Mathematics

Mathematics 2

Intermediate 1

4727

Spring 1999

HIGHER STILL

Mathematics

Mathematics 2

Intermediate 1

Support Materials

STAFF NOTES

Introduction

These support materials for Mathematics were developed as part of the Higher Still Development Programme in response to needs identified at needs analysis meetings and national seminars.

Advice on learning and teaching may be found in *Achievement for All* (SOEID 1996), *Effective Learning and Teaching in Mathematics* (SOEID 1993) and in the Mathematics Subject Guide.

This support package provides student material to cover the content of Mathematics 2 within the Intermediate 1 course. The depth of treatment is therefore more than is required to demonstrate competence in the unit assessment; that is, it goes beyond minimum grade C. The content of Mathematics 2 (Int 1) is set out in the landscape pages of content in the Arrangements document where the requirements of the unit Mathematics 2 (Int 1) are also stated. Students may have met some of the work in A (Integers) and B (Time, Distance Speed), but the work in part C (Pythagoras) and in the Statistic part of the unit may be new to most students.

The material is designed to be directed by the teacher/lecturer, who will decide on the ways of introducing topics and in the use of exercises for consolidation and for formative assessment. The use of calculators will be necessary for Part C and possibly part B, but students should be encouraged to set down all working and, where appropriate, use mental calculations. The use of computers is obviously highly desirable for some of the statistical content of the course.

An attempt has been made to have the ‘easy’ questions at the start of each exercise, leading to more testing questions towards the end of the exercise. While students may tackle most of the questions individually, there are opportunities for collaborative working. Staff may wish to discuss points raised with individuals, groups and the whole class.

The specimen assessment questions at the end of the package are **not** intended to be only at minimum grade C. The National Assessment Bank packages for Mathematics 2 (Int 1) contain questions that meet the requirements of this unit.

This package gives opportunities to practise core skills, particularly the components of the Numeracy core skill, Using Number and Using Graphical Information, and Problem Solving. Information on the core skills embedded in the unit, Mathematics 2 (Int 1) and in the Intermediate 1 course is given in the final version of the Arrangements document. General advice and details of the Core Skills Framework can be found in the Core Skills Manual (HSDU June 1998).

Brief notes of advice on the teaching of each topic are given.

Format of Student Material

- Exercises on Integers
  Checkup for Integers

- Exercises on Time, Distance Speed
  Checkup for Time, Distance Speed

- Exercises on Pythagoras
  Checkup for Pythagoras

- Specimen Assessment Questions

- Answers

INTEGERS

A. Coordinates in all four Quadrants

Students should be shown how to extend the number line, both vertically and horizontally, to include negative integers.

The set of positive and negative whole numbers, plus zero should be defined to students as ‘integers’.

Students should now be shown how to extend the *x* and *y* - axes to include negatives. Various points can be plotted at random in all 4 quadrants and students asked to give their coordinates.

Exercise 1 may now be attempted.

B. Addition and Subtraction of integers

A thermometer, drawn on the board (vertically) could be used to help students deal with addition and subtraction of integers. Questions could be asked, such as:

1. By how many degrees did a thermometer drop from 3°C o –5°C? (8°C)
2. By how many degrees did a thermometer rise from –8°C o –1°C? (7°C)
3. What was the new temperature when the thermometer :-
   (a) fell from 2°C by 8°C? (–6°C)
   (b) rose from –20°C by 12°C ? (–8°C)

A technique which could be helpful to students when adding or subtracting integers is to:

(a) picture the first number (on a thermometer or a number line), then
(b) (i) if it is followed by + (a positive number) => then move UP
(ii) if it is followed by + (a negative number) => then move DOWN
(iii) if it is followed by – (a positive number) => then move DOWN

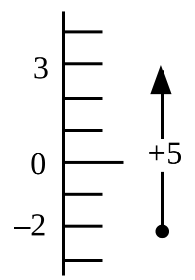
example of (i) –6 + 9 => (picture –6, then move UP by 9 => 3)
(ii) 2 + (–6) => (picture 2, then move DOWN by 6 => –4)
(iii) 5 – 7 => (picture 5, then move DOWN by 7 => –2)

Exercise 2A may now be attempted.

Double Negatives

Note: the examples in exercise 2B are of the type indicated by the different font and are, as such, beyond grade C.

The example 3 – (–2) can be shown graphically as the ‘distance’ from –2 up to 3, i.e. (+)5



This example can also be used :- –3 –(–7) = –3 + 7 = 4

Exercise 2B may now be attempted.

C. Multiplication and Division of Integers

Students should be shown that, initially, they can think of multiplication as an alternative to repeated addition. i.e.

$$5 \times (-3) = (-3) + (-3) + (-3) + (-3) + (-3) = -15$$

Several examples of this type should be shown to students, e.g. $7 \times (-4)$ etc.

It should also be indicated that $(-6) \times 8$ gives the same answer as $8 \times (-6) = -48$ and students lead to the rule:

(positive) x (negative) = (negative) x (positive) = (negative)

Students should now be encouraged to decide what happens with division of integers (positive denominator only).

$$\Rightarrow \quad \frac{-10}{5} = -5 \quad \text{and} \quad (-24) \div 4 = -6$$

Exercise 3A may now be attempted.

Double Negatives

Note: the examples in exercise 3B are of the type indicated by the different font and are, as such, beyond grade C.

The example below could be used to illustrate the rule

(positive) ÷ (negative) and (negative) ÷ (positive) = (negative)

Example: $30 \div (-5)$ does <u>not</u> give 6 since $6 \times (-5)$ does not take you back to 30!
hence $30 \div (-5)$ must give (-6)

Now, it should be shown what happens when two negatives are multiplied.
i.e. use the 'reverse' operation to show why $(-4) \times (-5) = (+)20$

$(-4) \times (-5) = 20$ because $20 \div (-5)$ <u>does</u> give (-4)!

Then, show what happens when two negatives are divided.
i.e. use the 'reverse' operation to show why $(-24) \div (-8) = (+)3$

i.e. $(-24) \div (-8) = 3$ because $3 \times (-8)$ <u>does</u> give (-24)!

Exercise 3B, questions 1 to 3 may now be attempted.

Students should be led through what happens when 3 or more integers are multiplied together. Each example can be done in stages:

i.e.

$$(-4) \times (-3) \times (-5) = 12 \times (-5) = -60$$

or

$$(-5) \times 2 \times (-7) = (-10) \times (-7) = 70$$

Exercise 3B, questions 4 and 5 may now be attempted.

The checkup exercise may also be attempted.

SPEED, DISTANCE AND TIME

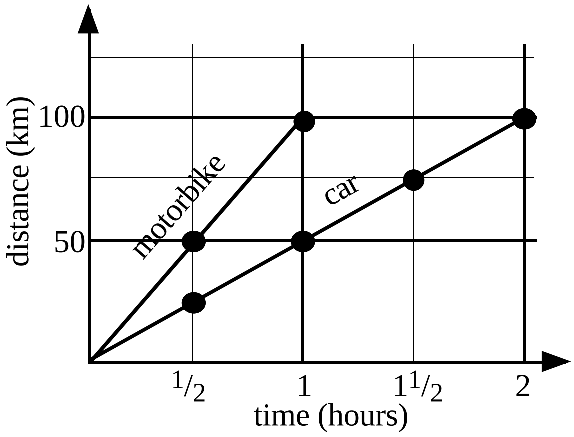
A. Interpret Distance - Time Graphs.

Distance - time graphs should be discussed with students using the example given opposite.

Point out that the ‘slope’ of the graph indicates the speed of the vehicle.

The steeper the slope, the faster the object.

To calculate speed:
Elicit from the students that to calculate the speed of a vehicle, you have to determine how far it travels in 1 hour (or minute or second).



Example: the car travelled 50 km in 1 hour => its speed is 50 km/hr

the motorbike travelled 100 km in 1 hour => its speed is 100 km/hr

Exercise 1 may now be attempted.

B. Time Intervals

Work done at Standard Grade on time intervals could be revised.

Example: How long is it from 10:45 am to 2:20 pm?

Method 1:

15 mins + 3 hrs + 20 mins = **3 hrs 35 mins**

10:45 am —> 11:00 am —> 2:00 pm —> 2:20 pm

If wished, this alternative method could be shown:

Method 2:

hrs	mins
^{13}14	^{60}20 ^{80}
10	45
3	35

(in 24 hour format!)

This example could also be used:
A ship leaves harbour at 10:50 pm and it arrives next morning at 6:15 am.
How long was the trip?

1 hr 10 mins + 6 hrs + 15 mins = **7 hrs 25 mins**

10:50 pm —> midnight —> 6:00 am —> 6:15 am

Exercise 2 may now be attempted.

C. Speed

Using easy values, it should be discussed how 'speed' is calculated given distance and time taken.

For example: If it takes me 2 hours to travel the 80 miles to Dundee
=> In 1 hour I would travel only 40 miles
=> My speed must be 40 m.p.h. (miles per hour).

This should lead to

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

Units, such as km/hr, metres/sec etc. should be discussed.

Exercise 3A, questions 1 to 3, may be attempted.

It should be emphasised that 2 hours 15 minutes is <u>not</u> entered into the calculator as 2·15 hours.

These examples can be used to illustrate this:

1 hour 30 mins	=	$1\frac{1}{2}$ hrs	=	1·5 hours
2 hours 15 mins	=	$2\frac{1}{4}$ hrs	=	2·75 hours
45 minutes	=	$\frac{3}{4}$ hr	=	0·75 hours

Example: I drive 90 kilometres in 2 hour and 45 minutes. Calculate my average speed.

$$\text{SPEED} = \frac{\text{Dist}}{\text{Time}} = \frac{90 \text{ km}}{2\text{hr } 45 \text{ min}} = \frac{90}{2{\cdot}75} = 40 \text{ km/hr}$$

Exercise 3A, questions 4 to 6, may be attempted.

Time

Using easy values, it should be discussed how 'time' is calculated given distance and the speed.

For example: If I travel the 80 miles to Dundee at an average speed of 40 m.p.h
=> It will take me 80 ÷ 40 hours to complete the journey
=> My time taken is therefore 2 hours.

This should lead to

$$\text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}}$$

Some simple examples should now be shown to students .

Exercise 3B may now be attempted.

Distance

Using easy values, it should be discussed how 'distance' is calculated given speed and time.

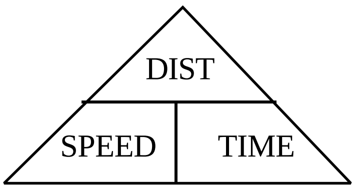
For example: If I travel at an average speed of 40 m.p.h for 2 hours
=> I will cover a distance of (2 x 40) miles
=> I will have travelled 80 miles.

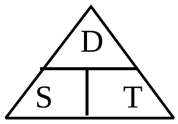
This should lead to **DISTANCE = SPEED x TIME**

Some simple examples should now be shown to students .

Exercise 3C may now be attempted.

The Time – Distance – Speed Triangle could be given as an help.





The checkup exercise may now be attempted.

THEOREM OF PYTHAGORAS

Introduction: Squares and Square Roots

Students shold be shown both the 'x^2' and '√' button on calculator.

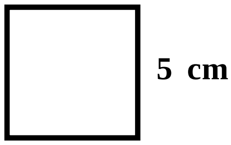
This exercise is simply based on revision of:

(i) the use of a calculator to square and square root.

(ii) the use of $A = L^2$, the formula for finding the area of a square.

The following examples can be used:

Example 1. Find the area of this square:

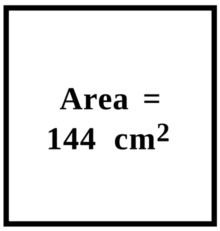


Ans. $A = L^2$

$= 5^2$

$= \underline{\underline{25 \text{ cm}^2}}$

Example 2. If the area of this square is 144 cm^2, find the length of one of its sides.



Ans. $A = L^2$

$144 = L^2$

$\sqrt{144} = L$

$L = \underline{\underline{12 \text{ cm}}}$

Exercise 1 may now be attempted

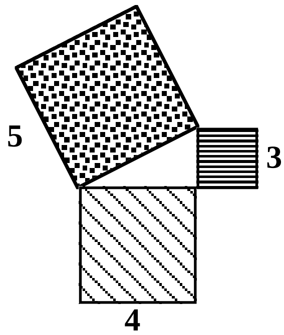
A. The Theorem of Pythagoras

Finding the length of one side of a right angled triangle, given the length of the other two sides.

(a) Finding the <u>longest</u> side:

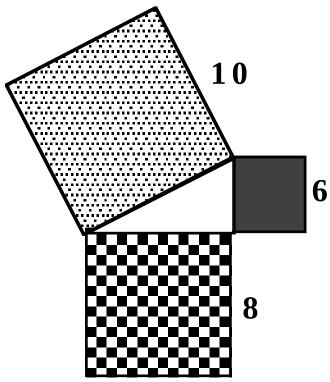
The term 'Hypotenuse' should be explained.

Then the following example can be given:



Ask the students to work out the area of each of the three squares.

Ans. 9, 16, 25



Ask them to do the same again with this shape.

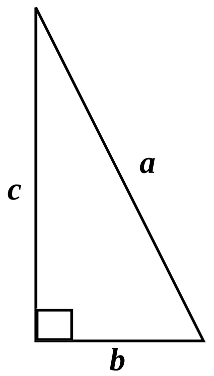
Ans. 36, 64, 100

Perhaps do another, with squares of sides 5, 12 and 13.

From these answers it can be deduced that:

the area of the square on the longest side = the area of the square on the shortest side + the area of the square on the middle side

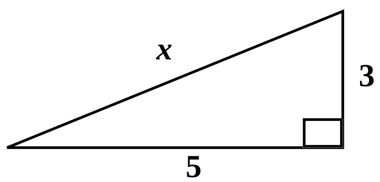
In summary, in a right angled triangle with sides *a*, *b* and *c*.



$$a^2 = b^2 + c^2$$

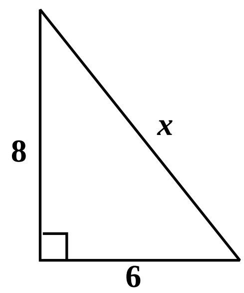
When calculating the longest side (hypotenuse) in a right angled triangle, use "Pythagoras <u>Plus</u>".

Example 1. Write an equation for *x*.



Ans. $x^2 = 5^2 + 3^2$

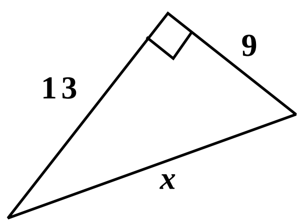
Example 2. Calculate *x*, the longest side (correct to 1 decimal place if necessary).



Ans.

$x^2 = 8^2 + 6^2$	(correct formula '+')
$= 64 + 36$	(square out)
$= 100$	(tidy)
$x = \sqrt{100}$	(bring in √)
$= \underline{\underline{10}}$	(use calc.)

Example 3. Calculate *x*, (correct to 1 decimal place if necessary).



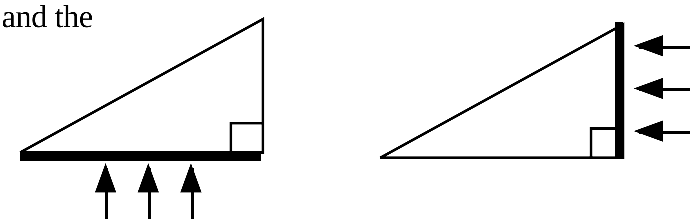
Ans.

$x^2 = 13^2 + 9^2$	(correct formula '+')
$= 169 + 81$	(square out)
$= 250$	(tidy)
$x = \sqrt{250}$	(bring in √)
$= \underline{\underline{15 \cdot 8}}$	(use calc. and round)

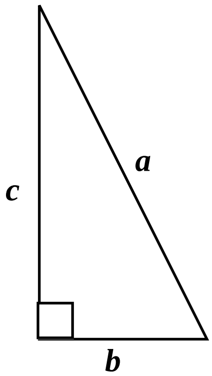
Exercise 2 may now be attempted

(b) Finding one of the <u>shorter</u> sides:

The difference between the hypotenuse and the shorter sides should be shown.



For this case, change the original formula to arrive at:



Of the 2 sides given, this number should always be the larger one.

$$b^2 = a^2 - c^2$$

or

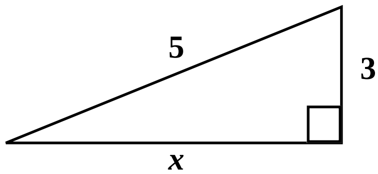
$$c^2 = a^2 - b^2$$

When calculating one of the shorter sides (**not** the hypotenuse) in a right angled triangle - use "Pythagoras <u>Minus</u>".

Example 1. Write an equation for *x:*

Ans. $x^2 = 5^2 - 3^2$

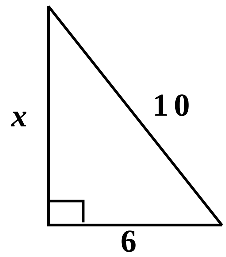
the larger of the two numbers given.



Example 2. Calculate the missing side *x*, (correct to 1 decimal place if necessary).

Ans.

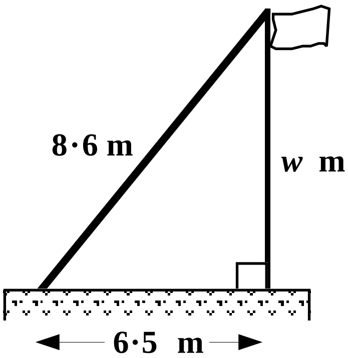
$x^2 = 10^2 - 6^2$	(correct formula "-")
$= 100 - 36$	(square out)
$= 64$	(tidy)
$x = \sqrt{64}$	(bring in √)
$= \underline{\underline{8}}$	(use calc.)



Example 3. Calculate *w*, correct to 1 decimal place.

Ans.

$w^2 = 8{\cdot}6^2 - 6{\cdot}5^2$	(correct formula "-")
$= 73{\cdot}96 - 42{\cdot}25$	(square out)
$= 31{\cdot}71$	(tidy)
$w = \sqrt{31{\cdot}71}$	(bring in √)
$= \underline{\underline{5{\cdot}6\ (m)}}$	(use calc. and round)



Exercise 3 may now be attempted

(c) A mixture of "Pythagoras <u>Plus</u>" and "Pythagoras <u>Minus</u>"

It should be explained to students that they must first make the following decision -

- if the hypotenuse is asked for . . . use the '+' formula
- if one of the shorter sides (**not** the hypotenuse) is asked for . . . use the '–' formula.

The following examples can be used:

Example 1. What length of cable is needed to secure this flag pole?

Ans. Hypotenuse asked for => 'Pythagoras Plus'

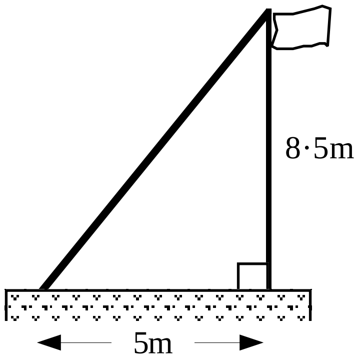
$x^2 = 8{\cdot}5^2 + 5^2$ (correct formula '+')

$= 72{\cdot}25 + 25$ (square out)

$= 97{\cdot}25$ (tidy)

$x = \sqrt{97{\cdot}25}$ (bring in $\sqrt{}$)

$= \underline{\underline{9{\cdot}9\ (m)}}$ (use calc. and round)



Example 2. A 6m pipe is resting against the top of a wall.The other end of the pipe is sitting on the ground, 3m from the foot of the wall.
What is the height of the wall ?

Ans. Shorter side asked for, (**not** hypotenuse) => 'Pythagoras Minus'

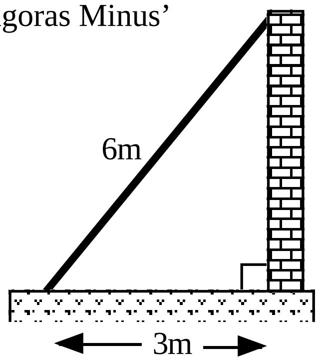
$x^2 = 6^2 – 3^2$ (correct formula "-")

$= 36 – 9$ (square out)

$= 27$ (tidy)

$x = \sqrt{27}$ (bring in $\sqrt{}$)

$= \underline{\underline{5{\cdot}2\ (m)}}$ (use calc. and round)



Exercise 4 Questions 1-12 may now be attempted
(Q13 to be used as an extension).

(d) Finding the distance between two coordinate points.

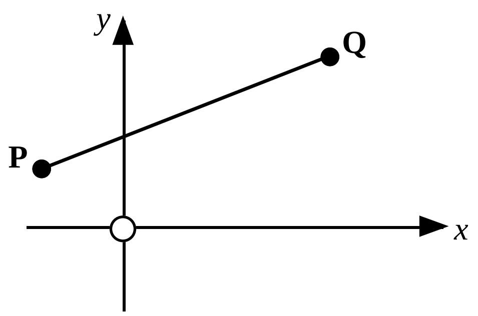
It may be that some revision on coordinates will be required first.

The following example can be used:

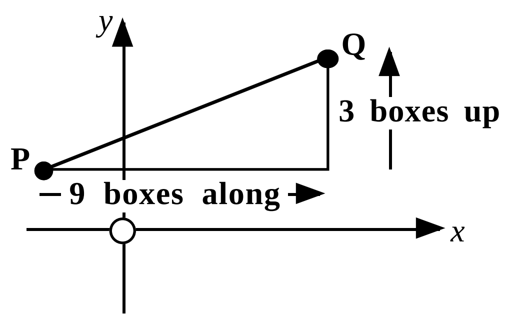
Example Find the distance PQ, between the points P(–2,3) and Q(7,6).

Ans.

Plot the points correctly on a diagram or sketch.

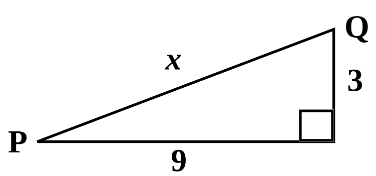


Make a right angled triangle by drawing a vertical line through Q and a horizontal one through P.



Proceed as if you are finding the hypotenuse of a right angled triangle

=> 'Pythagoras Plus'



$x^2 = 9^2 + 3^2$	(correct formula "+ ")
$= 81 + 9$	(square out)
$= 90$	(tidy)
$x = \sqrt{90}$	(bring in √)
$= \underline{\underline{9 \cdot 5}}$	(use calc. and round)

PQ has length 9·5 units

Exercise 5 Q1, Q2 and Q3 may now be attempted.
Q4 for extension (converse)

Then do the checkup for Theorem of Pythagoras.

STUDENT MATERIALS

CONTENTS

INTEGERS

By the end of this set of exercises, you should be able to

(a) plot and read coordinates in all four quadrants

(b) add and subtract positive and negative integers

(c) subtract a negative integer from an integer

(d) multiply two integers where one is positive and one is negative

(e) divide a negative integer by a positive integer

(f) multiply and divide two integers where both are negative

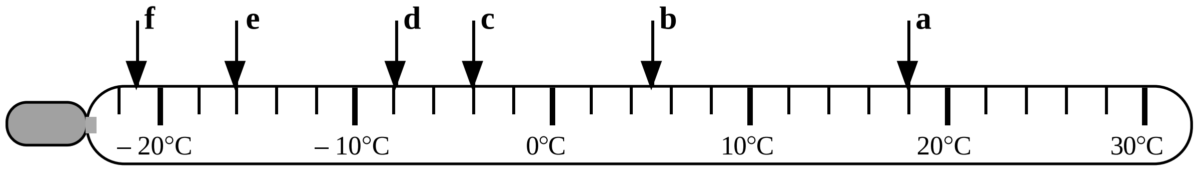
(g) multiply three or more integers.

INTEGERS

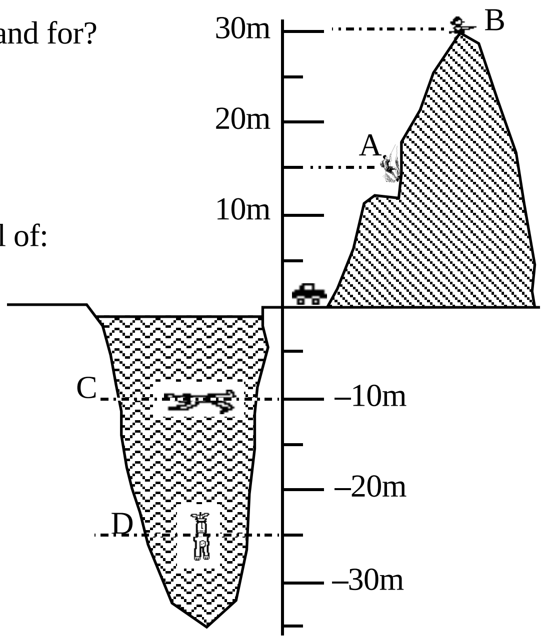
A. Coordinates

Exercise 1

1. Look at the thermometer below:



What numbers do the arrows labelled **a** to **f** stand for?



2. The diagram shows a small hill next to a deep lake.

   Use positive and negative numbers to give the height (in metres) above or below ground level of:

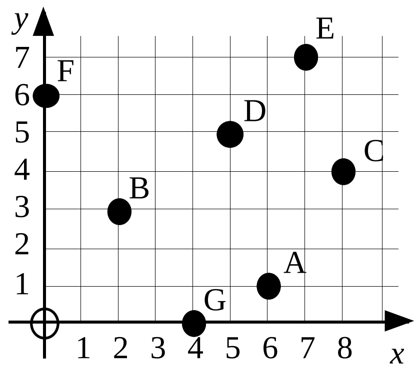
   (a) the car tyre (b) climber A
   (c) climber B (d) diver C
   (e) diver D
   (f) the foot of the lake.

3. Shown is a Cartesian (or coordinate) diagram.

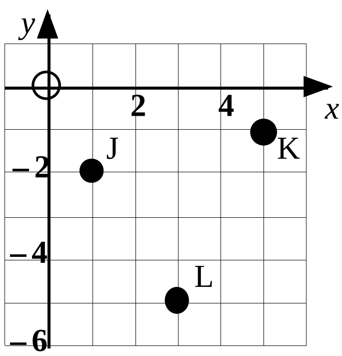
   The horizontal line O*x* is the *x* - axis
   The vertical line O*y* is the *y* - axis.
   The point where they meet (O) is the origin.

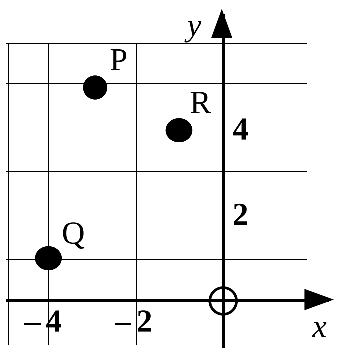
   The coordinates of A are :- A(6,1)

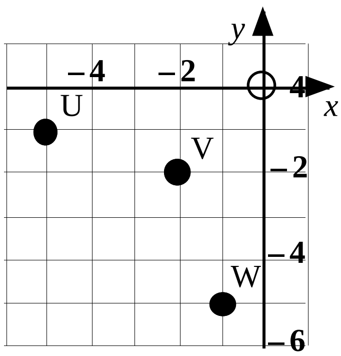
   Write down the coordinates of the other 6 points.



4. Write down the coordinates of the following points:

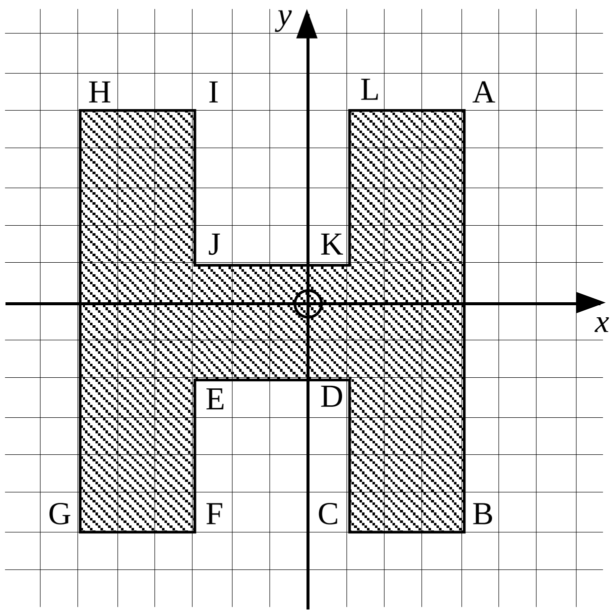






5. Write down the coordinates of all the points A to L which make up the letter H in this diagram.

(each box represents 1 unit on the horizontal and vertical axis)

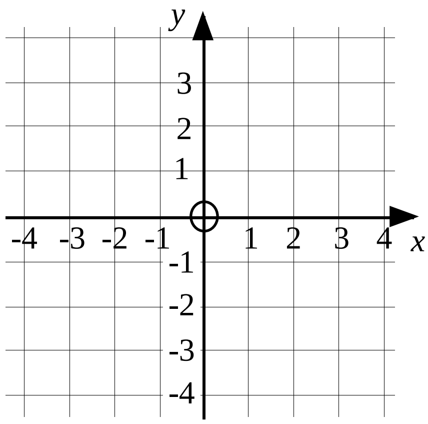


6. (a) Use a ruler to draw the set of axes shown opposite.

(b) Plot the points A(2,1), B(2,–2), C(–3,–2), and D(–3,1).

(c) Join up the four points.

(d) What shape do you get?



7. For each of the following:

(i) draw a set of axes

(ii) plot the points and join them up

(iii) write down the name of the shape produced.

(a) (–1,–3), (1,4), (3,–3)

(b) (–5,2), (–6,6), (–2,5), (2,–2)

(c) (–1,4), (–5,1), (–3,–4), (1,–4), (3,1).

8. (a) Draw a set of axes and plot the three points, P(2,3), Q(–4,3) and R(–4,–1).

(b) Find a fourth point (call it S) such that PQRS is a rectangle.

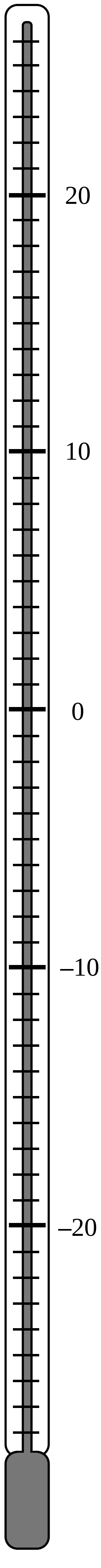
(c) Write down the coordinates of S.

(d) Draw the two diagonals PR and QS and write down the coordinates of the point at which the diagonals cross.

B. Add and Subtract Integers

Exercise 2A

Use the thermometer scale on the right to help with this exercise.



1. (a) By how many degrees did the temperature drop from:

(i) 12°C to 4°C	(ii) 5°C to 0°C	(iii) 3°C to –2°C
(iv) 0°C to –6°C	(v) 10°C to –10°C	(vi) –4°C to –10°C
(vii) 1°C to –13°C	(viii) –10°C to –25°C	(ix) $-1^1/_2$°C to $-7^1/_2$°C

(b) By how many degrees did the temperature rise from:

(i) 2°C to 11°C	(ii) 0°C to 20°C	(iii) 10°C to 70°C
(iv) –3°C to 0°C	(v) –6°C to –1°C	(vi) –10°C to 2°C
(vii) –20°C to 10°C	(viii) –11°C to 9°C	(ix) –1°C to 16°C

(c) What was the new reading when the temperature:

(i) fell from 12°C by 7°C	(ii) fell from 6°C by 6°C
(iii) fell from 3°C by 5°C	(iv) fell from –1°C by 9°C
(v) rose from –1°C by 3°C	(vi) rose from –7°C by 2°C
(vii) rose from –9°C by 9°C	(viii) rose from –18°C by 22°C
(ix) fell from 11°C by 20°C	(x) rose from –30°C by 16°C

2. Find:

(a) 5 + 7	(b) 0 + 6	(c) –2 + 5
(d) –8 + 8	(e) –1 + 7	(f) –8 + 2
(g) –10 + 7	(h) –4 + 9	(i) –9 + 4
(j) –27 + 27	(k) –5 + 18	(l) $-^1/_2 + 2^1/_2$

3. Find:

(a) 8 – 4	(b) 7 – 7	(c) 5 – 6
(d) 3 – 10	(e) 1 – 15	(f) 0 – 3
(g) –2 – 5	(h) –6 – 3	(i) –4 – 9
(j) –20 – 30	(k) 10 – 40	(l) –5 – 12

4. Find:

(a) 10 + (–3)	(b) 25 + (–16)	(c) 12 + (–12)
(d) 4 + (–6)	(e) 0 + (–9)	(f) –3 + (–5)
(g) –6 + (–8)	(h) 20 + (–29)	(i) –50 + (–50)

5. Find:

(a) –3 + 6	(b) 8 – 10	(c) 6 + (–5)
(d) –9 + 3	(e) 0 – 7	(f) 5 + (–9)
(g) –6 + 6	(h) –2 – 8	(i) 0 + (–3)
(j) –17 + 11	(k) 9 – 9	(l) –9 + (–9)
(m) –9 + 9	(n) 9 + (–9)	(o) –9 – 9

Exercise 2B

1. Remember:

	8 – (–2)
=	8 + 2
=	10

note

Find:

(a) 6 – (–3) (b) 10 – (–2)

(c) 11 – (–10) (d) 7 – (–7) (e) 0 – (–9)

(f) 1 – (–2) (g) 31 – (–29) (h) 6·5 – (–2·5)

2. Remember:

	–4 – (–7)
=	–4 + 7
=	3

note

Find:

(a) –2 – (–5) (b) –3 – (–7)

(c) –11 – (–1) (d) –4 – (–4) (e) 0 – (–12)

(f) –1 – (–2) (g) –2 – (–1) (h) $-2\frac{1}{2} - (-3\frac{1}{2})$

C. Multiply and Divide Integers

Exercise 3A

1. Find:

(a) 3 x 2 (b) 3 x –2 (c) 5 x 4

(d) 5 x –4 (e) 6 x –7 (f) 3 x –9

(g) 8 x –2 (h) 10 x –5 (i) 8 x –5

(j) 9 x –7 (k) 7 x –9 (l) 11 x –3

(m) 15 x –2 (n) 20 x –7 (o) 50 x –4

2. Remember: –3 x 6 is the same as 6 x –3 = –18

Find:

(a) –2 x 5 (b) –5 x 8 (c) –3 x 7

(d) –6 x 9 (e) –4 x 10 (f) –10 x 5

(g) –9 x 7 (h) –8 x 8 (i) –7 x 0

(j) -1 x 1 (k) –6 x 12 (l) –8 x 7

(m) –3 x 30 (n) –20 x 3 (o) –100 x 9

3. Find:

(a) –10 ÷ 2 (b) –20 ÷ 4 (c) –21 ÷ 7

(d) –60 ÷ 3 (e) –36 ÷ 9 (f) –100 ÷ 10

(g) –18 ÷ 6 (h) –49 ÷ 7 (i) –81 ÷ 9

(j) –42 ÷ 7 (k) –32 ÷ 4 (l) –120 ÷ 3

(m) $\frac{-27}{3}$ (n) $\frac{-36}{4}$ (o) $\frac{-55}{11}$

Double Negatives

Remember:

positive x negative = negative
negative x positive = negative
negative x negative = positive

negative ÷ positive = negative
positive ÷ negative = negative
negative ÷ negative = positive

Exercise 3B

1. Find:

(a)	-2×-3	(b)	-4×-6	(c)	-9×-4
(d)	-5×-5	(e)	-8×-3	(f)	-7×-6
(g)	-10×-9	(h)	-2×-30	(i)	-6×-8

2. Find:

(a)	$12 \div (-3)$	(b)	$-15 \div 5$	(c)	$30 \div (-6)$
(d)	$21 \div (-7)$	(e)	$40 \div (-8)$	(f)	$-48 \div 8$
(g)	$25 \div (-5)$	(h)	$\frac{-18}{2}$	(i)	$\frac{32}{-4}$

3. Find:

(a)	$-20 \div (-4)$	(b)	$-18 \div (-3)$	(c)	$-56 \div (-7)$
(d)	$-80 \div (-8)$	(e)	$-35 \div (-5)$	(f)	$-110 \div (-10)$
(g)	$\frac{-30}{-5}$	(h)	$\frac{-63}{-9}$	(i)	$\frac{-88}{-8}$

4. Find:

(a)	$2 \times -3 \times 4$	(b)	$-5 \times 2 \times -6$	(c)	$-5 \times -5 \times 2$
(d)	$-1 \times -2 \times -3$	(e)	$-4 \times -5 \times -3$	(f)	$-2 \times -2 \times 3 \times 3$
(g)	$-5 \times -2 \times 4 \times -6$	(h)	$-1 \times -1 \times -1 \times -1$	(i)	$-2 \times -2 \times -2 \times -2 \times -2$

5. Find:

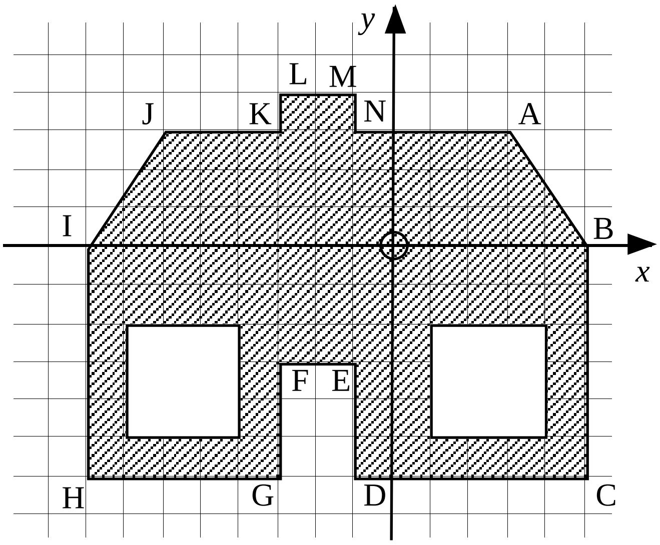
(a)	8×-3	(b)	-2×7	(c)	$-10 \div 2$
(d)	-8×-7	(e)	$-15 \div (-5)$	(f)	-9×9
(g)	0×-21	(h)	$-7 \div 7$	(i)	7×-7
(j)	$-7 \div (-7)$	(k)	$80 \div (-4)$	(l)	$-33 \div 11$
(m)	$-54 \div (-6)$	(n)	-8×-3	(o)	$48 \div (-3)$
(p)	8×-8	(q)	-2×11	(r)	$-45 \div (-9)$
(s)	$0 \div (-16)$	(t)	$-1 \div (-1)$	(u)	$1000 \div (-10)$
(v)	$-2 \times -3 \times -4$	(w)	$-5 \times 2 \times -10$	(x)	$-1 \times -1 \times -1 \times -1 \times -1$

Mathematics 2 (Intermediate 1)

Checkup for Integers

1. Write down the coordinates of all the points A to N which make up the outline of this house starting with:

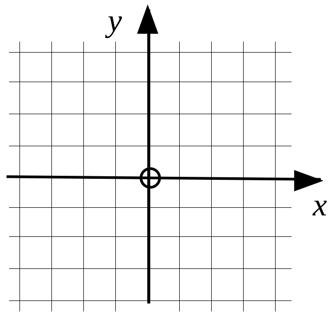
   A(3,3)



2. (a) Draw a set of axes and plot the three points A(–2,5), B(1,–2) and C(–2,–4).

   (b) Find a new fourth point, D, such that ABCD is a kite with axis of symmetry AC.

   (c) Write down the coordinates of point D.



3. Find:

(a) –5 + 7	(b) 6 – 9	(c) 8 + (–5)
(d) –10 + 4	(e) 0 – 12	(f) 4 + (–7)
(g) –8 + 8	(h) –3 – 7	(i) 0 + (–5)
(j) –19 + 12	(k) 6 – 6	(l) –6 + (–6)
(m) –6 + 6	(n) 6 + (–6)	(o) –6 – 6
(p) 5 – (–2)	(q) 9 – (–9)	(r) 22 – (–8)
(s) –3 – (–7)	(t) –5 – (–5)	(u) –9 – (–2)

4. Find:

(a) 6 x –2	(b) –3 x 5	(c) –12 ÷ 4
(d) –7 x –8	(e) –18 ÷ (–3)	(f) –5 x 5
(g) 0 x –8	(h) –4 ÷ 4	(i) 4 x –4
(j) –4 ÷ (–4)	(k) 60 ÷ (–3)	(l) –35 ÷ 7
(m) –56 ÷ (–7)	(n) –9 x –3	(o) 48 ÷ (–4)
(p) 3 x –3	(q) –3 x 10	(r) –42 ÷ (–3)
(s) 0 ÷ (–11)	(t) –2 ÷ (–2)	(u) 1000 ÷ (–100)
(v) –3 x –5 x 2	(w) –6 x –3 x –2	(x) –2 x –2 x –2 x –2

SPEED, DISTANCE AND TIME

By the end of this set of exercises, you should be able to

(a) interpret Distance Time Graphs

(b) solve problems involving speed, distance and time

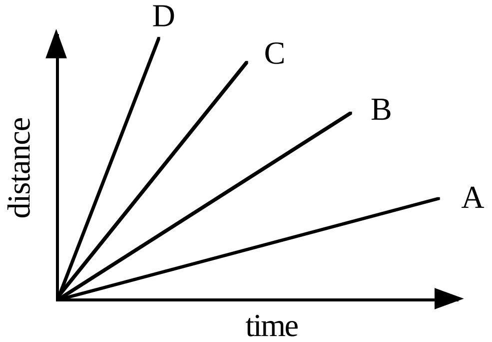
SPEED, DISTANCE AND TIME

A. Distance–Time Graphs

Exercise 1

1. This distance time graph indicates four journeys taken by:

   – a lorry – a cyclist – a racing car – a walker.



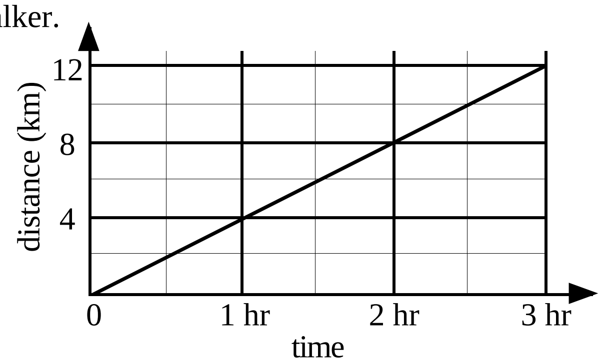
Match the above four journeys with the corresponding lines A, B, C and D.

2. This graph shows the journey taken by a walker.

   (a) How far did this walker travel in:

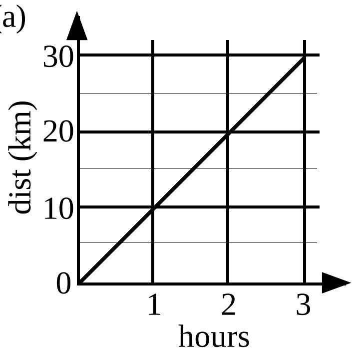
   (i) $^1/_2$ hour (ii) 1 hour
   (iii) 2 hours (iv) $2^1/_2$ hours?

   (b) What is his speed in kilometres per hour?

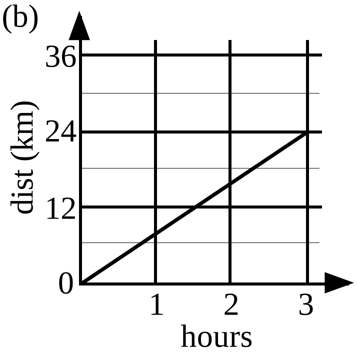


3. Calculate the speeds represented by the following graphs:

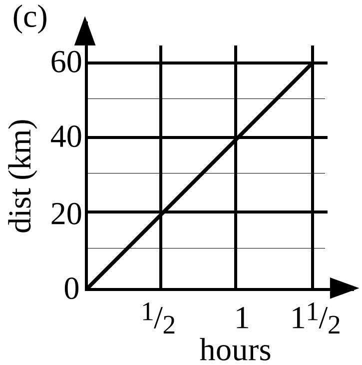
(a)



(b)



(c)



4. This graph indicates a lorry driver's journey from Allerton to Brockleish, 80 miles away.

He set off from Allerton at 11:00 along the motorway and stopped for some lunch before completing the rest of the trip along a dual carriageway.

(a) For how long was he driving on the motorway?

(b) For how long did he stop for lunch?

(c) At what time did he set off after lunch?

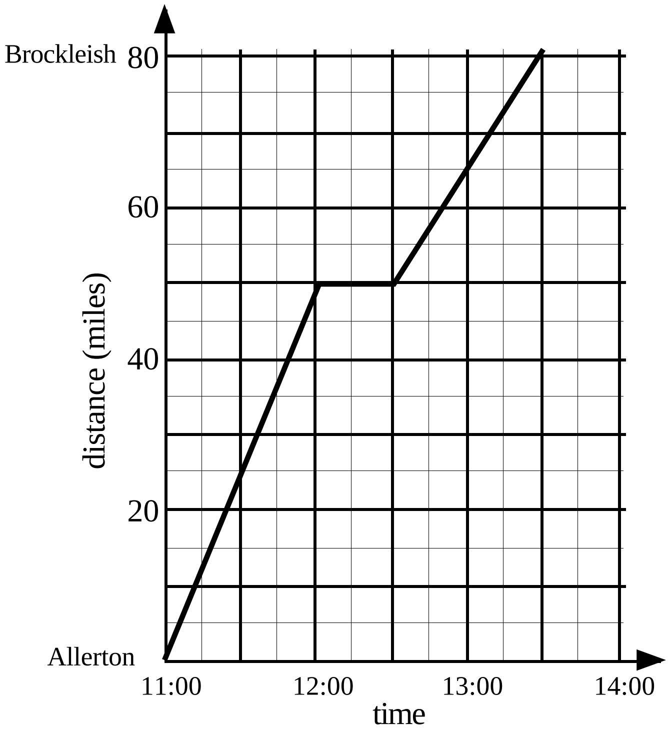
(d) When did he arrive at Brockleish?

(e) Calculate the speed of the lorry:

(i) on the motorway.

(ii) between 12:00 and 12:30.

(iii) on the dual carriageway.

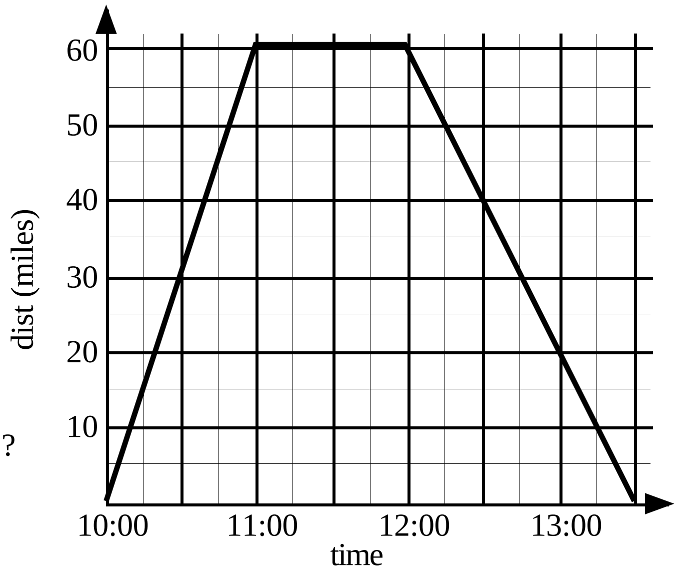


5. A motorist drove from his house to the coast, stayed there till it began to rain, then drove home via a different route.

(a) For how long did he remain at the coast?

(b) Calculate his speed for both the outward and the return journey.

(c) Which of the two trips do you think was done on the motorway ? Give a reason for your answer.



6. Lucy set off at 9:00 am on her bike to cycle to Newton Abbey, 30 kilometres away.

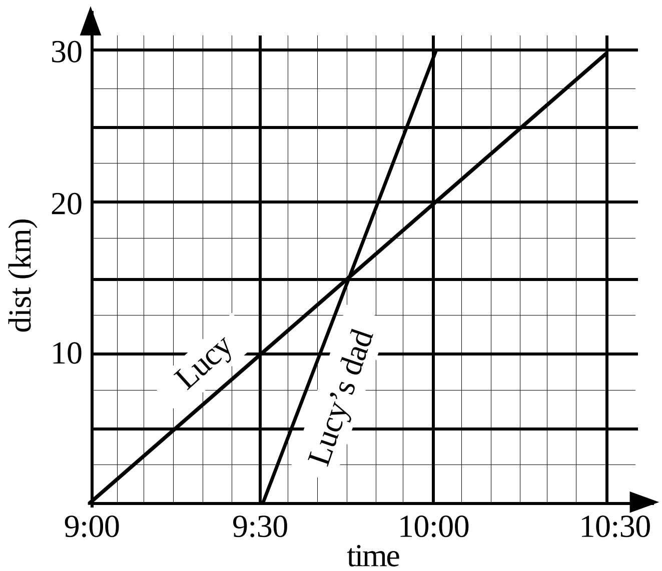
Her father left the house at 9:30 and drove to Newton Abbey.

(a) Calculate Lucy's speed.

(b) Calculate her dad's speed.

(c) At what time did Lucy's dad overtake her?

(d) How far away from home was she when her father passed her?



7. Tobbart's Timber Merchants has two depots, one at Carswell and the other, 160 kilometres away in Daewood.

Sam sets off from Carswell at 8:00 with a load of timber for Daewood.

George sets off at 9:00 from Daewood heading for Carswell.

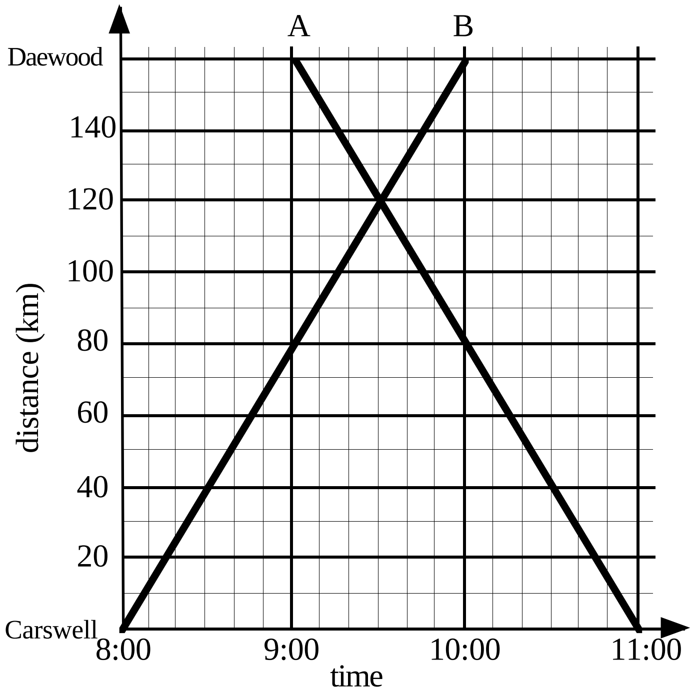
(a) Which line, A or B, represents Sam's journey?

(b) Calculate:

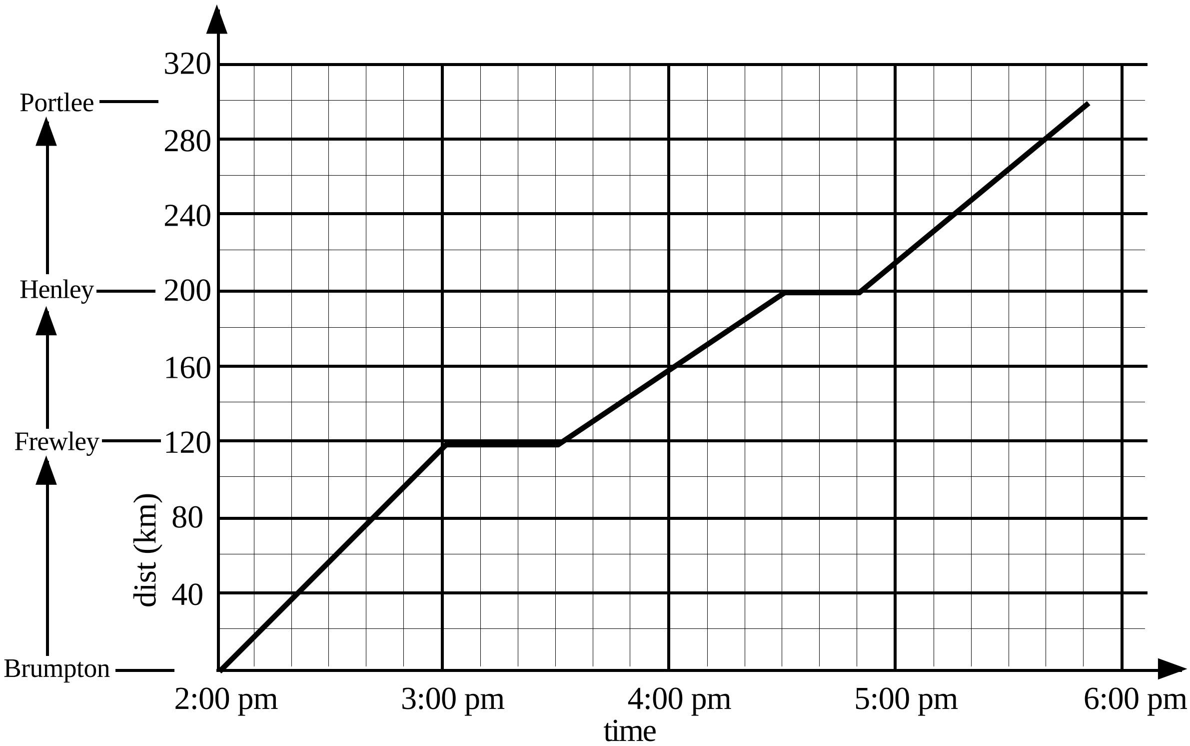
(i) Sam's speed.

(ii) George's speed.

(c) At what time could the two drivers wave to each other?



8. The graph shows a train journey from Brumpton to Portlee.



(a) Copy and complete this timetable:

Brumpton		Frewley		Henley		Portlee
depart		*arrive*	*leave*	*arrive*	*leave*	*arrive*
2:00pm	—>	…	…	…	…	…

(b) How far is it from: (i) Brumpton to Frewley (ii) Henley to Portlee?

(c) Calculate the average speed of the train:

(i) from Brumpton to Frewley (ii) from Frewley to Henley

(iii) from Henley to Portlee (iv) from Brumpton to Henley (including the stop).

Time Intervals

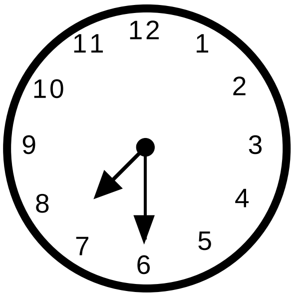
Exercise 2

1. How long is it from:

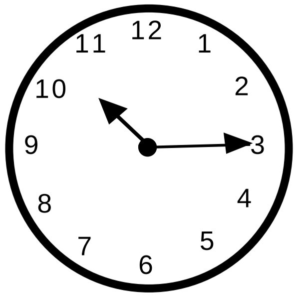
(a) 9:15 am to 11:15 am	(b) 8:00 pm to 9:30 pm	(c) noon to 3:15 pm
(d) 10 past 7 to $^1/_4$ to 9	(e) 10:20 pm to midnight	(f) 11:15 am to 2:30 pm?

2. The clocks indicate the start and finish time of a concert in the Glasgow Royal Concert Hall one <u>evening</u>.

   How long did the concert last ?



Concert Starts



Concert Ends

3. Shown is part of the bus timetable from Stetford to Walsail:

	Stetford →	**Leeward** →	**Painton** →	**Selby** →	**Walsail**
Early Bus	8:15 am	9:20 am	11:30 am	12:15 pm	2:50 pm
Late Bus	10:30 am	11:35 am			5:05 pm

   (a) How long does the early bus take to travel from :-

   (i) Stetford to Leeward (ii) Painton to Selby (iii) Stetford to Walsail?

   (b) Assuming the late bus travels at the same speed as the early one, when would you expect it to arrive at:

   (i) Painton (ii) Selby?

4. (a) A plane leaves Glasgow Airport at 11:45 pm. It touches down in Palma Airport in Majorca at 3:10 am (British time) the next morning.

   How long did the flight take?

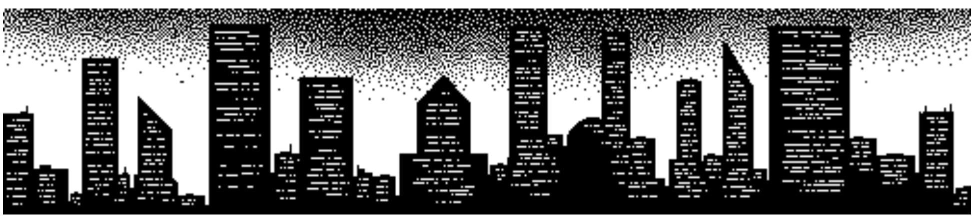
   (b) The transcontinental express train leaves London at 10:50 pm on Monday night and arrives at Prague at 8:15 am on Tuesday morning (British time).

   How long did the trip take?

   (c) Concord left Heathrow at 11:25 pm on Saturday and arrived in New York's Kennedy airport at 2:15 am (British time) on Sunday morning.

   (i) How long did the flight take?

   (ii) New York is 5 hours <u>behind</u> Britain. What time was it really, (New York time), when the plane touched down in New York?



B. Time Distance Speed

Exercise 3A

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

1. A man went for a walk. After 3 hours he found he had travelled 18 kilometres. Calculate his average walking speed.

2. Calculate the average speeds for the following journeys in kilometres per hour:
   (a) A cyclist covers a distance of 72 kilometres in 2 hours.
   (b) A plane flies 4800 kilometres in 4 hours.
   (c) A ship sails 320 kilometres in just 8 hours.
   (d) A train travels 249 kilometres in 3 hours
   (e) A marathon runner runs 36 kilometres in 2 hours.

3. Calculate the average speed of:
   (a) a motor-cyclist who travels 150 miles in 2 hours.
   (b) a runner who does the 100 metre race in 10 seconds.
   (c) a jet which flies 2700 miles in 3 hours.
   (d) a snail which crawls 172 centimetres in 4 hours.

4. Remember:

30 minutes	= $^1/_2$ hour	= 0·5 hour
2 hours 30 minutes	= $2^1/_2$ hours	= 2·5 hours

   Calculate the average speed of:
   (a) a lorry which travels 25 kilometres in 30 minutes.
   (b) a van which covers 150 miles in $2^1/_2$ hours.
   (c) a car which travels 66 kilometres in 1 hour 30 minutes.
   (d) a yacht which sails 56 miles in 3 hours 30 minutes.

5. Remember:

15 minutes	= $^1/_4$ hour	= 0·25 hour
1 hours 45 minutes	= $1^3/_4$ hours	= 1·75 hours

   Calculate the average speed of:
   (a) a runner who covers 18 kilometres in 45 minutes.
   (b) a short car journey of 12 miles in 15 minutes.
   (c) a motor-cyclist who travels 65 miles in 1 hour 15 minutes.
   (d) a car travelling the 165 miles to Manchester in 3 hours 45 minutes.
   (e) a plane which flies 720 kilometres in $2^1/_4$ hours.

6. (a) A driver leaves Paisley at 9:45 am. By 12:45 pm he finds he has covered 165 miles. Calculate his average speed.
   (b) A plane leaves London at 2:45 pm and flies the 500 miles to Aberdeen. Calculate the average speed of the plane if it touches down in Aberdeen at 4:00 pm.
   (c) A loch pleasure steamer sets off at 9:45 am, sails 30 kilometres round the loch and arrives back at the harbour at 12:15 pm. Calculate the average speed of the steamer.

Exercise 3B

$$\text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}}$$

1. A man's average walking speed is 6 kilometres per hour. How long would it take him to cover a distance of 24 kilometres?

2. Calculate the time taken (in hours) for each of the following journeys:
   (a) a car travelling 90 kilometres at an average speed of 45 kilometres per hour.
   (b) a cyclist covering 96 kilometres at an average speed of 24 kilometres per hour.
   (c) a train journey of 240 kilometres with an average speed of 80 kilometres per hour.
   (d) a plane flying a distance of 2500 kilometres at a speed of 500 kilometres per hour.

3. Calculate the time taken by:
   (a) a motor-cyclist driving 120 miles at an average speed of 60 miles per hour.
   (b) a runner in a 400 metre race with an average speed of 8 metres per second.
   (c) a jet flying 4800 miles at a speed of 800 miles per hour.
   (d) an old sailing boat travelling 120 miles at an average speed of 15 m.p.h.

4. Remember: 2·5 hours means $2\frac{1}{2}$ hours = 2 hours 30 minutes

   How long, in hours and minutes, did the following journeys take :-

   (a) a lorry travelling 45 miles at an average speed of 30 miles per hour.
   (b) a van which covers 150 kilometres at an average speed of 60 kilometres per hour.
   (c) a journey of 12 kilometres by bike at an average speed of 24 kilometres per hour.
   (d) a speedboat sailing 48 kilometres at a speed of 32 kilometres per hour.

5. Remember: 1·25 hours means $1\frac{1}{4}$ hours = 1 hours 15 minutes
   0·75 hours means $\frac{3}{4}$ hour = 45 minutes

   How long, in hours and minutes, do the following journeys take:

   (a) a jogger who covers 6 miles at an average speed of 8 miles per hour.
   (b) a car journey of 90 kilometres at an average speed of 40 kilometres per hour.
   (c) a plane journey of 2550 miles at an average speed of 600 miles per hour.
   (d) a boat ride over 45 miles at a speed of 20 miles per hour.

6. (a) A train leaves Glasgow at 8:45 am. It travels 125 miles to Carlisle at an average speed of 50 miles per hour. How long did it take and when did it arrive at Carlisle?
   (b) A plane sets off from Gatwick at 2:30 pm. It flies 1950 miles at a steady speed of 600 miles per hour. When will it arrive at its destination?
   (c) A driver sets off from Manchester at 10:50 pm on Monday night to drive the 345 miles to Inverness. If his average speed is 60 m.p.h., when should he reach Inverness?

Exercise 3C

DISTANCE = SPEED x TIME

1. A woman's average walking speed is 5 kilometres per hour.
   How far will she be able to walk in 3 hours?

2. Calculate the distance travelled by:
   (a) a car travelling for 2 hours at an average speed of 37 kilometres per hour.
   (b) a cyclist pedalling for 4 hours at an average speed of 17 kilometres per hour.
   (c) a train journey of 3 hours when the average speed is 75 kilometres per hour.
   (d) a plane flying for 6 hours at a speed of 540 kilometres per hour.

3. Calculate the distance travelled by:
   (a) a motor-cyclist driving for 3 hours at an average speed of 72 miles per hour.
   (b) a sprinter running for 20 seconds at an average speed of 10 metres per second.
   (c) a jet flying for 5 hours at a speed of 750 miles per hour.
   (d) an tug-boat sailing for 4 hours at an average speed of 12 m.p.h.

4. Remember:

2 hours 30 minutes	= $2^1/_2$ hours	= 2·5 hours
1 hours 45 minutes	= $1^3/_4$ hours	= 1·75 hours

   What distances are covered by the following:
   (a) a lorry travelling for 30 minutes at an average speed of 40 miles per hour.
   (b) a van trip lasting 2 hours 30 minutes at an average speed of 50 kilometres per hour.
   (c) a $1^1/_2$ hour bike trip at an average speed of 18 kilometres per hour.
   (d) a speedboat running for 1 hour 15 minutes at its top speed of 36 kilometres per hour.
   (e) a jogger who runs for 45 minutes at an average speed of 8 miles per hour.
   (f) a car journey lasting 2 hours 15 minutes at an average speed of 48 kilometres per hour.
   (g) a plane journey of $3^3/_4$ hours at an average speed of 520 miles per hour.
   (h) a boat ride of 15 minutes at a speed of 24 miles per hour.

5. (a) A plane left Paris at 8:45 am and arrived at Glasgow at 10:45 am, flying at an average speed of 310 miles per hour.
   How long did it take and how far is it from Paris to Glasgow?

   (b) A ship left harbour at 11:50 am and sailed at a steady speed of 32 miles per hour. It arrived at the port of Liverpool at 3:20 pm. How far is it from the harbour to Liverpool?

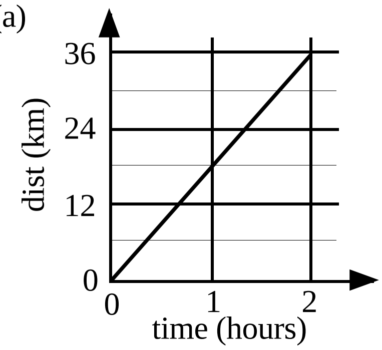
   (c) A driver sets off from Land's End at an average speed of 50 miles per hour, at 7:30 pm on Wednesday night on a sponsored run to John O'Groats.
   At precisely 11:30 am on Thursday the car pulled into John O'Groats hotel.
   How far is it from Land's End to John O'Groats?

Mathematics 2 (Intermediate 1)

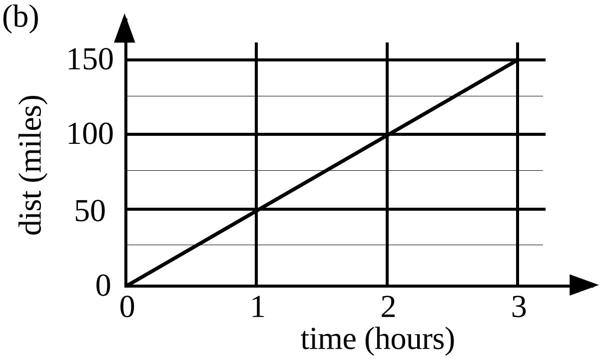
Checkup for Distance Time Speed

1. Calculate the speeds represented by the following distance time graphs:

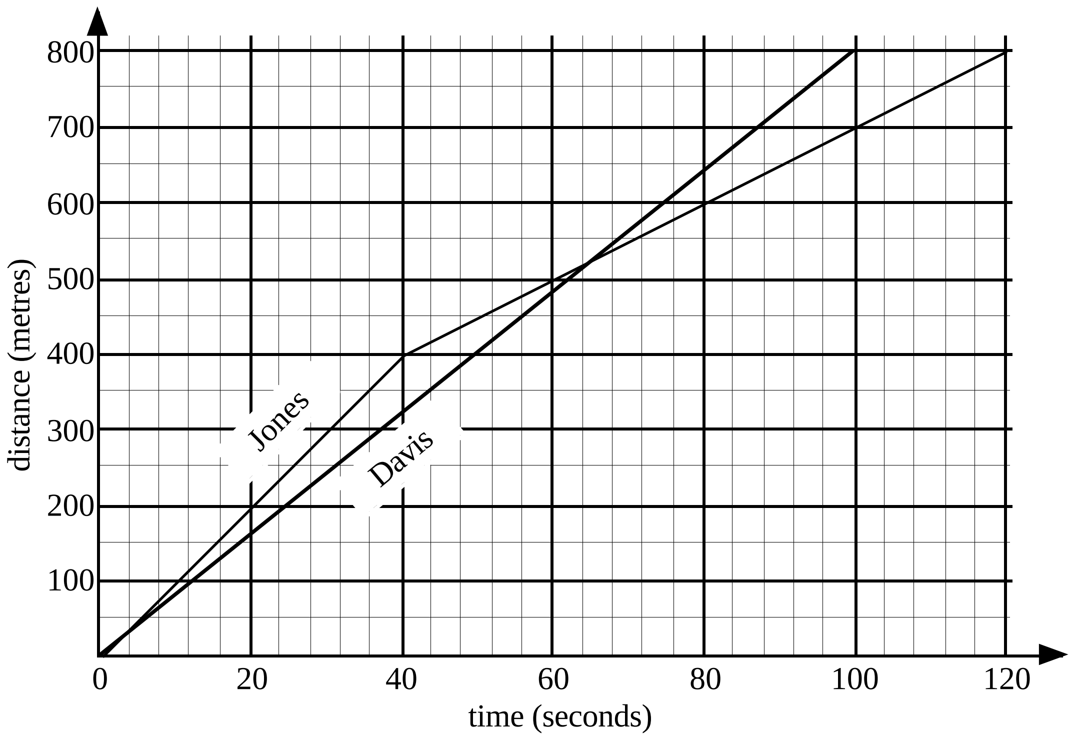
(a)



(b)



2. This graph shows how two amateur 800 metre runners, Jones and Davis, paced themselves over the two 400 metre laps.



(a) Who was the faster over the first 400 metres?

(b) How long did each of them take to run the first lap?

(c) Who won the 800 metre race?

(d) How long did each runner take to complete the 2nd lap?

(e) For how many seconds (approximately) was Jones in the lead?

(f) By how many seconds did the winner beat the runner up?

(g) Calculate Davis' speed in metres per second.

3. (a) An end of term Primary school service started at 11:20 am and finished at 1:30 pm! For how long did the service last?

(b) A plane left Manchester airport at 9:50 pm and landed in Ibiza at 1:15 am (British time) next morning. How long was the flight?

4. (a) A woman drove 180 miles in 3 hours. Calculate the average speed of her car.

(b) A train travelled 400 kilometres at an average speed of 80 kilometres per hour. How long did the journey take?

(c) A ship, sailing at a steady speed of 33 miles per hour, took 4 hours from the mainland to Stravaar Island. How far did it travel?

(d) A pilot took off from an air field at 9:45 am and flew 450 kilometres to a rendevous point, arriving there at 11:00 am.

How long did the flight take and what was the pilot's average speed?

(e) A hill walker covered a distance of 21 kilometres at an average walking speed of 6 kilometres per hour. How long did it take him?

(f) A satellite flew around one of Jupiter's moons in 1 hour 15 minutes.

If the average speed of the satellite was 16 400 kilometres per hour, calculate the length of its orbit.

THE THEOREM OF PYTHAGORAS

By the end of this set of exercises, you should be able to

(a) solve problems in right-angled triangles using the Theorem of Pythagoras

Squares and Square Roots

Exercise 1

1. Find the value of:

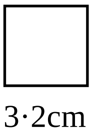
(a) 5^2	(b) 8^2	(c) 10^2	(d) 1^2
(e) $3{\cdot}5^2$	(f) $0{\cdot}3^2$	(g) $25{\cdot}2^2$	(h) 100^2

2. Find the value of:

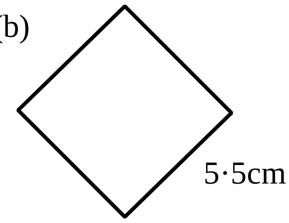
(a) $\sqrt{25}$	(b) $\sqrt{81}$	(c) $\sqrt{100}$	(d) $\sqrt{20{\cdot}25}$
(e) $\sqrt{110{\cdot}25}$	(f) $\sqrt{324}$	(g) $\sqrt{10\,000}$	(h) $\sqrt{1}$

3. Calculate the area of these squares, giving your answers correct to 1 decimal place:

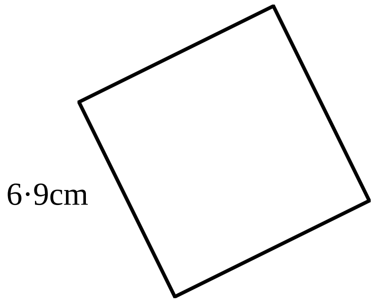
(a)



(b)

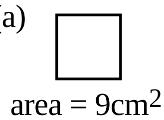


(c)

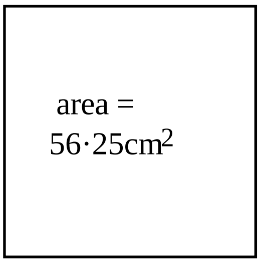


4. Calculate the length of a side in each of these squares:

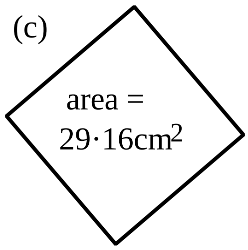
(a)



(b)



(c)



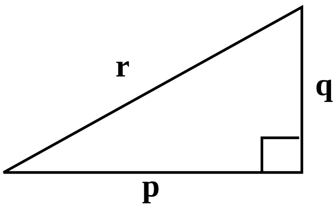
The Theorem of Pythagoras

Exercise 2

1. Use Pythagoras' Theorem to write an equation for each of these triangles:

(the first one has been done for you)

(a)

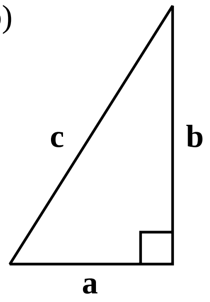


$$\mathbf{r^2 = p^2 + q^2}$$

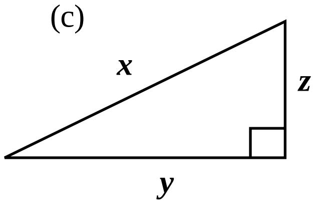
* ***r*** *is the* <u>*longest*</u> *side.*

contd...

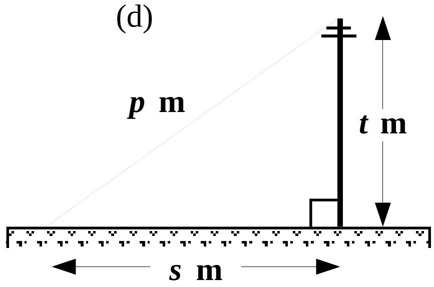
(b)



(c)

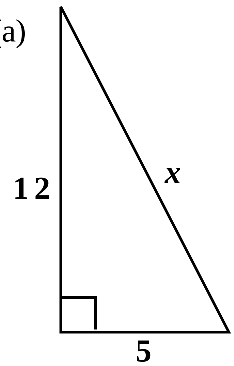


(d)

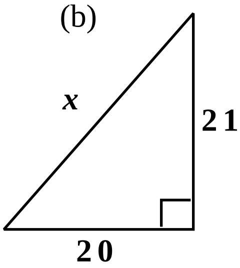


2. Calculate the length of the missing side *x*. Give your answers correct to one decimal place.

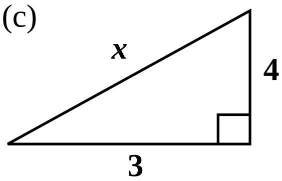
(a)



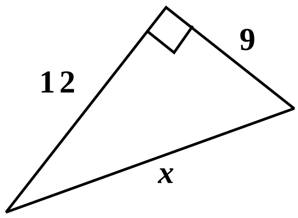
(b)



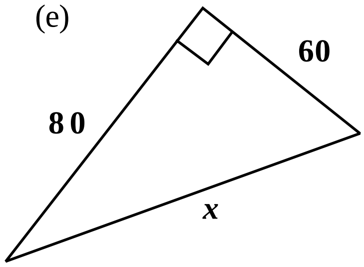
(c)



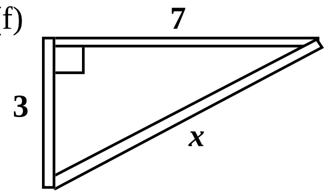
(d)



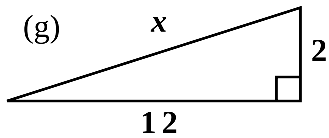
(e)



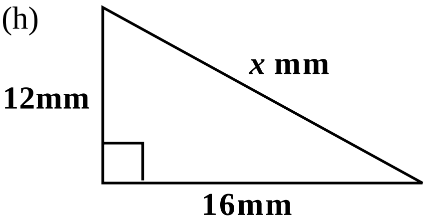
(f)



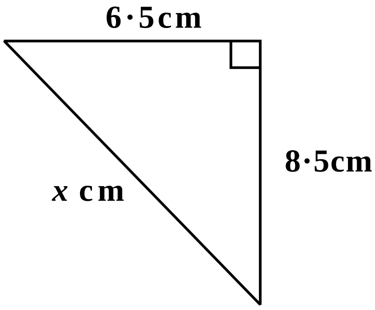
(g)



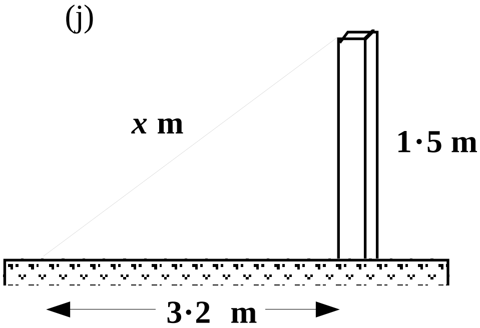
(h)



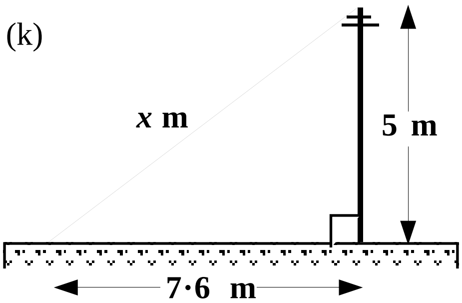
(i)



(j)



(k)

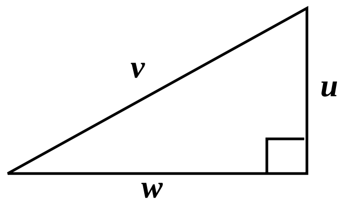


Exercise 3

1. Use Pythagoras' Theorem to write an equation which can be used to calculate the required side in each of the following triangles:

(the first one has been done for you)

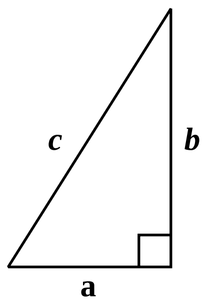
(a)



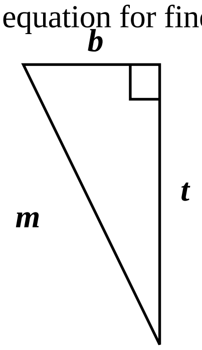
$u^2 = v^2 - w^2$

where u is one of the two shorter sides.

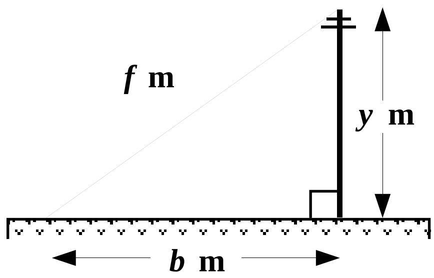
(b) Write an equation for finding a here.



(c) Write an equation for finding b here.

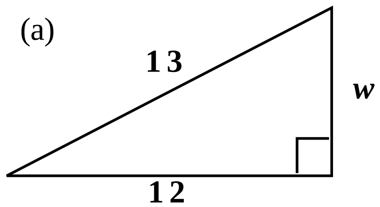


(d) Write an equation for finding y here.

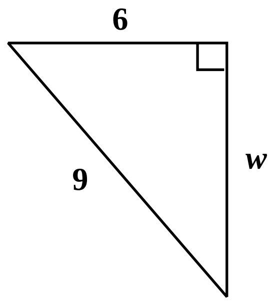


2. Calculate the length of the missing side w, giving your answers correct to one decimal place.

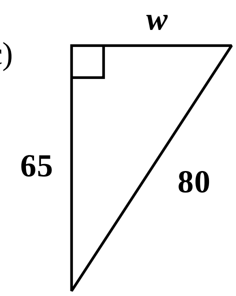
(a)



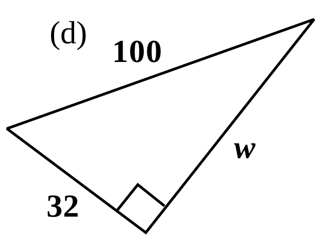
(b)



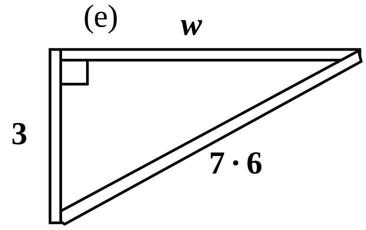
(c)



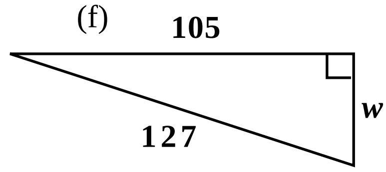
(d)



(e)

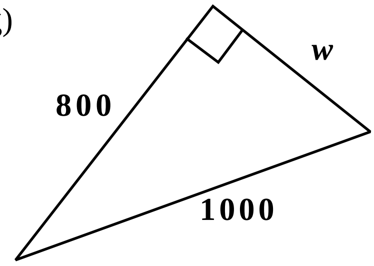


(f)

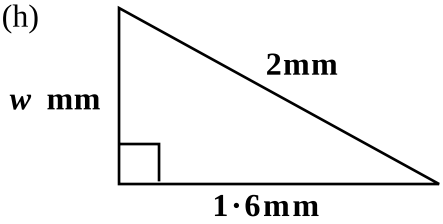


contd...

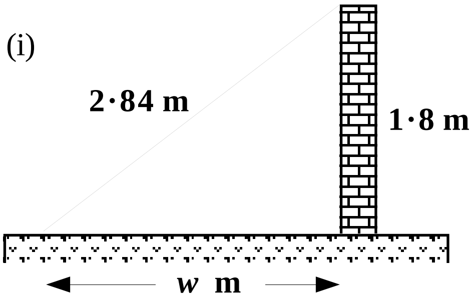
(g)



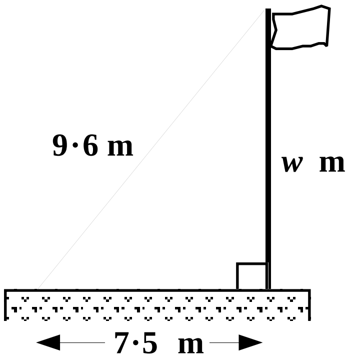
(h)



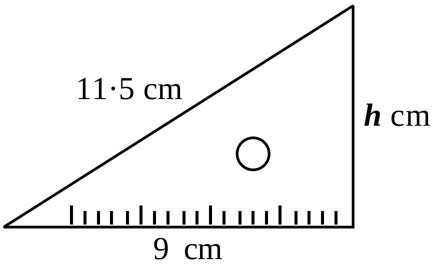
(i)



(j)



Exercise 4 In this exercise, give all your answers correct to 1 decimal place.

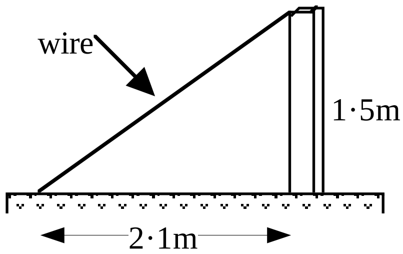
1. 

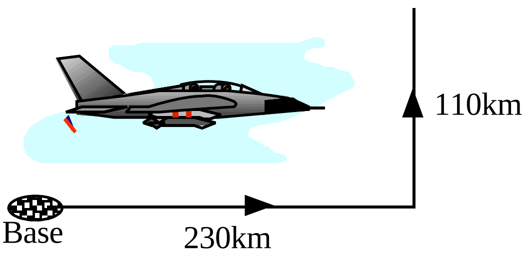
Calculate the height of this set square.

2. A 1·5 metre wooden post is cemented vertically into the ground and requires support until the cement dries.

   Due to marshy conditions, the nearest spot where a peg can be hammered in to hold a supporting wire is 2·1 metres from the post.

   What is the minimum length of wire which will be required?



3. 

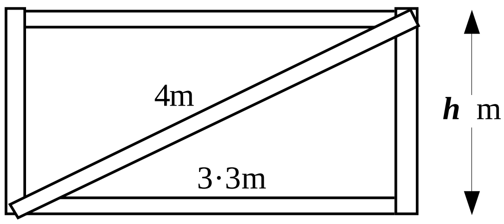
A fighter pilot flies 230 kilometres due East from base.

He then flies 110 kilometres due North.

How far is he now from base?

4. A gate which is 3·3 metres wide has a 4 metre wooden diagonal support.

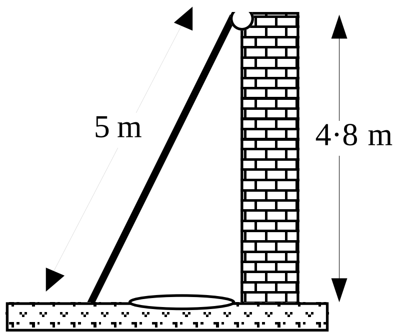
   Calculate the height of the gate?



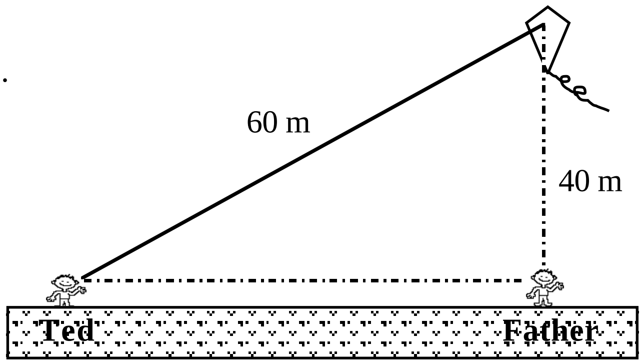
5. In order to rescue a cat who finds herself stuck in a gutter at the top of a 4·8 metre brick wall, the rescuer places his 5 metre ladder against the side of the wall.

For safety, he has to place the foot of the ladder <u>at least</u> 1 metre from the base of the wall.

Find how far the base of his ladder is from the the wall and state if it is safe for him to complete the rescue.



6.

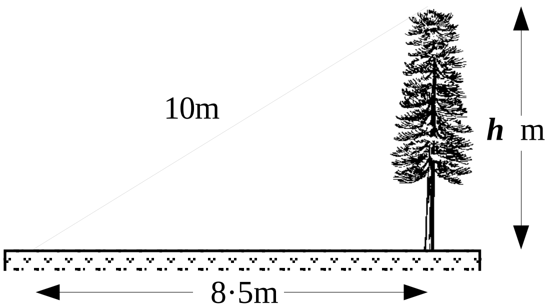


Ted and his father are flying a kite.

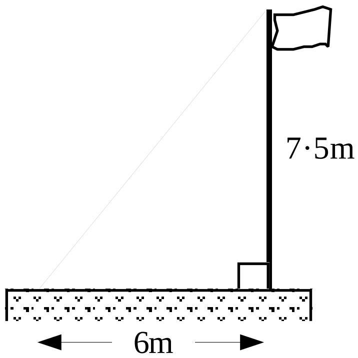
When Ted has let out 60 metres of string the kite is 40 metres above the ground and directly above his father.

How far away from his father is Ted standing?

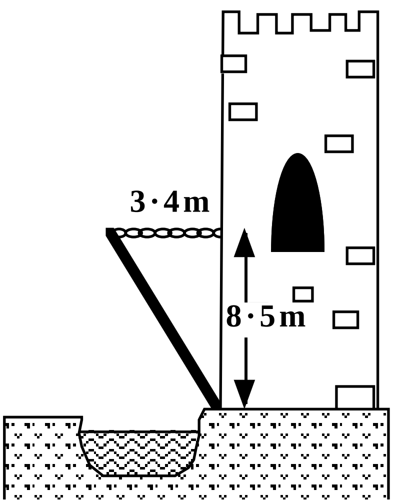
7. Calculate the height of this tree which is supported by a 10 metre rope tied down 8·5 metres from the foot of the tree.



8. What length of cable is needed to secure this flag pole?



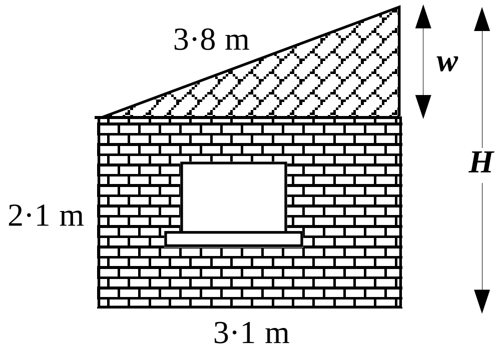
9.



The drawbridge of a castle is supported by a 3·4 metre chain which is attached to a bolt 8·5 metres up the castle wall.

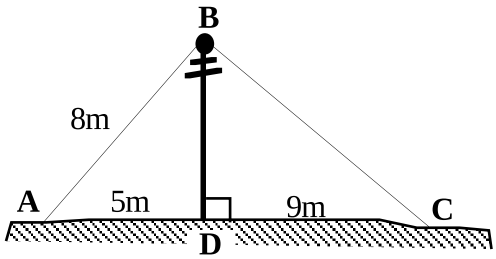
Calculate the length of the drawbridge.

10. The picture shows an end view of a house extension. The top part is made of timber.



(a) Calculate the height (***w*** metres) of the timber part of the extension.

(b) What is the overall height (***H*** metres) of the extension?

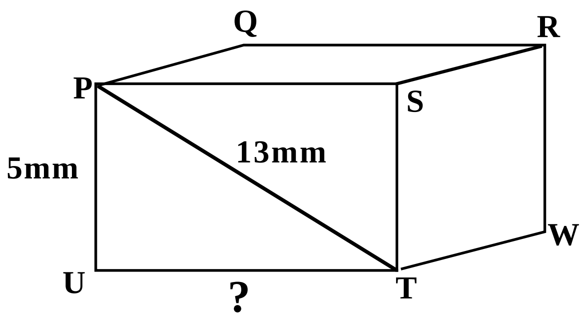
11. 

This picture shows a telegraph pole with 2 wires connecting the top of the pole to the ground.

(a) Calculate the height of the telegraph pole.

(b) Use the answer to part (a) to work out the length of wire BC.

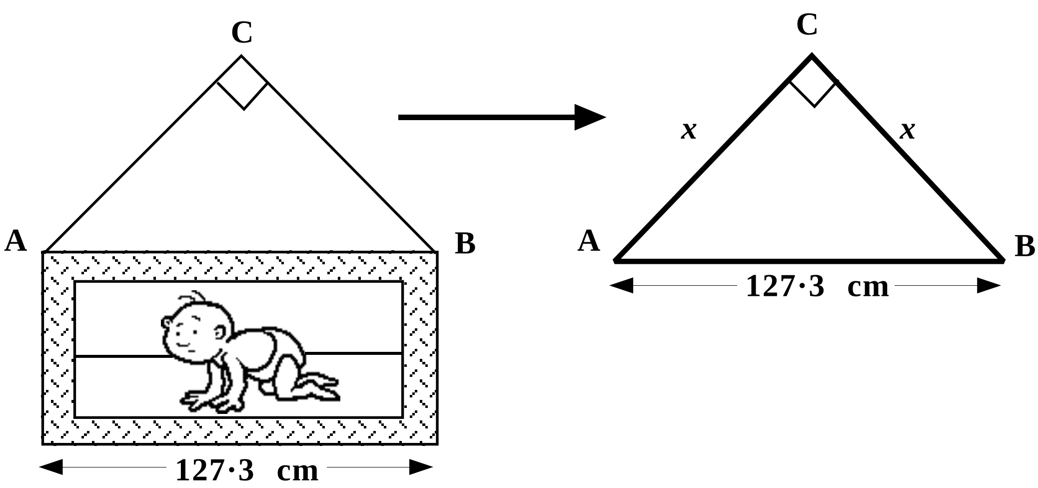
12. PSTU is a rectangular face of this cuboid.
PU = 5mm and PT = 13mm.
Calculate the length of the line UT.



13. HARD!

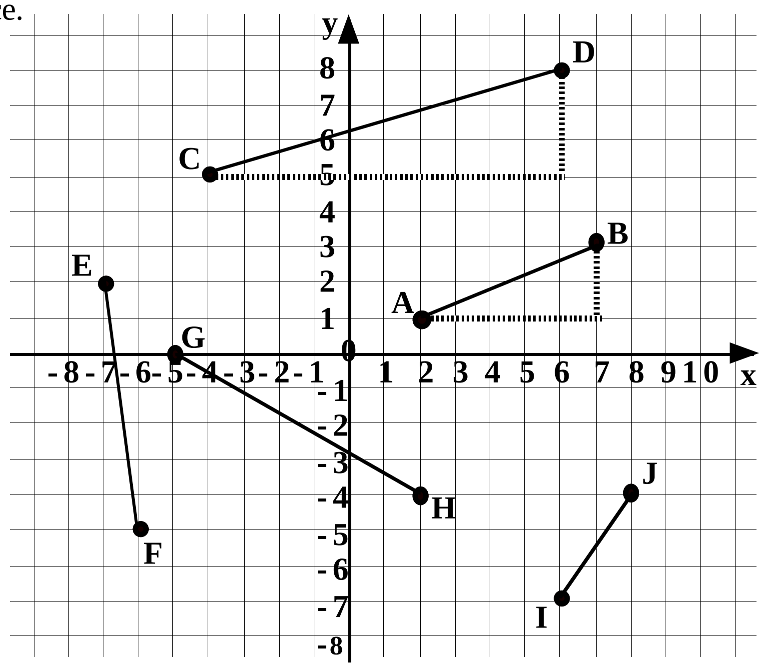
The baby portrait is 127·3 centimetres in length. (Line AB = 127·3 centimetres)

Calculate the length of AC, half of the string used for hanging the picture.



Exercise 5

1. Calculate the lengths of the 5 lines, AB, CD, EF, GH and IJ giving your answer correct to 1 decimal place.



2. Plot these points on a coordinate diagram and calculate the lengths of the lines joining them.

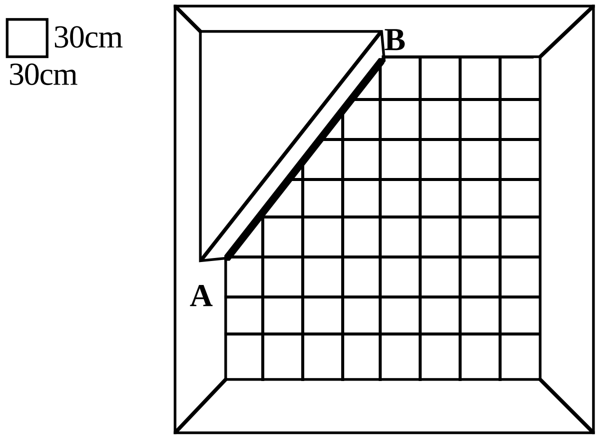
   (a) P(1,2) and Q(9,8)   (b) R(2,–1) and S(–3,11)

3. Part of a tiled bathroom wall is shown.

   A piece of sloping ceiling cuts across the tiles.

   The square tiles measure 30 centimetres by 30 centimetres.

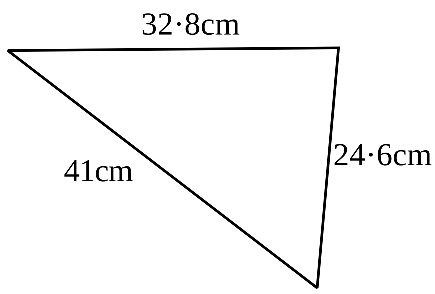
   Calculate the length of the sloping ceiling from **A** to **B**.



4. Charles was asked to draw a triangle with sides 32·8 centimetres, 24·6 centimetres and 41 centimetres.

   He drew a triangle with the correct measurements but sketched it like the one shown below.

   Explain why Charles' triangle should really have had a right angle in it.

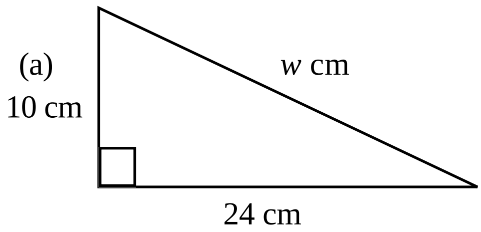


Mathematics 2 (Intermediate 1)

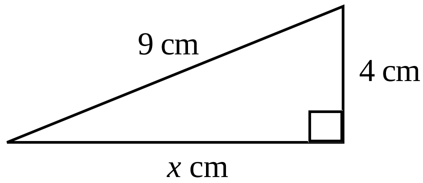
Checkup for The Theorem of Pythagoras

1. Calculate the length of the unknown side, correct to 1 decimal place, in each of these triangles:

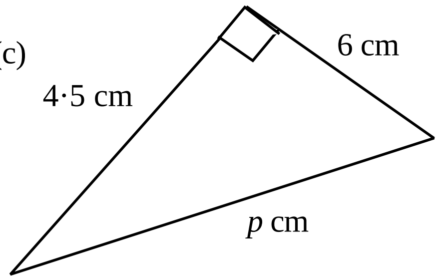
(a)



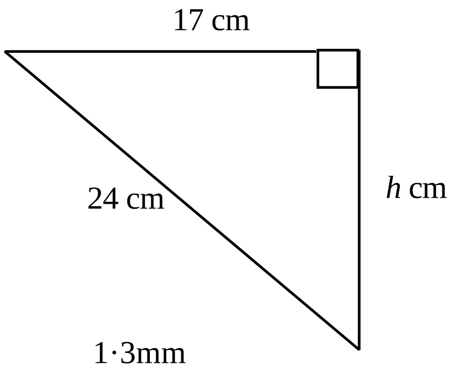
(b)



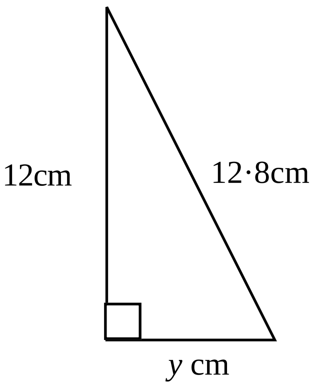
(c)



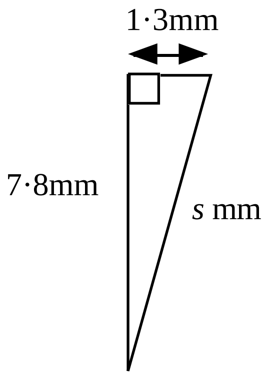
(d)



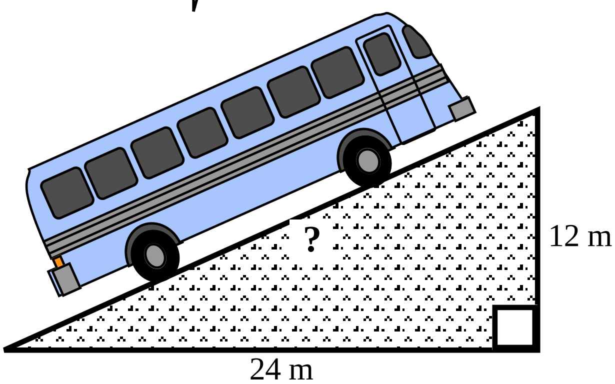
(e)



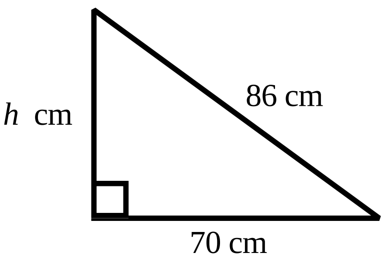
(f)



2. A bus is sitting on a giant ramp.
Find the length of the ramp.



3.



This diagram shows the sail on a model yacht.

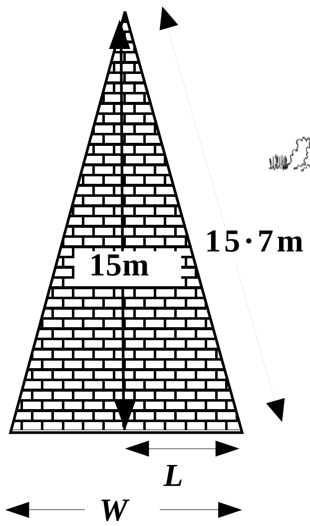
Calculate the height of the sail.

4. A church steeple is in the shape of an isosceles triangle. It is 15 metres high and has a sloping edge of 15·7 metres.

Calculate:

(a) the length ***L*** (metres).

(b) the width ***W*** (metres) of the steeple.



5. Plot these points on a coordinate diagram and calculate the lengths of the lines joining them.

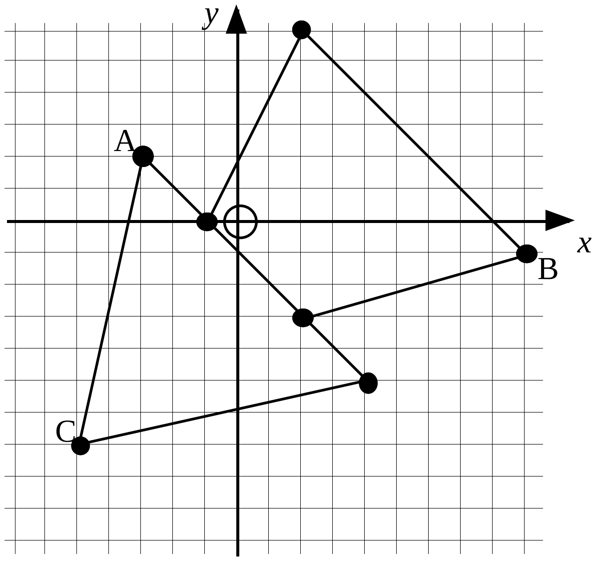
(a) A(1,0) and B(7,8)

(b) C(–4,–2) and D(–7,–6)

MATHEMATICS 2 (INTERMEDIATE 1)

Specimen Assessment Questions

1. Write down the coordinates of the three points A, B and C of the arrow shown opposite.



2. (a) Draw a set of axes and plot the three points P(–4,–4), Q(–3,1) and R(2,2).

   (b) Find and plot a fourth point, (call it S), such that when P, Q, R and S are joined, the quadrilateral formed is a **rhombus**.

3. Find the following:

(a) –7 + 9	(b) –11 + 3	(c) 5 + (–6)
(d) –3 + (–7)	(e) 8 – 10	(f) –2 – 5

4. Find the following:

(a) 5 x (–3)	(b) 20 x (–5)	(c) (–4) x 7	(d) (–21) x 0

5. Find the following:

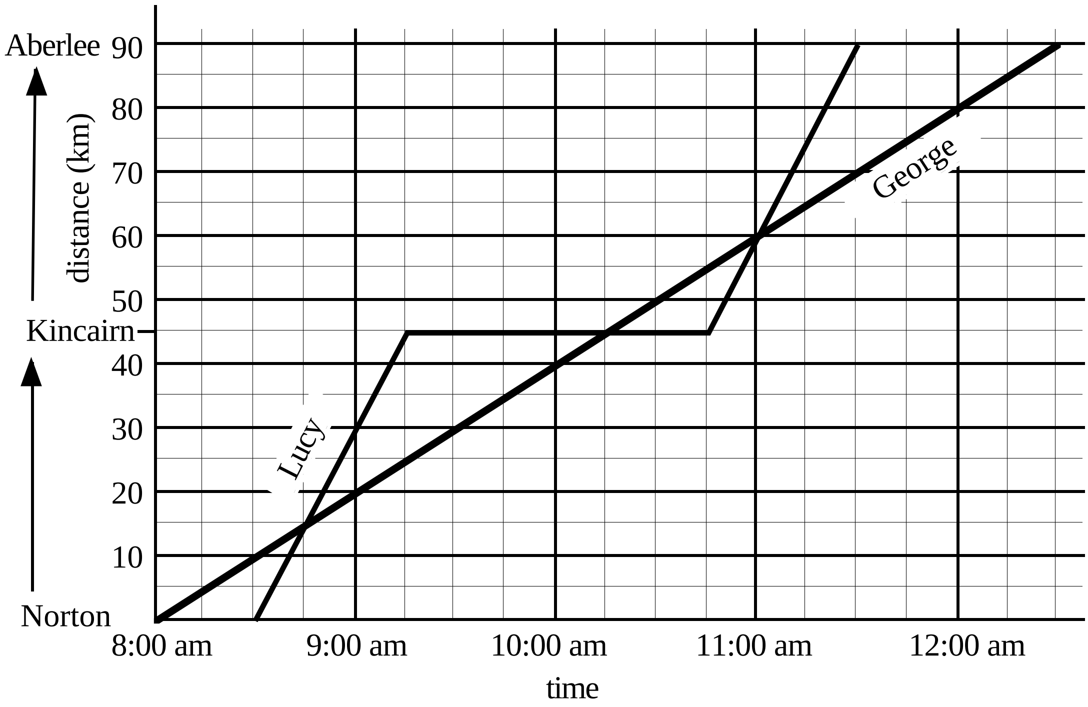
(a) (–18) ÷ 3	(b) 72 ÷ (–9)	(c) $\frac{-54}{6}$	(d) $\frac{98}{-7}$

6. Find the following:

(a) 7 – (–2)	(b) 10 – (–15)	(c) –3 – (–8)	(d) –12 – (–2)
(e) –7 x (–4)	(f) –5 x (–9)	(g) 15 ÷ (–3)	(h) 49 ÷ (–7)
(i) $\frac{-42}{-6}$	(j) $\frac{-72}{-8}$	(k) –3 x (–5) x 2	(l) –4 x (–4) x (–4)

7. My daughter arrived at a disco at 10:45 pm on Saturday night and left for a party at 2:25 am on Sunday morning. For how long was she at the disco?

8. George left Norton at 8:00 am and spent the morning cycling the 90 miles to Aberlee. Lucy left Norton in her car at 8:30 am and drove to Aberlee, but she stopped off at Kincairn to buy a new dress.
The graph below shows the two journeys.



(a) How far had George cycled when Lucy first overtook him?

(b) At what time did Lucy reach Kincairn and how long did she spend there?

(c) When did George overtake Lucy's car?

(d) Calculate George's average speed.

(e) Calculate Lucy's average speed from Norton to Kincairn.

9. (a) A ship takes 5 hours to sail the 120 miles between two ports. Calculate the average speed of the ship.

(b) A train, travelling at an average speed of 96 kilometres per hour, takes 3 hours to travel from Dirkby to Cornwell. Calculate the distance between the two towns.

(c) Travelling at an average speed of 420 m.p.h., how long should a plane take to travel the 1680 miles between two international airports?

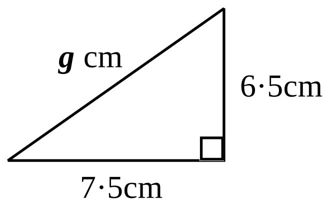
(d) It took me 2 hours 30 minutes to drive 240 kilometres from Glasgow to Aviemore. Calculate my average speed for the trip.

(e) Concord flew at a steady speed of 1200 kilometres per hour for 2 hours and 15 minutes. What distance did it cover in that time?

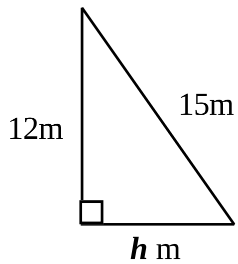
(f) My bus left Edinburgh at 9:30 am and travelled the 154 kilometres to Carlisle at an average speed of 88 kilometres per hour. At what time did I arrive at Carlisle?

10. Calculate the value of ***g*** and ***h*** in the following right-angled triangles:

(a)

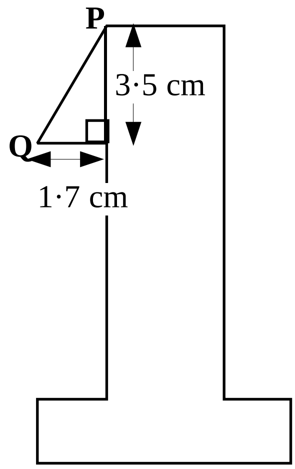


(b)

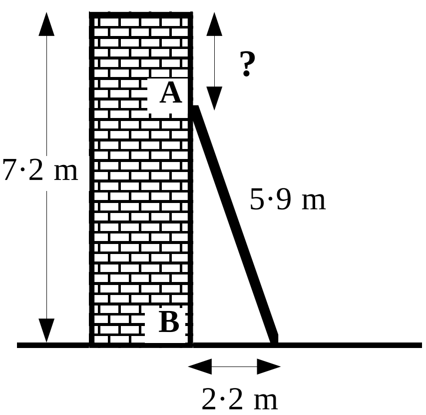


11. Alexis is an engineer and stays at Number 1 Arran View. She makes this number plate out of brass.

Calculate the length of the sloping edge PQ.



12.



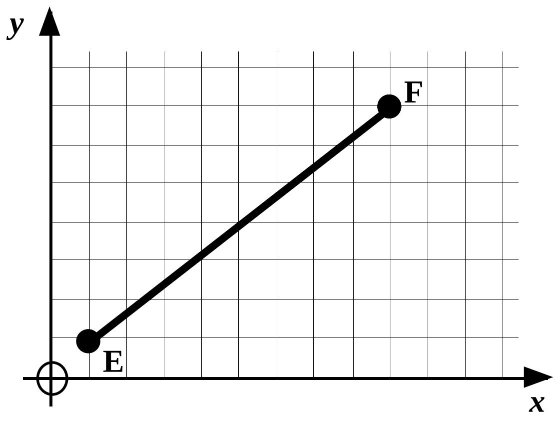
A wall is 7·2 metres high.

A 5·9 metre ladder is placed against it with its base 2·2 metres from the foot of the wall.

(a) Calculate the height AB which the ladder reaches up the wall.

(b) What is the distance from the top of the ladder to the top of the wall?

13. In the coordinate diagram E is the point (1,1) and F has coordinates (9,7). Calculate the length of the line EF.



ANSWERS TO MATHEMATICS 2 (INT 1)

Integers

Exercise 1

1. a = 18 b = 5 c = –4 d = –8 e = –16 f = –21

2. (a) 0 m (b) 15 m (c) 30 m (d) –10 m (e) –25 m (f) –35 m

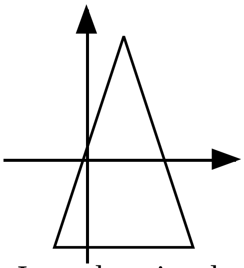
3. B(2,3) C(8,4) D(5,5) E(7,7) F(0,6) G(4,0)

4. J(1,–2) K(5,–1) L(3,–5) P(–3,5) Q(–4,1) R(–1,4)
U(–5,–1) V(–2,–2) W(–1,–5)

5. A(4,5) B(4,–6) C(1,–6) D(1,–2) E(–3,–2) F(–3,–6)
G(–6,–6) H(–6,5) I(–3,5) J(–3,1) K(1,1) L(1,5)

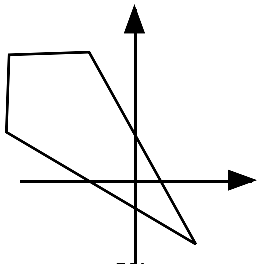
6. (a) diagram (b) points plotted (c) shape drawn (d) rectangle

7. (a)



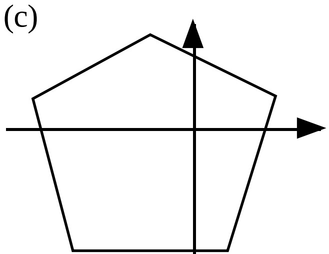
Isosceles triangle

(b)



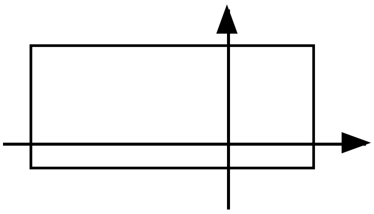
Kite

(c)



Pentagon

8. (a)



(b) / (c) S(2,–1)

(d) (–1,1)

Exercise 2A

1. (a) (i) 8 (ii) 5 (iii) 5 (iv) 6 (v) 20 (vi) 6 (vii) 14 (viii) 15 (ix) 6

(b) (i) 9 (ii) 20 (iii) 60 (iv) 3 (v) 5 (vi) 12 (vii) 30 (viii) 20 (ix) 17

(c) (i) 5°C (ii) 0°C (iii) –2°C (iv) –10°C (v) 2°C
(vi) –5°C (vii) 0°C (viii) 4°C (ix) –9°C (x) –14°C

2. (a) 12 (b) 6 (c) 3 (d) 0 (e) 6 (f) –6
(g) –3 (h) 5 (i) –5 (j) 0 (k) 13 (l) 2

3. (a) 4 (b) 0 (c) –1 (d) –7 (e) –14 (f) –3
(g) –7 (h) –9 (i) –13 (j) –50 (k) –50 (l) –17

4. (a) 7 (b) 9 (c) 0 (d) –2 (e) –49 (f) –8
(g) –14 (h) –9 (i) –100

5. (a) 3 (b) –2 (c) 1 (d) –6 (e) –7 (f) –4
(g) 0 (h) –10 (i) –3 (j) –6 (k) 0 (l) –18
(m) 0 (n) 0 (o) –18

Exercise 2B

1. (a) 9 (b) 12 (c) 21 (d) 14 (e) 9 (f) 3 (g) 60 (h) 9

2. (a) 3 (b) 4 (c) –10 (d) 0 (e) 12 (f) 1 (g) –1 (h) 1

Exercise 3A

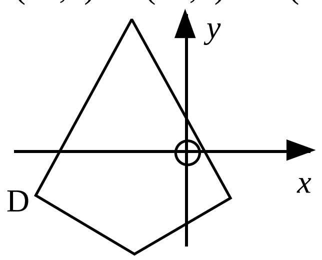
1. (a) 6 (b) –6 (c) 20 (d) –20 (e) –42 (f) –27 (g) –16 (h) –50
(i) –40 (j) –63 (k) –63 (l) –33 (m) –30 (n) –140 (o) –200
2. (a) –10 (b) –40 (c) –21 (d) –54 (e) –40 (f) –50 (g) –63 (h) –64
(i) 0 (j) –1 (k) –72 (l) –56 (m) –90 (n) –60 (o) –900
3. (a) –5 (b) –5 (c) –3 (d) –20 (e) –4 (f) –10 (g) –3 (h) –7
(i) –9 (j) –6 (k) –8 (l) –40 (m) –9 (n) –9 (o) –5

Exercise 3B

1. (a) 6 (b) 24 (c) 36 (d) 25 (e) 24 (f) 42 (g) 90 (h) 60 (i) 48
2. (a) –4 (b) –3 (c) –5 (d) –3 (e) –5 (f) –6 (g) –5 (h) –9 (i) –8
3. (a) 5 (b) 6 (c) 8 (d) 10 (e) 7 (f) 11 (g) 6 (h) 7 (i) 11
4. (a) –24 (b) 60 (c) 50 (d) –6 (e) –60 (f) 36 (g) –240 (h) 1 (i) –32
5. (a) –24 (b) –14 (c) –5 (d) 56 (e) 3 (f) –81 (g) 0 (h) –1 (i) –49
(j) 1 (k) –20 (l) –3 (m) 9 (n) 24 (o) –16 (p) –64 (q) –22 (r) 5
(s) 0 (t) 1 (u) –100 (v) –24 (w) 100 (x) –1

Checkup for Integers

1. A(3,3) B(5,0) C(5,–6) D(–1,–6) E(–1,–3) F(–3,–3) G(–3,–6)
H(–8,–6) I(–8,0) J(–6,3) K(–3,3) L(–3,4) M(–1,4) N(–1,3)
2. (a) & (b)



(c) D(–5,–2)

3. (a) 2 (b) –3 (c) 3 (d) –6 (e) –12 (f) –3 (g) 0 (h) –10 (i) –5
(j) –7 (k) 0 (l) –12 (m) 0 (n) 0 (o) –12 (p) 7 (q) 18 (r) 30
(s) 4 (t) 0 (u) –7
4. (a) –12 (b) –15 (c) –3 (d) 56 (e) 6 (f) –25 (g) 0 (h) –1 (i) –16
(j) 1 (k) –20 (l) –5 (m) 8 (n) 27 (o) –12 (p) –9 (q) –30 (r) 14
(s) 0 (t) 1 (u) –10 (v) 30 (w) –36 (x) 16

ANSWERS TO MATHEMATICS 2 (INT 1)

Speed Distance Time

Exercise 1

1. A — walker B — cyclist C — lorry D — racing car
2. (a) (i) 2 km (ii) 4 km (iii) 8 km (iv) 10 km (b) 4 km/hr
3. (a) 10 km/hr (b) 8 km/hr (c) 40 km/hr
4. (a) 1 hr (b) $^1/_2$ hr (c) 12:30 (d) 13:30
(e) (i) 50 mph (ii) 0 mph (iii) 30 mph
5. (a) 1 hr (b) Out – 60 mph, In – 40 mph (c) the outward journey
6. (a) 20 km/hr (b) 60 km/hr (c) 9:45 am (d) 15 km

cont'd ....

7. (a) B (b) (i) 80 km/hr (ii) 80 km/hr (c) 9:30 am

8. (a) 2:00 pm —> (3:00 pm 3:30 pm) —> (4:30 pm 4:50 pm) —> 5:50 pm
(b) (i) 120 km (ii) 100 km
(c) (i) 120 km/hr (ii) 80 km/hr (iii) 100 km/hr (iv) 80 km/hr

Exercise 2

1. (a) 2 hr (b) 1 hr 30 mins (c) 3 hr 15 mins
(d) 1 hr 35 mins (e) 1 hr 40 mins (f) 3 hr 15 mins

2. 2 hr 45 mins

3. (a) (i) 1 hr 5 mins (ii) 45 mins (iii) 6 hr 35 mins
(b) (i) 1:45 pm (ii) 2:30 pm

4. (a) 3 hr 25 mins (b) 9 hr 25 mins (c) (i) 2 hr 50 mins (ii) 9:15 pm

Exercise 3A

1. 6 km/hr

2. (a) 36 km/hr (b) 1200 km/hr (c) 40 km/hr (d) 83 km/hr (e) 18 km/hr

3. (a) 75 mph (b) 10 m/sec (c) 900 mph (d) 43 cm/hr

4. (a) 50 km/hr (b) 60 mph (c) 44 km/hr (d) 16 mph

5. (a) 24 km/hr (b) 48 mph (c) 52 mph (d) 44 mph (e) 320 km/hr

6. (a) 55 mph (b) 400 mph (c) 12 kph

Exercise 3B

1. 4 hr

2. (a) 2 hr (b) 4 hr (c) 3 hr (d) 5 hr

3. (a) 2 hr (b) 50 secs (c) 6 hr (d) 8 hr

4. (a) 1 hr 30 mins (b) 2 hr 30 mins (c) 30 mins (d) 1 hr 30 mins

5. (a) 45 mins (b) 2 hr 15 mins (c) 4 hr 15 mins (d) 2 hr 15 mins

6. (a) 2 hr 30 mins –> 11:15 am (b) 5:45 pm (c) 4:35 am, Tuesday

Exercise 3C

1. 15km

2. (a) 74 km (b) 68 km (c) 225 km (d) 3240 km

3. (a) 216 miles (b) 200 metres (c) 3750 miles (d) 48 miles

4. (a) 20 miles (b) 125 km (c) 27 km (d) 45 km
(e) 6 miles (f) 108 km (g) 1950 miles (h) 6 miles

5. (a) 2 hr –> 620 miles (b) 112 miles (c) 800 miles

Checkup

1. (a) 18 km/hr (b) 50 mph

2. (a) Jones (b) 40 and 50 seconds (c) Davis (d) 50 and 80 secs
(e) about 64 - 65 secs (f) 20 seconds (g) 8 metres/sec

3. (a) 2 hr 10 mins (b) 3 hr 25 mins

4. (a) 60 mph (b) 5 hr (c) 132 miles
(d) 1hr 15 mins and 360 km/hr (e) $3^1/_2$ hr (f) 20 500 km

ANSWERS TO MATHEMATICS 2 (INT 1)

Theorem of Pythagoras

Exercise 1

1. (a) 25 (b) 64 (c) 100 (d) 1 (e) 12·25
   (f) 0·09 (g) 635·04 (h) 10000

2. (a) 5 (b) 9 (c) 10 (d) 4·5 (e) 10·5
   (f) 18 (g) 100 (h) 1

3. (a) $10{\cdot}2\text{cm}^2$ (b) $30{\cdot}3\text{cm}^2$ (c) $47{\cdot}6\text{cm}^2$

4. (a) 3cm (b) 7·5cm (c) 5·4cm

Exercise 2

1. (a) done (b) $c^2 = a^2 + b^2$ (c) $x^2 = y^2 + z^2$ (d) $p^2 = t^2 + s^2$

2. (a) 13(·0) (b) 29(·0) (c) 5(·0) (d) 15(·0) (e) 100(·0)
   (f) 7·6 (g) 12·2 (h) 20(·0) (i) 10·7 (j) 3·5
   (k) 9·1

Exercise 3

1. (a) done (b) $a^2 = c^2 - b^2$ (c) $b^2 = m^2 - t^2$ (d) $y^2 = f^2 - b^2$

2. (a) 5(·0) (b) 6·7 (c) 46·6 (d) 94·7 (e) 7(·0)
   (f) 71·4 (g) 600(·0) (h) 1·2 (i) 2·2 (j) 6(·0)

Exercise 4

1. 7·2cm 2. 2·6m 3. 255(·0)km 4. 2·3m 5. 1·4m, safe to complete rescue
6. 44·7m 7. 5·3m 8. 9·6m 9. 9·2m
10. (a) 2·2m (b) 4·3m 11. (a) 6·2m (b) 11(·0)m
12. 12mm 13. 90(·0)cm

Exercise 5

1. AB = 5·4, CD = 10·4, EF = 7·1, GH = 8·1, IJ = 3·6

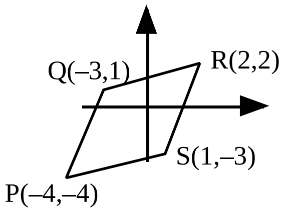
2. (a) 10 (b) 13 3. 192·1 cm 4. $32{\cdot}8^2 + 24{\cdot}6^2 = 41^2$

Check Up

1. (a) 26(·0) cm (b) 8·1cm (c) 7·5cm (d) 16·9 cm (e) 4·5cm
   (f) 7·9mm
2. 26·8m 3. 50(·0)cm 4. (a) 4·6m (b) 9·2/9·3m 5. (a) 10 (b) 5

ANSWERS TO MATHEMATICS 2 (INT 1)
Specimen Assessment Questions

1. A(–3,2), B(9,–1), C(–5,–7)
2. (a) & (b)



   (c) S(1,–3)
3. (a) 2 (b) –8 (c) –1 (d) –10 (e) –2 (f) –7
4. (a) –15 (b) –100 (c) –28 (d) 0
5. (a) –6 (b) –8 (c) –9 (d) –14
6. (a) 9 (b) 25 (c) 5 (d) –10 (e) 28 (f) 45
   (g) –5 (h) –7 (i) 7 (j) 9 (k) 30 (l) –64
7. 3 hours 40 minutes
8. (a) 15 km (b) 9:15 am, $1\frac{1}{2}$ hours (c) 10:15 am (d) 20 km/hr (e) 60 km/hr
9. (a) 24 mph (b) 288 km (c) 4 hours
   (d) 96 km/hr (e) 2700 km (f) 11:15 am
10. (a) 9·9cm (b) 9 m
11. 3·9cm
12. (a) 5·5m (b) 1·7m
13. 10 units