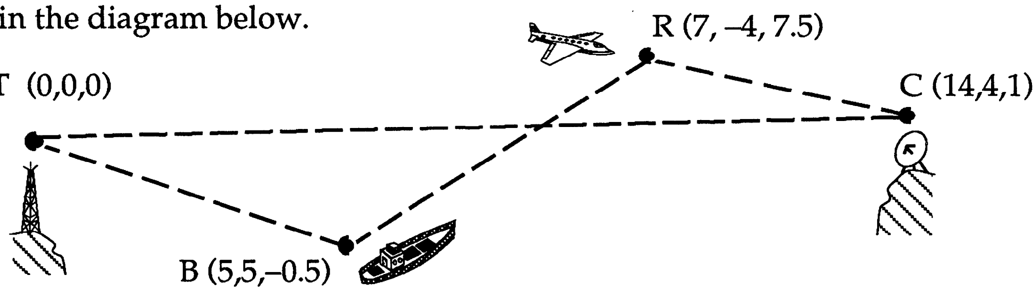
•9 $y = -36$, **and** $y = \frac{400}{27}(14.8)$

•10

	$-\frac{8}{3}^-$	$-\frac{8}{3}$	$-\frac{8}{3}^+$	2^-	2	2^+
$f'(x)$	+	0	–	–	0	+
	⋰	…	⋱	⋱	…	⋰

•11 max at $\left(-\frac{8}{3}, \frac{400}{27}\right)$, min at $(2, -36)$

Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point (5, 5, –0·5), the centre C of the dish on the top of a mountain is the point (14, 4, 1) and the reflector R on the aircraft is the point (7, –4, 7·5).

(*a*) Find the distance from the bridge of the ship to the reflector on the aircraft. **(3)**

(*b*) Three minutes earlier the aircraft was at the point M(–2, 4, 8·5). Find the speed of the aircraft in kilometres per hour. **(2)**

(*c*) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. **(3)**

(*d*) Calculate the size of angle TCR. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	3	3.1			3				3.1.3		Source
(*b*)	2	3.1			2				3.1.3		**1992 Paper 2**
(*c*)	3	3.1			3				3.1.10		**Qu.2**
(d)	5	3.1			5				3.1.11		

(*a*) •1 Strategy: use vectors or 3-D distance formula

•2 $\overrightarrow{BR} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$ *or* $BR^2 = 2^2 + 7^2 + 4^2$

•3 answer

(*b*) •4 $|\overrightarrow{MR}| = \sqrt{115.25}$ or equivalent

•5 answer

(*c*) •6 know to use a scalar product

•7 $\overrightarrow{TC}.\overrightarrow{BR} = 0$

•8 communication: 0 ⇔ perpendicularity

(*d*) •9 Strategy: know to use

$\cos T\hat{C}R = \dfrac{\overrightarrow{TC}.\overrightarrow{RC}}{|TC||RC|}$ or equiv.

•10 $\overrightarrow{TC} = \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$ and $\overrightarrow{RC} = \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix}$

•11 $\sqrt{161}$ and $\sqrt{65}$

•12 $\overrightarrow{TC}.\overrightarrow{RC} = 82$

•13 36.7°

Biologists calculate that when the concentration of a particular chemical in a sea loch reaches 5 milligrams per litre (mg/l) the level of pollution endangers the life of the fish.

A factory wishes to release waste containing this chemical into the loch. It is claimed that the discharge will not endanger the fish.

The Local Authority is supplied with the following information:

1. The loch contains none of this chemical at present.
2. The factory manager has applied to discharge effluent once per week which will result in an increase in concentration of 2.5 mg/l of the chemical in the loch.
3. The natural tidal action will remove 40% of the chemical from the loch every week.

(*a*) Show that this level of discharge would result in fish being endangered. **(3)**

When this result is announced, the company agrees to install a cleaning process that reduces the concentration of chemical released into the loch by 30%.

(*b*) Show the calculations you would use to check this revised application. Should the Local Authority grant permission ? **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.4
(*a*)	3	1.4			3				1.4.3		Source
(*b*)	5	1.4			5				1.4.3,	1.4.5	**1992 Paper 2 Qu.3**

(*a*)
- •1 0.6 stated/implied
- •2 $u_{n+1} = 0.6u_n + 2.5$
- •3 communication: ie 6.25 $\Rightarrow$ danger

(*b*)
- •4 0.7 x 2.5 = 1.75
- •5 2.8, 3.43, 3.808
- •6 $u_{n+1} = 0.6u_n + 1.75$
- •7 limit = 4.375
- •8 communication: ie 4.375 $\Rightarrow$ allow/disallow

(*a*) For a particular radioactive substance the mass m (in grams) at time t (in years) is given by

$$m = m_0 e^{-0.02t}$$

where m_0 is the original mass.

If the original mass is 500 grams, find the mass after 10 years. **(2)**

(*b*) The half-life of any material is the time taken for half of the mass to decay.
Find the half-life of this substance. **(3)**

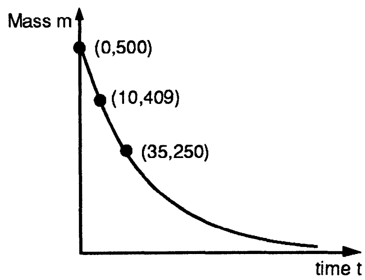
(*c*) Illustrate **ALL** of the above information on a graph. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.3
(*a*)	2	3.3			2				3.3.4		Source
(*b*)	3	3.3			1	2			3.3.4		**1992 Paper 2**
(*c*)	3	1.2			1	2			1.2.5		**Qu.4**

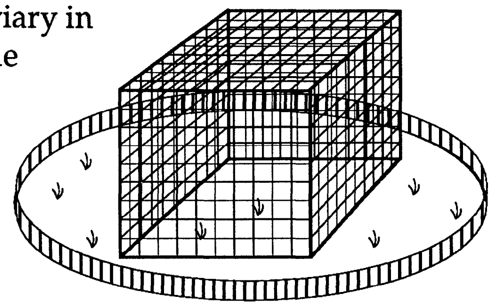
(*a*)
•1 $m = 500e^{-0.02\times10}$
•2 409.37 grams

(*b*)
•3 $250 = 500e^{-0.02t}$
•4 $\ln 250 = \ln 500 - 0.02t \times 1$ or equiv.
•5 34.7 years

(*c*)
•6 any **two** of the 3 points
•7 the remaining point
•8 a decreasing curve



The owners of a zoo intend to build a new aviary in the shape of a cuboid with a square floor. The volume of the aviary will be 500 m^3.



(*a*) If x metres is the length of one edge of the floor, show that the area A square metres of netting required is given by

$$A = x^2 + \frac{2000}{x}.$$
(4)

(*b*) Find the dimensions of the aviary to ensure that the cost of netting is minimised. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.3
(*a*)	4	0.1	2	2					0.1		Source
(*b*)	6	1.3	4	2					1.3.15		1992 Paper 2 Qu.5

(*a*) •1 introduce height specific to this cuboid

•2 $h = \frac{500}{x^2}$

•3 $A = x^2 + 4xh$

•4 $A = x^2 + 4x.\frac{500}{x^2}$ explicitly stated

(*b*) •5 $A'(x) = \ldots\ldots$

•6 $2x - 2000x^{-2}$

•7 $A'(x) = 0$ specifically stated

•8 $x = 10$

•9 justify minimum e.g. with table

•10 dimensions of 10 by 10 by 5

The *vector product*, $\boldsymbol{a} \times \boldsymbol{b}$, of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined by

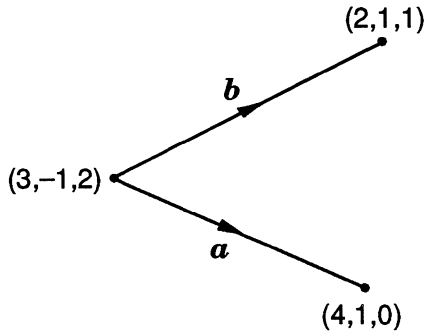
$$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

EXAMPLE

$$\text{when } \boldsymbol{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \text{ then } \boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} 2\times2-3\times0 \\ 3\times(-1)-1\times2 \\ 1\times0-2\times(-1) \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix}$$

(*a*) If $\boldsymbol{a}$ and $\boldsymbol{b}$ are as shown in the diagram and $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$, evaluate $\boldsymbol{c}$. **(3)**

(*b*) By considering $\boldsymbol{a.c}$ and $\boldsymbol{b.c}$, what can be concluded about $\boldsymbol{c}$? **(4)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	3	0.1					3		0.1		Source
(*b*)	4	3.1					4		3.1.9,	3.1.10	**1992 Paper 2 Qu.6**

(*a*) •1 $a = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ **and** $b = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$

•2 substitute in the rule for a x b

•3 answer

(*b*) •4 evaluate $\underline{a}.\underline{c}$

•5 evaluate $\underline{b}.\underline{c}$

•6 a statement that $\underline{a}$ is perpendicular to $\underline{c}$

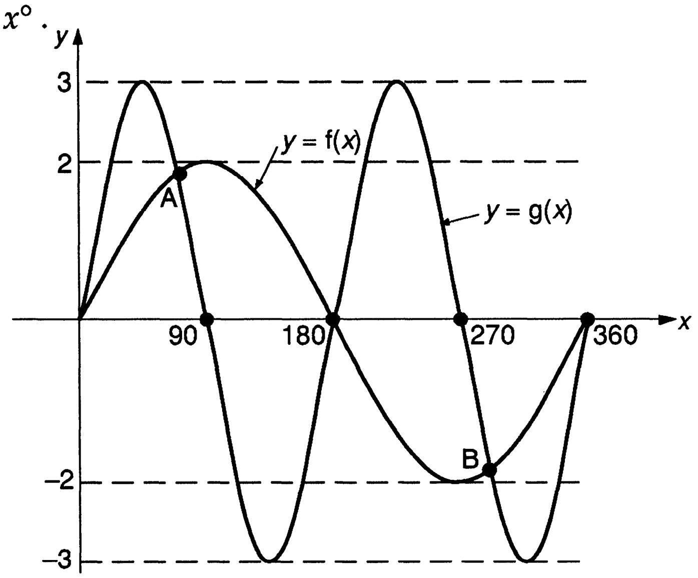
•7 a statement that $\underline{b}$ is perpendicular to $\underline{c}$

(*a*) Solve the equation $3\sin 2x° = 2\sin x°$ for $0 \le x \le 360$ **(4)**

(*b*) The diagram below shows parts of the graphs of sine functions f and g.
State expressions for $f(x)$ and $g(x)$. **(1)**

(*c*) Use your answers to part (*a*) to find the co-ordinates of A and B. **(2)**

(*d*) Hence state the values of x in the interval $0 \le x \le 360$ for which $3\sin 2x° < 2\sin x°$. **(3)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.3
(*a*)	4	2.3			4				2.3.5	Source
(*b*)	1	1.2			1				1.2.7	1992 Paper 2
(*c*)	2	1.2			2				1.2.9	Qu.7
(*d*)	3	1.2			2	1			1.2.10	

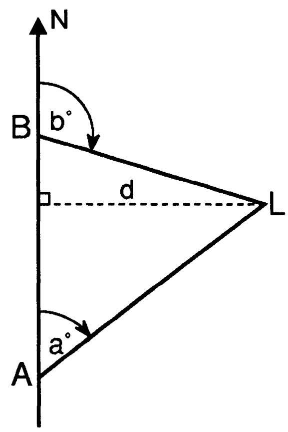
(*a*)
- •1 strategy: ie $\sin 2x = 2\sin x\cos x$
- •2 $\sin x = 0$ AND $\cos x = \frac{1}{3}$
- •3 0, 180 AND 360
- •4 70.5 AND 289.5 AND no other angles

(*b*)
- •5 $f(x) = 2\sin x°,\ g(x) = 3\sin 2x°$

(*c*)
- •6 $x = 70.5$ AND 289.5
- •7 $y = 1.89$ AND -1.89

(*d*)
- •8 70.5 AND 180
- •9 289.5 AND 360
- •10 use inequality signs logically to connect the points of intersection (ie **not** for $180 < x < 70.5$)

A ship is sailing due north at a constant speed. When at position A, lighthouse L is observed on a bearing of $a°$. One hour later, when the ship is at position B, the lighthouse is on a bearing of $b°$. The shortest distance between the ship and the lighthouse during this hour was d miles.



(*a*) Prove that $\text{AB} = \dfrac{d}{\tan a°} - \dfrac{d}{\tan b°}$. **(2)**

(*b*) Hence prove that $\text{AB} = \dfrac{d\sin(b-a)°}{\sin a° \sin b°}$. **(3)**

(*c*) Calculate the shortest distance from the ship to the lighthouse when the bearings $a°$ and $b°$ are 060° and 135° respectively and the constant speed of the ship is 14 miles per hour. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
(*a*)	2	0.1			1	1			0.1		Source
(*b*)	3	2.3				3			2.3.4		**1992 Paper 2**
(*c*)	3	0.1			3				0.1		**Qu.8**

(*a*) •[1] $\text{CA} = \frac{d}{\tan a}$

•[2] $\text{CB} = \frac{d}{\tan(180-b)}$

(*b*) •[3] $\text{AB} = \frac{d}{\frac{\sin a}{\cos a}} - \frac{d}{\frac{\sin b}{\cos b}}$

•[4] $\frac{d\cos a}{\sin a} - \frac{d\cos b}{\sin b}$

•[5] $\frac{d\sin b\cos a - d\cos b\sin a}{\sin a\sin b}$

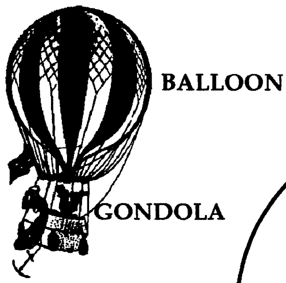
(*c*) •[6] AB = 14

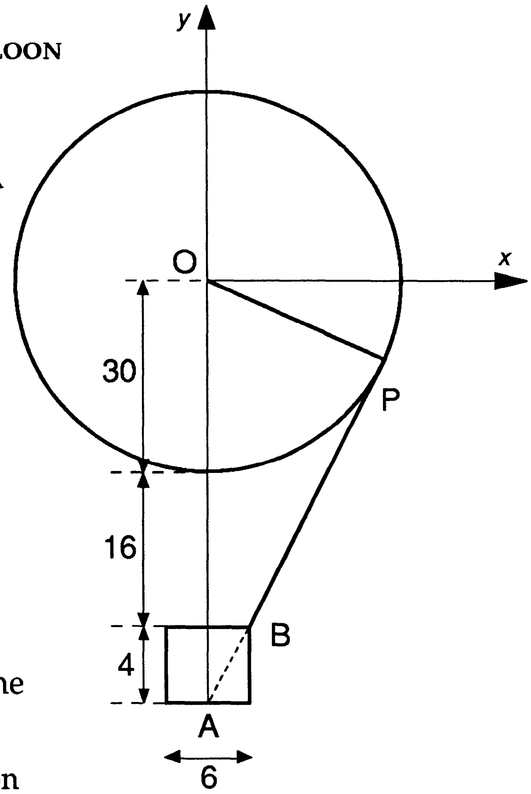
•[7] 1.577 or 0.634

(comes from AB = 1.577d or d = 0.634 AB)

•[8] 8.9 miles

A spherical hot-air balloon has radius 30 feet. Cables join the balloon to the gondola which is cylindrical with diameter 6 feet and height 4 feet. The top of the gondola is 16 feet below the bottom of the balloon.





Co-ordinate axes are chosen as shown in the diagram. One of the cables is represented by PB and PBA is a straight line.

(*a*) Find the equation of the cable PB. **(3)**

(*b*) State the equation of the circle representing the balloon. **(1)**

(*c*) Prove that this cable is a tangent to the balloon and find the co-ordinates of the point P. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.4
(*a*)	3	1.1					3		1.1.1,	1.1.7	Source
(*b*)	1	2.4					1		2.4.3		**1992 Paper 2**
(*c*)	5	2.4					2	3	2.4.4		**Qu.9**

(*a*) •1 Strategy: know to find m

•2 $m = \frac{4}{3}$

•3 $y + 46 = \frac{4}{3}(x - 3)$

(*b*) •4 $x^2 + y^2 = 900$ or equivalent

(*c*) •5 Strategy: know to substitute

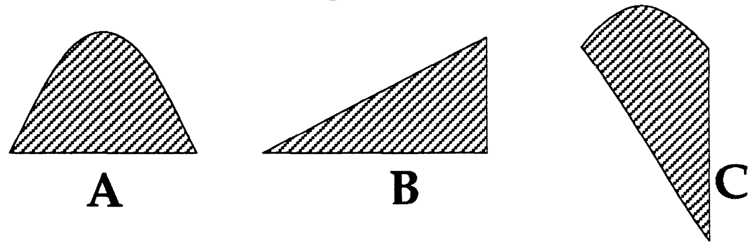
•6 $x^2 + \left(\frac{4}{3}x - 50\right)^2 = 900$

•7 $(x - 24)^2$ or evaluate the discriminant

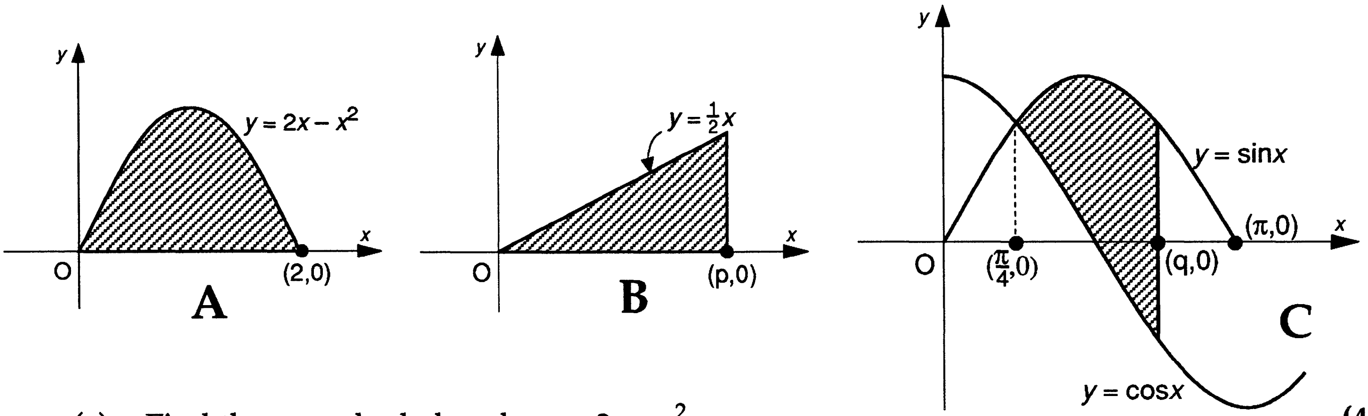
•8 communication relating to tangency

•9 $(24, -18)$

An artist has been asked to design a window made from pieces of coloured glass with different shapes. To preserve a balance of colour each shape must have the **same** area. Three of the shapes used are drawn below.



Relative to x,y–axes, the shapes are positioned as shown below.



(*a*) Find the area shaded under $y = 2x - x^2$. **(4)**

(*b*) Use the area found in part (*a*) to find the value of p. **(2)**

(c) Prove that q satisfies the equation $\cos q + \sin q = 0.081$ and hence find the value of q to 2 significant figures. **(10)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.4
(*a*)	4	2.2			4				2.2.6		Source
(*b*)	2	0.1			2				0.1		**1992 Paper 2**
(*c*)	10	3.4			2	8			3.4.2,	3.2.1, 2.2.7	**Qu.10**

(*a*) •1 strategy: know to integrate

•2 $\int_0^2 \left(2x - x^2\right)\, dx$

•3 $x^2 - \frac{1}{3}x^3$

•4 $1\frac{1}{3}$ *units*^{2}

(*b*) •5 strategy: use area to find p

•6 $p = \frac{4}{\sqrt{3}}$ or equivalent

(*c*) •7 $\int (\sin x - \cos x)\, dx$

•8 for the limits $\int_{\frac{\pi}{4}}^{q}$

•9 $[-\cos x - \sin x]$

•10 $-\cos q - \sin q + \sqrt{2}$

•11 $\sqrt{2} - \frac{4}{3} = 0.081$

•12 strategy: eg $k\cos(q - \alpha)$

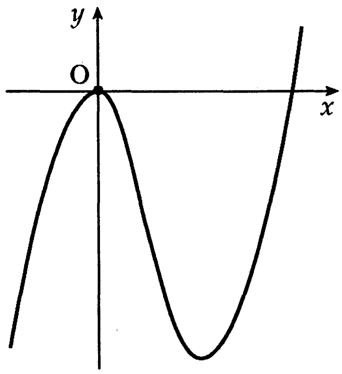
•13 $k = \sqrt{2}$

•14 $\alpha = \frac{\pi}{4}$

•15 $\cos\left(q - \frac{\pi}{4}\right) = \frac{0.081}{\sqrt{2}}$

•16 $q = 2.3$

(*a*) The diagram shows a part of the curve with equation $y = 2x^2(x-3)$. Find the coordinates of the stationary points on the graph and determine their nature. **(5)**



(*b*) State the range of values of *k* for which $y = k$ intersects the graph in three distinct points. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(*a*)	5	1.3					5		1.3.12		Source
(*b*)	2	1.2					2		1.2.1		**1991 Paper 2 Qu. 1**

(*a*) •1 $\frac{dy}{dx} = 6x^2 - 12x$

•2 $\frac{dy}{dx} = 0$

•3 $x = 0,\ x = 2$

•4

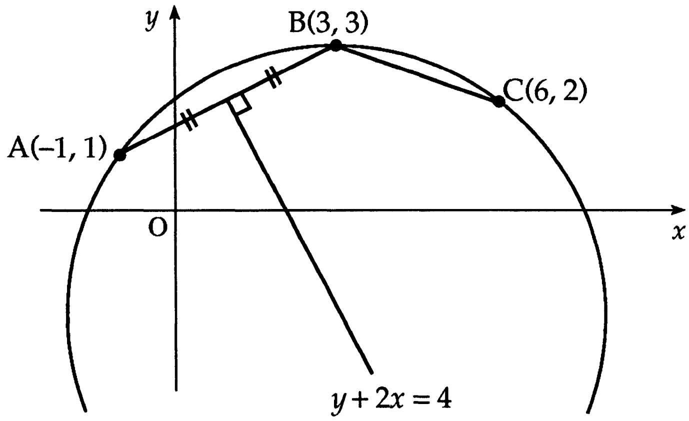
x	0^-	0	0^+	2^-	2	2^+
$\frac{dy}{dx}$	+	−	+	+	−	+

•5 max. at (0,0) min at (2,−8)

(*b*) •6 $k < 0$

•7 $k > -8$

(*a*) In the diagram, A is the point (–1, 1), B is (3, 3) and C is (6, 2). The perpendicular bisector of AB has equation $y + 2x = 4$. Find the equation of the perpendicular bisector of BC. **(4)**



(*b*) Find the centre and the equation of the circle which passes through A, B and C. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.4
(*a*)	4	1.1					4		1.1.9,	1.1.7	Source
(*b*)	6	2.4					6		2.4.3,	1.1.2	**1991 Paper 2 Qu. 2**

(*a*)
- •1 $m_{BC} = -\frac{1}{3}$
- •2 $m_{\perp} = 3$
- •3 $\text{midpoint}_{BC} = \left(\frac{9}{2}, \frac{5}{2}\right)$
- •4 $y - \frac{5}{2} = 3\left(x - \frac{9}{2}\right)$

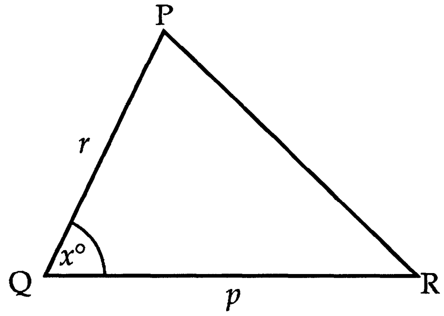
(*b*)
- •5 $y - 3x = -11$
- •6 perp. bisector passes thr' centre **stated explicitly**
- •7 using $y - 3x = -11$ **and** $y + 2x = 4$
- •8 (3,–2)
- •9 $r^2 = 25$
- •10 $(x-3)^2 + (y+2)^2 = 25$

The diagram shows an isosceles triangle PQR in which PR = QR and angle PQR = $x°$.

(*a*) Show that $\frac{\sin x°}{p} = \frac{\sin 2x°}{r}$. **(3)**

(*b*) (i) State the value of $x°$ when $p = r$.

(ii) Using the fact that $p = r$, solve the equation in (*a*) above, to justify your stated value of $x°$. **(5)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(*a*)	3	0.1					2	1	0.1		Source
(*b*)	5	2.3					5		2.3.5,	0.1	**1991 Paper 2 Qu. 3**

(*a*)
- •[1] $(180-2x)°$
- •[2] $\frac{\sin x°}{p} = \frac{\sin(180-2x)°}{r}$
- •[3] $\sin(180-2x)° = \sin 2x°$ **stated explicitly**

(*b*)
- •[4] $60°$
- •[5] $\sin x° = \sin 2x°$
- •[6] $\sin x°(2\cos x° - 1) = 0$
- •[7] $\sin x° = 0$ *and* $\cos x° = \frac{1}{2}$
- •[8] $x = 60$ is only answer **stated explicitly**

(*a*) On the same diagram, sketch the graphs of $y = \log_{10} x$ and $y = 2 - x$ where $0 < x < 5$.
Write down an approximation for the x-coordinate of the point of intersection. **(3)**

(*b*) Find the value of this x-coordinate, correct to 2 decimal places. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.2
(*a*)	3	1.2			3				1.2.5		Source
(*b*)	3	0.1			1	2			0.1		**1991 Paper 2 Qu. 4**

(*a*)
- •[1] graph of $y = 2 - x$ with two annotated points eg (2,0) and (0,2)
- •[2] graph of $y = \log_{10} x$ with one annotated point eg (1,0)
- •[3] any consistent approximation

(*b*)
- •[4] between 1.7 and 1.8
- •[5] between 1.75 and 1.80
- •[6] 1.76

Diagram 1 shows a christmas tree decoration which is made of coloured glass rods in the shape of a square-based prism topped by a square pyramid. Diagram 2 shows the decoration relative to the origin and rectangular coordinate axes OX, OY and OZ.

The vertex F has position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$

and the vertex V has position vector $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

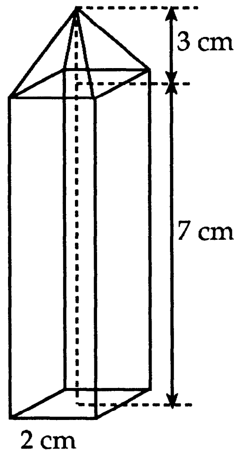


Diagram 1

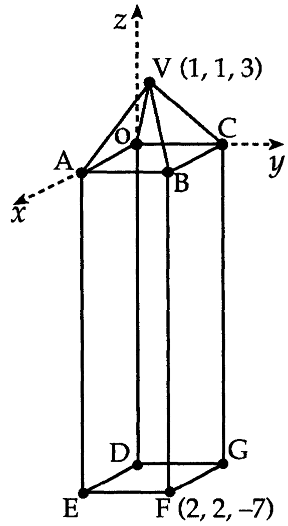


Diagram 2

(*a*) Find

(i) the components of the vectors represented by $\overrightarrow{VF}$ and $\overrightarrow{VE}$;

(ii) the size of angle EVF. **(7)**

(*b*) To make the decoration more attractive, triangular sheets of coloured glass VEF and VDG are added to it.

Calculate the area of the glass triangle VEF. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	3.1
											Source
(*a*)	7	3.1					7		3.1.11,	3.1.1	**1991 Paper 2**
(*b*)	3	0.1					3		0.1		**Qu. 5**

(*a*)

•[1] $\overrightarrow{VF} = \begin{pmatrix} 1 \\ 1 \\ -10 \end{pmatrix}$

•[2] E = (2,0,–7)

•[3] $\overrightarrow{VE} = \begin{pmatrix} 1 \\ -1 \\ -10 \end{pmatrix}$

•[4] $\cos E\hat{V}F = \frac{\overrightarrow{VE}.\overrightarrow{VF}}{|\overrightarrow{VE}|\,|\overrightarrow{VF}|}$ This may appear as $\frac{100}{102}$ **after the completion of •[5] and •[6].**

•[5] $\overrightarrow{VE}.\overrightarrow{VF} = 100$

•[6] $|\overrightarrow{VE}|\,|\overrightarrow{VF}| = 102$

•[7] 11.4°

(*b*)

•[8] $\frac{1}{2} VE \times VF \sin E\hat{V}F$

•[9] $\frac{1}{2} \times 102 \times \sin 11.4°$

•[10] 10.02

There is a rule known as the Product Rule which is used, as shown below, to differentiate any product of two functions of the same variable.

The Product Rule

If $P(x) = f(x).g(x)$, then $P'(x) = f'(x).g(x) + f(x).g'(x)$

Example: Find the derivative of $P(x) = x^2 \sin x$.

$P(x) = x^2 \sin x$ Choose $f(x) = x^2$ *and* $g(x) = \sin^2 x$

then $f'(x) = 2x$ *and* $g'(x) = \cos x$

so $P'(x) = 2x.\sin x + x^2.\cos x$

$P'(x) = 2x \sin x + x^2 \cos x$

Use the Product Rule to find the derivative of $P(x) = x^3 \cos x$ **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	0.1
(-)	5	0.1					5		0.1	Source **1991 Paper 2 Qu. 6**

(-) •[1] $f(x) = x^3$

•[2] $g(x) = \cos x$

•[3] $f'(x) = 3x^2$ **and** $g(x) = -\sin x$

•[4] $3x^2 \cos x$

•[5] $-x^3 \sin x$

(*a*) A tractor tyre is inflated to a pressure of 50 units.
Twenty-four hours later the pressure has dropped to 10 units.

If the pressure, P_t units, after t hours is given by the formula $P_t = P_0 e^{-kt}$, find the value of k, to three decimal places. **(5)**

(*b*) The tyre manufacturer advises that serious damage to the tyre will result if it is used when the pressure drops below 30 units.

If the farmer inflates the tyre to 50 units and drives the tractor for four hours, can the tractor be driven further without inflating the tyre and without risking serious damage to the tyre? **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.3
											Source
(*a*)	5	3.3			2	3			3.3.4		1991 Paper 2
(*b*)	4	3.3			1	3			3.3.4		Qu. 7

(*a*)
- •1 $10 = 50e^{-24k}$
- •2 $0.2 = e^{-24k}$
- •3 $-24k = \ln 0.2$
- •4 $-24k = -1.609$
- •5 $k = 0.067$

(*b*)
- •6 knowing to find P_4
- •7 $P_4 = 50e^{-0.067 \times 4}$
- •8 38
- •9 38 > 30 so can be driven further

The displacement, d units, of a wave after t seconds, is given by the formula $d = \cos 20t° + \sqrt{3}\sin 20t°$.

(*a*) Express d in the form $k\cos(20t - \alpha)°$, where $k > 0$ and $0 \le \alpha \le 360$. **(4)**

(*b*) Sketch the graph of d for $0 \le t \le 18$. **(4)**

(*c*) Find, correct to 1 decimal place, the values of t, $0 \le t \le 18$, for which the displacement is 1·5 units. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.4
(*a*)	4	3.4			4				3.4.1		Source
(*b*)	4	1.2			2	2			1.2.3		**1991 Paper 2**
(*c*)	3	2.3			1	2			2.3.1		**Qu. 8**

(*a*)
- •[1] $k\cos 20t°\cos\alpha° + k\sin 20t°\sin\alpha°$
- •[2] $k\cos\alpha° = 1$ *and* $k\sin\alpha° = \sqrt{3}$
- •[3] $k = 2$
- •[4] $\alpha = 60$

(*b*)
- •[5] endpoints: $(0,1)$ **or** $(18,1)$
- •[6] zeros: $(7.5,0)$ **and** $(16.5,0)$
- •[7] stationary points: $(3,2)$ **and** $(12,-2)$
- •[8] correct annotation of graph

(*c*)
- •[9] $2\cos(20t - 60)° = 1.5$
- •[10] $20t - 60 = 41.4 \Rightarrow t = 5.1$
- •[11] $20t - 60 = -41.4 \Rightarrow t = 0.9$

(*a*) At 12 noon a hospital patient is given a pill containing 50 units of antibiotic.
By 1 pm the number of units in the patient's body has dropped by 12%.
By 2 pm a further 12% of the units remaining in the body at 1 pm is lost.
If this fall-off rate is maintained, find the number of units of antibiotic remaining at 6 pm. **(4)**

(*b*) A doctor considers prescribing a course of treatment which involves a patient taking one of these pills every 6 hours over a long period of time.
The doctor knows that mopre than 100 units of this antibiotic in the body is regarded as too dangerous.
Should the doctor prescribe this course of treatment?
Give reasons for your answer. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.4
(*a*)	4	1.4			4				1.4.1		Source
(*b*)	6	1.4			4	2			1.4.3, 1.4.5		**1991 Paper 2 Qu. 9**

(*a*)
- •[1] use 0.88 or 88%
- •[2] $n = 6$
- •[3] $u_6 = 50 \times 0.88^6$
- •[4] 23.22

(*b*)
- •[5] adding 50
- •[6] $u_{n+1} = 0.88^6 u_n + 50$
- •[7] $-1 < 0.88^6$ (*or* $0.4644) < 1$ so limit exists
- •[8] $L = \frac{50}{1 - 0.88^6}$
- •[9] 93.4
- •[10] $93.4 < 100$ so safe to continue

Diagram 1 shows a rectangular plate of transparent plastic moulded into a parabolic shape and pegged to the ground to form a cover for growing plants. Triangular metal frames are placed over the cover to support it and prevent it blowing away in the wind.
Diagram 2 shows an end view of the cover and the triangular frame related to the origin O and axes Ox and Oy. (All dimensions are given in centimetres.)

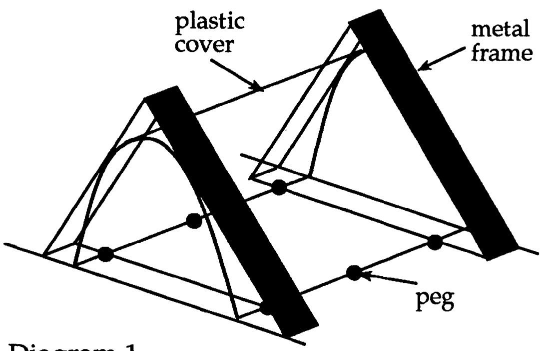


Diagram 1

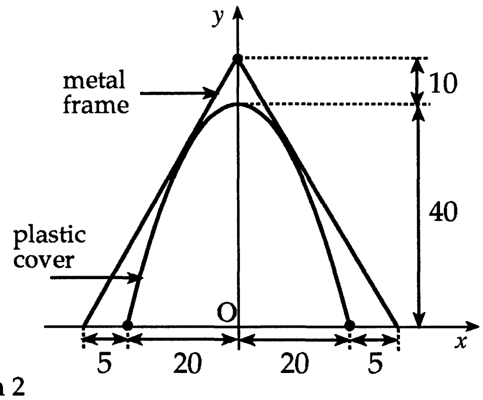


Diagram 2

(*a*) Show that the equation of the parabolic end is $y = 40 - \frac{x^2}{100}$, $-20 \le x \le 20$. **(4)**

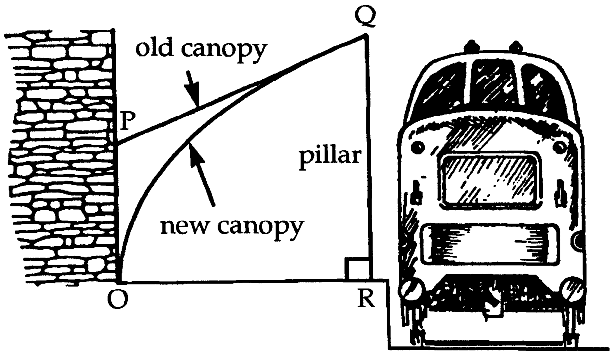
(*b*) Show that the triangular frame touches the cover without disturbing the parabolic shape. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.1
(*a*)	4	1.2					1	3	1.2.7		Source
(*b*)	7	2.1					3	4	2.1.8,	1.1.1, 1.1.7	**1991 Paper 2 Qu. 10**

(*a*)
- •1 $y = ax^2 + bx + c$
- •2 $(0,40) \Rightarrow c = 40$
- •3 symmetry $\Rightarrow b = 0$
- •4 $(20,0) \Rightarrow a = -\frac{1}{10}$

(*b*)
- •5 strategy: find equ of line and solve with parabola
- •6 *e.g.* gradient of left line = 2
- •7 $y = 2x + 50$
- •8 $2x + 50 = 40 - \frac{1}{10}x^2$
- •9 $x^2 + 20x + 100 = 0$
- •10 $b^2 - 4ac = 0$ *or* $(x-10)^2 = 0$
- •11 equal roots so line is tangent to parabola

The diagram shows a proposed replacement of the old platform canopy at the local railway station by a new parabolic canopy, while keeping the original pillars.
If OR and OP are taken as the x- and y- axes and Q has coordinates (1, 1), then OQ has equation $y = \sqrt{x}$ and PQ is the tangent at Q to the parabola.



The planners have received an objection that there is a reduction of more than 10% in the space under the canopy and wish to compare the two canopies.

(*a*) Find the equation of the tangent PQ and the coordinates of P. **(5)**

(*b*) Find the area of the trapezium OPQR. **(2)**

(*c*) Find the area under the parabola OQ. **(3)**

(*d*) Comment on the objection received. **(3)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(*a*)	5	1.3	5						1.3.9,	1.1.7	**1991 Paper 2 Qu. 11**
(*b*)	2	0.1	2						0.1		
(*c*)	3	2.2	3						2.2.6		
(*d*)	3	0.1	1	2					0.1		

(*a*)
- •1 $\frac{dy}{dx} = \ldots\ldots$
- •2 $\frac{1}{2}x^{-\frac{1}{2}}$
- •3 $m = \frac{dy}{dx}_{x=1} = \frac{1}{2}$
- •4 $y - 1 = \frac{1}{2}(x-1)$
- •5 $\mathrm{P} = (0, \frac{1}{2})$

(*b*)
- •6 method for area of trapezium
- •7 $\frac{3}{4}$

(*c*)
- •8 $\int_0^1 x^{\frac{1}{2}}\,dx$
- •9 $\frac{2}{3}x^{\frac{3}{2}}$
- •10 $\frac{2}{3}$

(*d*)
- •11 strategy: compare reduction with original
- •12 $\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$ *and* $\frac{\frac{1}{12}}{\frac{3}{4}} = \frac{1}{9}$
- •13 $\frac{1}{9} = 11.1\% > 10\%$ so objection correct

A function f is defined by the formula $f(x) = (x-1)^2(x+2)$ where $x \in \mathbf{R}$.

(*a*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **(3)**

(*b*) Find the stationary points of this curve $y = f(x)$ and determine their nature. **(7)**

(*c*) Sketch the curve $y = f(x)$. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	1.3
(*a*)	3	1.2	3						1.2.9	Source
(*b*)	7	1.3	7						1.3.15	**1990 Paper 2**
(*c*)	2	1.3	2						1.3.13	**Qu. 1**

(*a*) •1 $x = 1, -2$

•2 $(1,0)$ **and** $(-2,0)$

•3 $(0,2)$

(*b*) •4 $f(x) = x^3 - 3x + 2$

•5 $f'(x) = 3x^2 - 3$

•6 $f'(x) = 0$ **stated explicitly**

•7 $x = 1$ **and** -1

•8

x	-1^-	-1	-1^+	1^-	1	1^+
$f'(x)$	+	0	−	−	0	+

•9 max at $(-1, 4)$

•10 min at $(1,0)$

(*c*) •11 correct shape of sketch

•12 correct annotation of sketch(max, min, 2 axes intersections)

P, Q and R have coordinates (1, –2), (6, 3) and (9, 14) respectively and are three vertices of a kite PQRS.

(*a*) Find the equations of the diagonals of this kite and the coordinates of the point where they intersect. **(7)**

(*b*) Find the coordinates of the fourth vertex S. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.1
(*a*)	7	1.1					7		1.1.10,	0.1	Source **1990 Paper 2 Qu. 2**
(*b*)	2	0.1					2		0.1		

(*a*)
- •1 $m_{PR} = 2$
- •2 PR: *e.g.* $y + 2 = 2(x - 1)$
- •3 knowing to use $m_1 m_2 = 1$ for m_{QS}
- •4 $m_{QS} = -\frac{1}{2}$
- •5 QS: *e.g.* $y - 3 = -\frac{1}{2}(x - 6)$
- •6 knowing to solve simultaneously
- •7 $S = (4,4)$

(*b*)
- •8 $\vec{QM} = \vec{MS}$ or equivalent indication
- •9 $S = (2,5)$

The extract below is taken from the "OIL RIG NEWS".

> **RARE ILLNESS STRIKES RIG**
> **Storm prevents delivery of medicine**
> By noon on Tuesday 20th December 1988 50 of our oil rig personnel were laid low by a mystery illness.
> Our resident medical officer is expressing concern because the number of personnel affected is increasing each day by 8% of the previous day's total.

(*a*) If the daily rate of increase remained at 8% of the previous day's total, how many personnel were affected by noon on Sunday 25th December 1988? **(3)**

(*b*) An improvement in the weather conditions allowed a team of medics to fly out to the rig on the morning of Tuesday 27th December 1988.
At noon on that Tuesday, all personnel were innoculated and no new cases of the illness arose. Within the next 24 hours, 21% of those who had been affected had recovered.
If the daily rate of recovery of 21% of the previous day's total was maintained, how many personnel were still affected by the illness at noon on Saturday 31st December 1988? **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.4
(*a*)	3	1.4			3				1.4.2		Source
(*b*)	5	1.4			5				1.4.2		**1990 Paper 2 Qu. 3**

(*a*) •[1] $u_n = 1.08^n u_0$

•[2] $u_5 = 1.08^5 \times 50$

•[3] 73 or 74

(*b*) •[4] $u_7 = 1.08^7 \times 50$

•[5] $u_7 = 85$ or 86

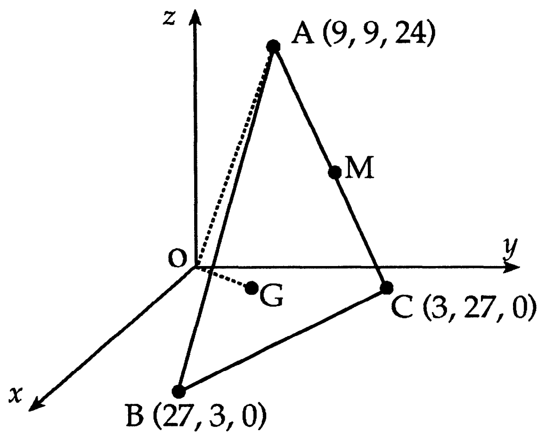
•[6] $v_n = 0.79^n v_0$

•[7] $v_4 = 33$ or 34

•[8] for consistent rounding

(*a*) Relative to mutually perpendicular axes Ox, Oy and Oz, the vertices of triangle ABC have coordinates A(9, 9, 24), B(27, 3, 0) and C(3, 27, 0). M is the mid-point of AC.
Find the coordinates of G which divides BM in the ratio 2:1. **(3)**

(*b*) Calculate the size of angle GOA. **(5)**



part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.1
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(*a*)	3	3.1			3				3.1.6		**1990 Paper 2 Qu. 4**
(*b*)	5	3.1			5				3.1.11		

(*a*)

•1 $M = (6,\ 18,\ 12)$

•2 e.g. $\overrightarrow{BG} = \frac{2}{3}\begin{pmatrix} -21 \\ 15 \\ 12 \end{pmatrix}$

•3 $G = (13,\ 13,\ 8)$

(*b*)

•4 $\cos A\hat{O}G = \dfrac{\overrightarrow{OA}.\overrightarrow{OG}}{|\overrightarrow{OA}|\,|\overrightarrow{OG}|}$

•5 $\overrightarrow{OA} = \begin{pmatrix} 9 \\ 9 \\ 24 \end{pmatrix}$ **and** $\overrightarrow{OG} = \begin{pmatrix} 13 \\ 13 \\ 8 \end{pmatrix}$

•6 $\overrightarrow{OA}.\overrightarrow{OG} = 426$

•7 $|\overrightarrow{OA}| = \sqrt{738}$ **and** $|\overrightarrow{OG}| = \sqrt{402}$

•8 38.5°

(*a*) Show that $2\cos(x+30)° - \sin x°$ can be written as $\sqrt{3}\cos x° - 2\sin x°$. **(3)**

(*b*) Express $\sqrt{3}\cos x° - 2\sin x°$ in the form $k\cos(x+\alpha)°$ where $k > 0$ and $0 \leq \alpha \leq 360$ and find the values of k and α. **(4)**

(*c*) Hence, or otherwise, solve the equation $2\cos(x+30)° = \sin x° + 1$, $0 \leq x \leq 360$. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.4
(*a*)	3	2.3			3				2.3.2,	1.2.11	Source
(*b*)	4	3.4			4				3.4.1		**1990 Paper 2**
(*c*)	3	3.4				3			3.4.2		**Qu. 5**

(*a*)

•[1] $\cos(x+30)° = \cos x° \cos 30° - \sin x° \sin 30°$

•[2] $\frac{\sqrt{3}}{2}\cos x° - \frac{1}{2}\sin x°$

•[3] $2 \times \left(\frac{\sqrt{3}}{2}\cos x° - \frac{1}{2}\sin x°\right) - \sin x°$

(*b*)

•[4] $k\cos x° \cos\alpha° - k\sin x° \sin\alpha°$

•[5] $k\sin\alpha° = \sqrt{3}$ **and** $k\sin\alpha° = 1$

•[6] $k = \sqrt{7}\ \overrightarrow{OG} = 426$

•[7] $\alpha = 49.1$

(*c*)

•[8] $\sqrt{7}\cos(x+49.1)° = 1$

•[9] $x = 18.7°$

•[10] $x = 243.1°$

(*a*) The function *f* is defined by $f(x) = x^3 - 2x^2 - 5x + 6$.

The function *g* is defined by $g(x) = x - 1$.

Show that $f(g(x)) = x^3 - 5x^2 + 2x + 8$. **(4)**

(*b*) Factorise fully $f(g(x))$. **(3)**

(*c*) The function *k* is such that $k(x) = \dfrac{1}{f(g(x))}$.

For what values of *x* is the function *k* not defined? **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(*a*)	4	1.2	4						1.2.6		Source
(*b*)	3	2.1	3						2.1.3		1990 Paper 2
(*c*)	2	1.2	2						1.2.1		Qu. 6

(*a*) •1 $f(g(x)) = f(x-1)$

•2 $(x-1)^3 - 2(x-1)^2 - 5(x-1) + 6$

•3 $(x-1)^3 = x^3 - 3x^2 + 3x - 1$

•4 $-2x^2 + 4x - 2 - 5x + 5 + 6$ and completing argument

(*b*) •5 *first "0" e.g.*

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 2 & 8 \\ & & 2 & -6 & -8 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

•6 $x^2 - 3x - 4 = (x+1)(x-4)$

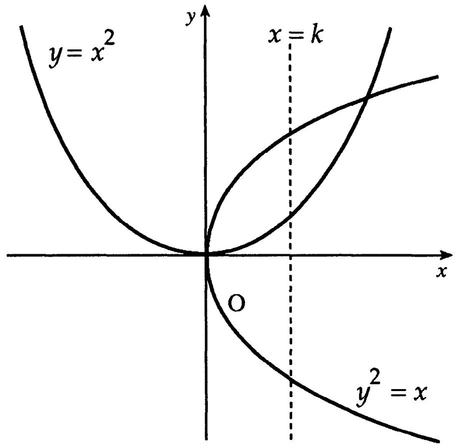
•7 $(x-2)(x+1)(x-4)$

(*c*) •8 denominator $(= (x-2)(x+1)(x-4)) \neq 0$

•9 $-1,\ 2,\ 4$

The diagram shows two curves with equations $y = x^2$ and $y^2 = x$.

The area completely enclosed between the two curves is divided in half by the line with equation $x = k$.



(*a*) Represent these two equal areas by two separate integrals each involving k. **(6)**

(*b*) Equate the integrals and show that k is given by the equation

$$2k^3 - 4k^{\frac{3}{2}} + 1 = 0.$$ **(4)**

(*c*) Use the substitution p^2 for k^3 to find the value of k. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	6	2.2	3	3					2.2.7		Source
(*b*)	4	2.2	2	2					2.2.5,	0.1	**1990 Paper 2**
(*c*)	4	0.1		4					0.1		**Qu. 7**

(*a*)
- •¹ strategy: equate functions
- •² $x^4 = x$
- •³ $x(x^3 - 1) = 0 \Rightarrow x = 0,\ x = 1$
- •⁴ $\int x^{\frac{1}{2}} - x^2\ dx$
- •⁵ $\int_0^k x^{\frac{1}{2}} - x^2\ dx$
- •⁶ $\int_k^1 x^{\frac{1}{2}} - x^2\ dx$

(*b*)
- •⁷ $\frac{2}{3}x^{\frac{3}{2}}$
- •⁸ $\frac{1}{3}x^3$
- •⁹ $\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3$ *or* $\frac{2}{3} - \frac{1}{3} - \left(\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3\right)$
- •¹⁰ $\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3 = \frac{2}{3} - \frac{1}{3} - \left(\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3\right)$ and completing proof

(*c*)
- •¹¹ $2p^2 - 4p + 1 = 0$
- •¹² strategy for solving: *e.g.* $p = \frac{4 \pm \sqrt{16 - 8}}{4}$
- •¹³ $p = 0.293,\ 1.707$
- •¹⁴ $k = 0.441$

A sports club awards trophies in the form of paperweights bearing the club crest. Diagram 1 shows the front view of one of these paperweights. Each is made from two different types of glass. The two circles are concentric and the base line is a tangent to the inner circle.

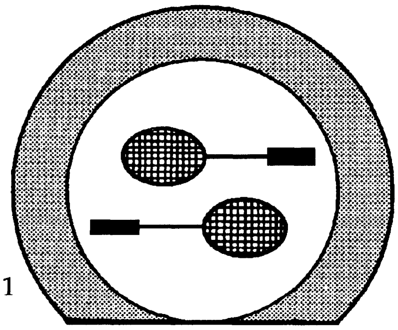


Diagram 1

(*a*) Relative to *x*, *y* coordinate axes, the equation of the outer circle is $x^2 + y^2 - 8x + 2y - 19 = 0$ and the equation of the base line is $y = -6$.

Show that the equation of the inner circle is $x^2 + y^2 - 8x + 2y - 8 = 0$. **(4)**

(*b*) An alternative form of the paperweight is made by cutting off a piece of glass from the original design along a second line with equation $3x - 4y + 9 = 0$ as shown in diagram 2.
Show that this line is a tangent to the inner circle and state the coordinates of the point of contact. **(7)**

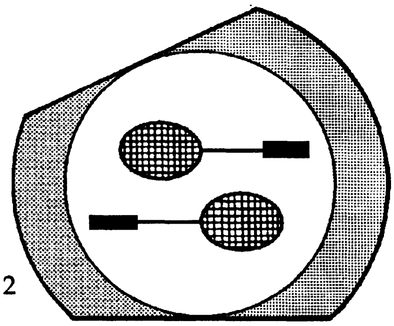


Diagram 2

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.4
(*a*)	4	2.4					4		2.4.3		Source
(*b*)	7	2.4					3	4	2.4.4		**1990 Paper 2 Qu. 8**

(*a*)
- •1 centre = (4,–1)
- •2 inner radius = 5
- •3 $(x-4)^2 + (y+1)^2 = 25$
- •4 completing argument

(*b*)
- •5 *e.g.* $x = \frac{4}{3}y - 3$
- •6 $\left(\frac{4}{3}y - 3\right)^2 + y^2 - 8\left(\frac{4}{3}y - 3\right) + 2y - 8 = 0$
- •7 $\frac{16}{9}y^2 - 8y + 9 + y^2 - \frac{32}{3}y + 24 + 2y - 8$
- •8 $y^2 - 6y + 9 = 0$
- •9 *e.g.* $(y-3)(y-3) = 0$
- •10 equal roots ⇒ line is a tangent
- •11 (1,3)

Polynomial equations often have roots which are not whole numbers.
One method of estimating the roots of such equations is to make repeated use of the following:

> If $x = p$ is an estimate of a root of the equation $f(x) = 0$, then $x = q$ will will be a closer estimate where $q = p - \frac{f(p)}{f'(p)}$.

> **Example**
>
> One of the roots of the equation $x^2 - 2x - 5 = 0$ is known to lie between 3 and 4.
>
> We have $f(x) = x^2 - 2x - 5$ and so $f'(x) = 2x - 2$.
>
> **Choose p = 3** (1st estimate) then $q = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{-2}{4} = 3.5$.
>
> **Choose p = 3.5** (2nd estimate) then $q = 3.5 - \frac{f(3.5)}{f'(3.5)} = 3.5 - \frac{0.25}{5} = 3.45$.
>
> **Choose p = 3.45** (3rd estimate) then $q = 3.45 - \frac{f(3.45)}{f'(3.45)} = 3.45 - \frac{0.0025}{4.9} = 3.449$.
>
> **Conclusion** The root, correct to 1 decimal place, is **x = 3.4**

(*a*) Show that the equation $x^3 - 2x^2 + 6x - 4 = 0$ has a root between 0 and 1. **(3)**

(*b*) Use the method described above to find this root correct to 1 decimal place. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	4
(*a*)	3	2.1			3				2.1.11		Source
(*b*)	6	0.1			3	3			0.1		**1990 Paper 2 Qu. 9**

(*a*)
- •1 $0^3 - 2\times0 + 6\times0 - 4 = -4$
- •2 $1^3 - 2\times1 + 6\times1 - 4 = 1$
- •3 $f(0) < 0$ and $f(1) > 0$ so $0 < \text{root} < 1$

(*b*)
- •4 $f'(x) = 3x^2 - 4x + 6$
- •5 *e.g.* 1st est = 0, 2nd est $= 0 - \frac{f(0)}{f'(0)} = 0.67$
- •6 3rd est $= 0.67 - \frac{f(0.67)}{f'(0.67)}$
- •7 0.7936
- •8 4th est $= 0.7936 - \frac{f(0.7936)}{f'(0.7936)} = 0.7932$
- •9 0.8

The Water Board of a local authority discovered it was able to represent the approximate amount of water $W(t)$, in millions of gallons, stored in a reservoir t months after the 1st May 1988 by the formula $W(t) = 1.1 - \sin\frac{\pi t}{6}$.
The board then predicted that under normal conditions this formula would apply for three years.

(*a*) Draw and label sketches of the graphs of $y = \sin\frac{\pi t}{6}$ and $y = -\sin\frac{\pi t}{6}$, for $0 \le t \le 36$ on the same diagram. **(4)**

(*b*) On a separate diagram and using the same scale on the t-axis as you used in part (*a*), draw a sketch of the graph of $W(t) = 1.1 - \sin\frac{\pi t}{6}$. **(3)**

(*c*) On the 1st April 1990 a serious fire required an extra $\frac{1}{4}$ million gallons of water from the reservoir to bring the fire under control.
Assuming that the previous trend continues from the new lower level, when will the reservoir run dry if water rationing is not imposed? **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.2
(*a*)	4	1.2		4					1.2.3		Source
(*b*)	3	1.2		3					1.2.4		**1990 Paper 2**
(*c*)	3	1.2		3					1.2.9		**Qu. 10**

(*a*)
- •1 correct scales
- •2 zeros
- •3 graph of $y = \sin\frac{\pi t}{6}$
- •4 graph of $y = -\sin\frac{\pi t}{6}$

(*b*)
- •5 indication of translation $\begin{pmatrix} 0 \\ 1.1 \end{pmatrix}$ to $y = -\sin\frac{\pi t}{6}$
- •6 for minima at $W = 0.1$
- •7 sketch

(*c*)
- •8 indicate on graph effect of fire at $t = 23$
- •9 $t = 26$ (±1)
- •10 about July(±1) 1990

A function f is defined by the formula $f(x) = 4x^2(x-3)$ where $x \in \mathbf{R}$.

(*a*) Write down the coordinates of the points where the curve with equation $y = f(x)$ meets the x- and y-axes. **(2)**

(*b*) Find the stationary points of $y = f(x)$ and determine the nature of each. **(6)**

(*c*) Sketch the curve $y = f(x)$. **(2)**

(*d*) Find the area completely enclosed by the curve $y = f(x)$ and the x-axis. **(4)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	2.2
			C	A/B	C	A/B	C	A/B	Main Additional	
(*a*)	2	1.2	2						1.2.9	Source
(*b*)	6	1.3	6						1.3.12	**1989 Paper 2**
(*c*)	2	1.3	2						1.3.13	**Qu. 1**
(*d*)	4	2.2	4						2.2.6	

(*a*)
- •1 (0,0)
- •2 (3,0)

(*b*)
- •3 $f'(x) = 12x^2 - 24x$
- •4 $f'(x) = 0$ **stated explicitly**
- •5 $x = 0,\ x = 2$
- •6

x	0^-	0	0^+	2^-	2	2^+
f'	$+$	0	$-$	$-$	0	$+$

- •7 max at (0,0)
- •8 min at (2,−16)

(*c*)
- •9 correct shape
- •10 (0,0),(3,0),(2,−16) annotated

(*d*)
- •11 $\int_0^3 \left(4x^3 - 12x^2\right) dx$
- •12 area $= -\int_0^3 \left(4x^3 - 12x^2\right) dx$
- •13 $\left[-x^4 + 4x^3\right]_0^3$
- •14 27

ABCD is a quadrilateral with vertices A(4, –1, 3), B(8, 3, –1), C(0, 4, 4) and D(–4, 0, 8).

(*a*) Find the coordinates of M, the midpoint of AB. **(1)**

(*b*) Find the coordinates of the point T, which divides CM in the ratio 2:1. **(3)**

(*c*) Show that B, T and D are collinear and find the ratio in which T divides BD. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
(*a*)	1	0.1					1		0.1		Source
(*b*)	3	3.1					3		3.1.6		**1989 Paper 2**
(c)	4	3.1					4		3.1.7,	3.1.6	**Qu. 2**

(*a*) •1 $(6,1,1)$

(*b*) •2 *e.g.* $\overrightarrow{CM} = \begin{pmatrix} 6 \\ -3 \\ -3 \end{pmatrix}$

•3 $\overrightarrow{CT} = \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}$

•4 $T = (4,2,2)$

(*c*) •5 *e.g.* $\overrightarrow{BT} = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}$

•6 $\overrightarrow{TD} = \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} = 2 \times \overrightarrow{BT}$

•7 TD is parallel to BT, T is common point so B, T, D collinear

•8 BT:TD = 1:2

(*a*) (i) Make a sketch of the graph of $y = x^3$, where $-3 \leq x \leq 3,\ x \in \mathbf{R}$.

(ii) On the same diagram, draw the graph of $y = 6x + 1$. **(3)**

(*b*) State the number of roots which the equation $x^3 = 6x + 1$ has in the interval $-3 \leq x \leq 3$. **(1)**

(*c*) Calculate the value of the positive root, correct to 3 significant figures. **(4)**

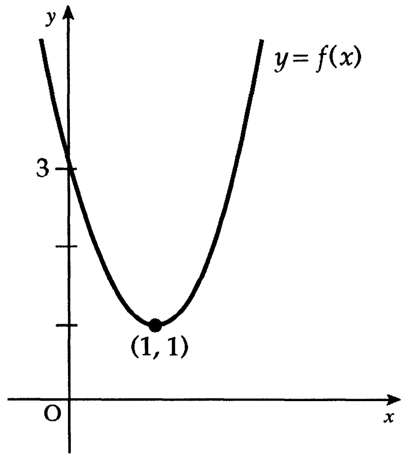
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	3	0.1	3						0.1		Source
(*b*)	1	0.1	1						0.1		**1989 Paper 2**
(*c*)	4	2.1	1	3					2.1.11		**Qu. 3**

(*a*)
- •1 suitable choice of scales
- •2 sketch of $y = x^3$ from $x = -3$ to $x = 3$
- •3 sketch of $y = 6x + 1$ from $x = -3$ to $x = 3$

(*b*)
- •4 3 roots

(*c*)
- •5 1st estimate: between 2 and 3
- •6 2nd estimate: between 2.5 and 2.6
- •7 3rd estimate: between 2.53 and 2.534
- •8 2.53

The diagram shows a sketch of the parabola $y = f(x)$.



(*a*) Copy the sketch of $y = f(x)$. On your diagram, draw the parabola with equation $y = -f(x) + 3$. **(4)**

(*b*) State the values of x for which $3 - f(x) \geq 0$. **(2)**

(*c*) If $g(x) = 3 - f(x)$, express $g(x)$ in terms of x. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.2
(*a*)	4	1.2		4					1.2.4		Source
(*b*)	2	1.2		2					1.2.1		**1989 Paper 2**
(*c*)	3	1.2		3					1.2.7		**Qu. 4**

(*a*)
- •1 inverted shape
- •2 passing through origin
- •3 annotating $(1,2)$
- •4 annotating $(2,0)$

(*b*)
- •5 endpoints of $0 \leq x \leq 2$
- •6 "less than signs" of $0 \leq x \leq 2$

(*c*)
- •7 $g(x) = ax(x-2)$
- •8 $(1,2) \Rightarrow 2 = a(1-2)$
- •9 $g(x) = -2x(x-2)$

An ear-ring is to be made from silver wire and is designed in the shape of two touching circles with two tangents to the outer circle as shown in Diagram 1.

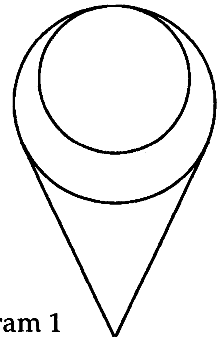


Diagram 1

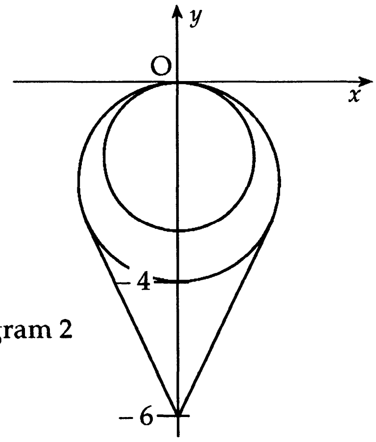


Diagram 2

Diagram 2 shows a drawing of this ear-ring related to the coordinate axes.

The circles touch at (0, 0).

The equation of the inner circle is $x^2 + y^2 + 3y = 0$.

The outer circle intersects the y-axis at (0, –4).

The tangents meet the y-axis at (0, –6).

Find the total length of silver wire required to make this ear-ring. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.4
(-)	6	2.4			6				2.4.2, 2.4.4	Source **1989 Paper 2 Qu. 5**

(-)

- $\bullet^1$ radius of inner circle $= \frac{3}{2}$
- $\bullet^2$ centres are $\left(0, -1\frac{1}{2}\right)$ **and** $(0, -2)$
- $\bullet^3$ circumferences are 3π **and** 4π
- $\bullet^4$ *e.g.* $\text{tgt}^2 = 4^2 - 2^2$
- $\bullet^5$ $\text{tgt} = \sqrt{12}$
- $\bullet^6$ 29

Some environmentalists are concerned that the presence of chemical nitrates in drinking water presents a threat to health.

The World Health Organisation recommends an upper limit of 50 milligrams per litre (mg/l) for nitrates in drinking water, although it regards levels up to 100 mg/l as safe.

A sub-committee of a Local Water Authority is considering a proposal affecting a small loch which supplies a nearby town with drinking water. The proposal is that a local factory be permitted to make a once-a-week discharge of effluent into the loch, provided that a cleaning treatment of the loch is carried out before each discharge of effluent.

The Water Engineer has presented the following data:

1. The present nitrate level in the loch is 20 mg/l.
2. The cleaning treatment removes 55% of the nitrates from the loch.
3. Each discharge of effluent will result in an addition of 26 mg/l to the nitrate presence in the loch.

and advises the sub-committee that the proposal presents no long-term danger from nitrates to the drinking water supply.

(*a*) Show the calculations you would use to check the engineer's advice. **(5)**

(*b*) Is the engineer's advice acceptable? **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	1.4
(*a*)	5	1.4			5				1.4.1, 1.4.5	Source **1989 Paper 2 Qu. 6**
(*b*)	1	0.1			1				0.1	

(*a*)
- •[1] $u_0 = 20$
- •[2] $u_1 = 35$
- •[3] three further values eg 41.75, 44.78, 46.15
- •[4] 46.76, 47.04, 47.17 looks like approaching a limit
- •[5] five more lead to 47.27'something' $\Rightarrow$ limit = 47.27

(*b*)
- •[6] $47.27 < 50$ so level safe

Diagram 1 is an artist's impression of a new warehouse based on the architect's plans. The warehouse is in the shape of a cuboid and is supported by three identical parabolic girders spaced 30 metres apart.
With coordinate axes as shown in Diagram 2, the shape of each girder can be described by the equation $y = 9 - \frac{1}{4}x^2$.

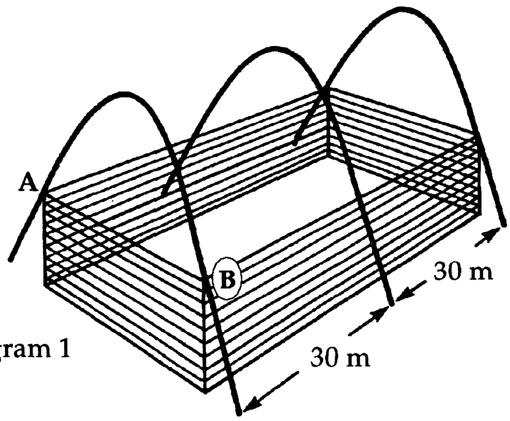


Diagram 1

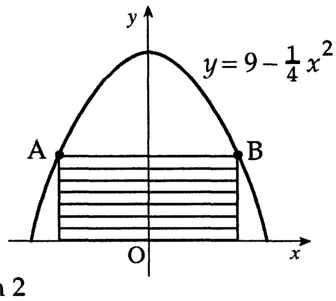


Diagram 2

(*a*) Given that AB is $2x$ metres long, show that the shaded area in Diagram 2 is $\left(18x - \frac{1}{2}x^2\right)$ square metres. **(2)**

(*b*) The architect wished to fit into the girders the cuboidal warehouse which had the maximum volume. Find the value of this maximum volume. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(*a*)	2	0.1					2		0.1		Source
(*b*)	6	1.3					3	3	1.3.15		1989 Paper 2 Qu. 7

(*a*)
- •[1] B $= (x, y)$ where $y = 9 - \frac{1}{4}x^2$
- •[2] area $= 2x\left(9 - \frac{1}{4}x^2\right)$

(*b*)
- •[3] $V = 1080x - 30x^3$
- •[4] $\frac{dV}{dx} = 1080 - 90x^2$
- •[5] $\frac{dV}{dx} = 0$ **stated explicitly**
- •[6] $x = 2\sqrt{3}$
- •[7]

x	$2\sqrt{3}^-$	$2\sqrt{3}$	$2\sqrt{3}^+$
$\frac{dV}{dx}$	+	0	−

- •[8] max at $x = 2\sqrt{3}$ of $1440\sqrt{3}$

A function f is **EVEN** if $f(-x) = f(x)$

e.g. when $f(x) = x^2$, f is **EVEN** because $f(-x) = (-x)^2 = x^2 = f(x)$.

A function f is **ODD** if $f(-x) = -f(x)$

e.g. when $f(x) = x^3$, f is **ODD** because $f(-x) = (-x)^3 = -x^3 = -f(x)$.

(*a*) Given that $g(x) = \cos x$ and $h(x) = \sin 2x$, decide for each of the functions g and h whether it is **EVEN** or **ODD**.
Justify your decisions. **(4)**

(*b*) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$ and $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2x \, dx$. **(5)**

(*c*) On separate diagrams, draw rough sketches of the graphs of $y = \cos x$ and $y = \sin 2x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. **(2)**

(*d*) If $v(x) = x\cos x$, check whether the function v is **EVEN** or **ODD** and suggest a value for $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x \, dx$. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	0.1
(*a*)	4	0.1	4						0.1		Source
(*b*)	5	3.2	2	3					3.2.1,	3.2.4	**1989 Paper 2**
(*c*)	2	1.2	2						1.2.3		**Qu. 8**
(*d*)	2	0.1		2					0.1		

(*a*)
- •1 $\cos(-x) = \cos x$
- •2 g is EVEN
- •3 $\sin(-2x) = -\sin(2x)$
- •4 h is ODD

(*b*)
- •5 $\sin x$
- •6 $[\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2$
- •7 $-\cos 2x$
- •8 $\times\frac{1}{2}$
- •9 $\left[-\frac{1}{2}\cos 2x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$

(*c*)
- •10 sketch of $g(x) = \cos x$
- •11 sketch of $h(x) = \sin 2x$

(*d*)
- •12 $v(x)$ is ODD
- •13 0

The formula $d = 200 + 80(\cos 30t° + \sqrt{3}\sin 30t°)$ gives an approximation to the depth of water, d, measured in centimetres, in a harbour t hours after midnight.

(*a*) Express $f(t) = \cos 30t° + \sqrt{3}\sin 30t°$ in the form $k\cos(30t - \alpha)°$ and state the values of k and α, where $0 \le \alpha \le 360$. **(4)**

(*b*) (i) Use your result from part (*a*) to help you sketch the graph of $f(t)$ for $0 \le t \le 12$.

(ii) Hence, on a separate diagram, sketch the graph of d for $0 \le d \le 12$. **(6)**

(*c*) What is the "low-water" time at the harbour during the time interval shown on your graph? **(1)**

(*d*) If the local fishing fleet needs at least 1·5 metres depth of water to enter the harbour without risk of running aground, between what hours must it avoid entering the harbour during the time interval shown on your graph? **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.4
(*a*)	4	3.4			4				3.4.1		Source
(*b*)	6	1.2			2	4			1.2.3,	1.2.4	**1989 Paper 2**
(*c*)	1	0.1				1			0.1		**Qu. 9**
(*d*)	2	0.1				2			0.1		

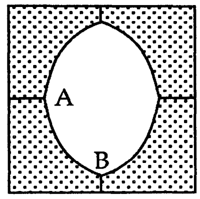
(*a*)
- •1 $k\cos 30t°\cos\alpha° + k\sin 30t°\sin\alpha°$
- •2 $k\cos\alpha° = 1$ **and** $k\sin\alpha° = \sqrt{3}$
- •3 $k = 2$
- •4 $\alpha = 60$

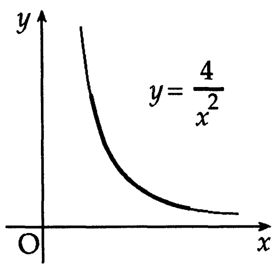
(*b*)
- •5 maximum at (2, 2)
- •6 minimum at (8, −2)
- •7 endpoints: (0, 1) **or** (12, 1)
- •8 graph correctly annotated with 3 points
- •9 sketch with original amplitude increased by factor of 60
- •10 sketch with original graph translated $\begin{pmatrix} 0 \\ 200 \end{pmatrix}$

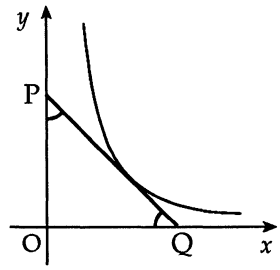
(*c*)
- •11 0800 hours

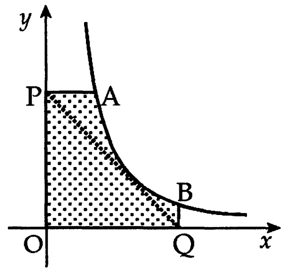
(*d*)
- •12 5.6 hours and 10.4 hours
- •13 e.g. between 5am and 11am

The makers of "OLO", the square mint with the not-so-round hole, commissioned an advertising agency to prepare an illustration to the specification described in (i) to (iii) below. The finished illustration will look like the diagram on the right.









(i) The curve AB in the finished illustration is part of the curve with equation $y = \frac{4}{x^2}$.

(ii) A tangent to this curve, making equal angles with both axes, is to be drawn as shown (line PQ)

(iii) Straight lines perpendicular to the axes are to be drawn from P and Q as shown. The shaded part forms $\frac{1}{4}$ of the finished illustration.

(*a*) State the gradient of PQ and hence find the point of contact of the tangent PQ with the curve. **(5)**

(*b*) Find the equation of PQ and the coordinates of A and B. **(4)**

(*c*) Calculate the shaded area of the finished illustration. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.2
(*a*)	5	1.1	3	2					1.1.3,	1.1.10	Source
(*b*)	4	1.1	1	3					1.1.7,	0.1	**1989 Paper 2**
(*c*)	6	2.2		6					2.2.6		**Qu. 10**

(*a*)
- •1 $m_{PQ} = -1$
- •2 $f(x) = 4x^{-2}$
- •3 $f'(x) = -8x^{-3}$
- •4 $-8x^{-3} = -1$
- •5 $x = 2$ and $f(2) = 1$

(*b*)
- •6 $x + y = 3$
- •7 $\frac{4}{x^2} = 3$
- •8 $x \approx 1.15$
- •9 A(1.15,3), B(3,0.44)

(*c*)
- •10 suitable division of area
- •11 rectangle $OPA'C' = 3 \times 1.15 = 3.45$
- •12 curved area $QBA'C' = \int_{1.15}^{3} \frac{4}{x^2}\,dx$
- •13 $\left[-\frac{4}{x}\right]_{1.15}^{3}$
- •14 2.15
- •15 $(3.45 + 2.15) \times 4 = 22.4$