Mathematics

Mathematics 3
Intermediate 2

4953

Spring 1999

HIGHER STILL

Mathematics

Mathematics 3
Intermediate 2

Support Materials

STAFF NOTES

INTRODUCTION

These support materials for Mathematics were developed as part of the Higher Still Development Programme in response to needs identified at needs analysis meetings and national seminars.

Advice on learning and teaching may be found in *Achievement for All* (SOEID 1996), *Effective Learning and Teaching in Mathematics* (SOEID 1993) and in the Mathematics Subject Guide.

This support package provides student material to cover the content of Mathematics 3 within the Intermediate 2 course. The depth of treatment is therefore more than is required to demonstrate competence in the unit assessment; that is, it goes beyond minimum grade C.
The content of Mathematics 3 (Int 2) is set out in the landscape pages of content in the Arrangements document where the requirements of the unit Mathematics 3 (Int 2) are also stated. Students are unlikely to have met much of the materials of this unit before - Algebraic Operations, Quadratic Functions and non-Right Angled Triangle work.

The material is designed to be directed by the teacher/lecturer, who will decide on the ways of introducing topics and in the use of exercises for consolidation and for formative assessment. The use of a scientific calculator will be necessary for Part C but students should be encouraged to set down all working and, where appropriate, use mental calculations. The use of computers or graphics calculators is obviously highly desirable for some of the graphical aspects in Parts B and C of the course.

An attempt has been made to have the ‘easy’ questions at the start of each exercise, leading to more testing questions towards the end of the exercise. While students may tackle most of the questions individually, there are opportunities for collaborative working. Staff may wish to discuss points raised with individuals, groups and the whole class.

The specimen assessment questions at the end of the package are **not** intended to be only at minimum grade C. The National Assessment Bank packages for Mathematics 3 (Int 2) contain questions that meet the requirements of this unit.

This package gives opportunities to practise core skills. Information on the core skills embedded in the unit, Mathematics 3 (Int 2) and in the Intermediate 2 course is given in the final version of the Arrangements document. General advice and details of the Core Skills Framework can be found in the Core Skills Manual (HSDU June 1998).

Brief notes of advice on the teaching of each topic are given.

The introductory notes for teachers/lecturers comprise pages 1-16 inclusive.

Format of Student Material

- Exercises on Algebraic Operations
  Checkup for Algebraic Operations

- Exercises on Quadratic Functions
  Checkup for Quadratic Functions

- Exercises on Further Trigonometry
  Checkup for Further Trigonometry

- Specimen Assessment Questions

- Answers for all exercises

ALGEBRAIC OPERATIONS

A. Reducing algebraic fractions to their simplest form

Students should be shown the method for 'cancelling down' a vulgar fraction, then fractions with letters can be introduced. * Before this exercise some revision of factorisation may be necessary.

Example 1 Simplify $^{35}/_{49}$

Ans. $^{35}/_{49} = {}^{5}/_{7}$

Example 2 Simplify $^{6a}/_{9a}$

Ans. $^{6a}/_{9a} = {}^{2}/_{3}$

Example 3 Simplify $\dfrac{16xy^2}{32x^2y}$

Ans. $\dfrac{16xy^2}{32x^2y} = \dfrac{y}{2x}$

Example 4 Simplify $\dfrac{p^2 - pq}{p}$

Ans. $\dfrac{p^2 - pq}{p} = \dfrac{p(p-q)}{p} = p - q$

Note the factorisation in the numerator !

Exercise 1 Questions 1 and 2 may now be attempted.

The following two examples could be used to illustrate the need for factorising the numerator and/or denominator before cancelling.

Example 5 Simplify $\dfrac{v^2 - 1}{v - 1}$

Ans. $\dfrac{v^2 - 1}{v - 1}$

$= \dfrac{(v-1)(v+1)}{v-1}$

$= v + 1$

Example 6 Simplify $\dfrac{2a^2 + a - 1}{2a^2 + 5a - 3}$

Ans. $\dfrac{2a^2 + a - 1}{2a^2 + 5a - 3}$

$= \dfrac{(2a-1)(a+1)}{(2a-1)(a+3)}$

$= \dfrac{(a+1)}{(a+3)}$

Exercise 1 Question 3 may now be attempted.

B. Applying the four rules to algebraic fractions

The following eight examples can be used to illustrate the four rules. It may be that additional examples will be required to be shown to the class in order to reinforce the method.

Example 1 Simplify $\dfrac{2}{3} \times \dfrac{1}{8}$

Ans. $\dfrac{2}{3} \times \dfrac{1}{8}$

$= \dfrac{2}{24}$

$= \dfrac{1}{12}$

Example 2 Simplify $\dfrac{a^2}{b} \times \dfrac{b}{a}$

Ans. $\dfrac{a^2}{b} \times \dfrac{b}{a}$

$= \dfrac{a^2b}{ba}$

$= a$

contd.

Example 3 Simplify $\frac{3}{5} \div \frac{6}{5}$

Ans. $\frac{3}{5} \div \frac{6}{5}$

$= \frac{3}{5} \times \frac{5}{6}$

$= {}^{1}/_{2}$ (by cancelling)

Example 4 Simplify $\frac{c^2}{d} \div \frac{c}{d}$

Ans. $\frac{c^2}{d} \div \frac{c}{d}$

$= \frac{c^2}{d} \times \frac{d}{c}$

$= c$ (by cancelling)

Exercise 2 Questions 1 and 2 may now be attempted.

Example 5 Simplify $\frac{1}{3} + \frac{1}{5}$

Ans. $\frac{1}{3} + \frac{1}{5}$

$= \frac{5}{15} + \frac{3}{15}$

$= {}^{8}/_{15}$

Example 6 Simplify $\frac{2}{3} - \frac{3}{5}$

Ans. $\frac{2}{3} - \frac{3}{5}$

$= \frac{10}{15} - \frac{9}{15}$

$= {}^{1}/_{15}$

Example 7 Simplify $\frac{4r}{5} + \frac{s}{2}$

Ans. $\frac{4r}{5} + \frac{s}{2}$

$= \frac{8r}{10} + \frac{5s}{10}$

$= \frac{8r + 5s}{10}$

Example 8 Simplify $\frac{1}{v} - \frac{3}{w}$

Ans. $\frac{1}{v} - \frac{3}{w}$

$= \frac{w}{vw} - \frac{3v}{vw}$

$= \frac{w - 3v}{vw}$

Exercise 2 Questions 3 and 4 may now be attempted.

Example 9 Simplify $\frac{x+2}{3} + \frac{x-1}{4}$

Ans. $\frac{x+2}{3} + \frac{x-1}{4}$

$= \frac{4(x+2)}{12} + \frac{3(x-1)}{12}$

$= \frac{4x + 8 + 3x - 3}{12}$

$= \frac{7x + 5}{12}$

Exercise 2 Question 5 may now be attempted.

This example should be followed by the simplification of $\frac{x+2}{3} - \frac{x-1}{4}$

C. Changing the subject of a formula

It should be emphasised that ALL WORKING and ALL STEPS should be shown.

The following examples could be used to introduce changing the subject of a formula. The examples are a mixture of 'change side, change sign', cross multiplication, dividing and finding the square root.

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Example 1 Change $x + 3 = m$ to x.

Ans. $x + 3 = m$

$x = m - 3$

Example 2 Change $a/b = p$ to b

Ans. $a/b = p$

$bp = a$ (cross x)

$b = a/p$

Example 3 Change $ax - c = d$ to x.

Ans. $ax - c = d$

$ax = d + c$

$x = \frac{d + c}{a}$

Example 4 Change $A = \pi d^2$ to d

Ans. $A = \pi d^2$

$\pi d^2 = A$

$d^2 = A/\pi$

$d = \sqrt{A/\pi}$

Question 6 of Exercise 3 provides 6 harder examples, this example could be used as extension:

Example: Change $y = \frac{v - z}{z}$ to z.

Ans.

$$\begin{aligned} y &= \frac{v - z}{z} \\ zy &= v - z \\ zy + z &= v \\ z(y + 1) &= v \\ z &= \frac{v}{y + 1} \end{aligned}$$

Exercise 3 may now be attempted.

D. Simplifying surds

Irrational numbers could be introduced by first revising sets of numbers. E.g.
Real nos. - all the numbers which can be represented on a number line.
Whole nos. 0, 1, 2, 3, 4, 5, 6, ...
Integers ....-3, -2, -1, 0, 1, 2, 3, 4, ...
Rational nos. 8, -2, 1/2, -3/4 etc. numbers which can be expressed as a fraction.
Then explaining that numbers like √2, √3, π.... cannot be expressed as a fraction, therefore these are irrational.

contd.

A SURD is a special kind of irrational number. It is a square root, a cube root, etc. which cannot be expressed as a rational number.

$\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{10}$ are all surds, whereas $\sqrt{25}$ and $\sqrt[3]{8}$ are not surds as $\sqrt{25} = 5$ and $\sqrt[3]{8} = 2$.

The following examples could be used to show students how to simplify surds:

Example 1 Express $\sqrt{18}$ in its simplest form.

Ans. $\sqrt{18} = \sqrt{(\mathbf{9} \times 2)} = \underline{\underline{3\sqrt{2}}}$ Explain why $\sqrt{18} = \sqrt{(6 \times 3)}$ is not used.

(↑ $\mathbf{9}$: Largest square number which divides into 18)

Example 2 Simplify $\sqrt{8} + 5\sqrt{2}$

Ans. $\sqrt{8} + 5\sqrt{2} = \sqrt{(\mathbf{4} \times 2)} + 5\sqrt{2} = \mathbf{2}\sqrt{2} + 5\sqrt{2} = \underline{\underline{7\sqrt{2}}}$

Exercise 4 may now be attempted.

Example 3 Simplify $\sqrt{6} \times \sqrt{6}$

Ans. $\sqrt{6} \times \sqrt{6} = \sqrt{36} = \underline{\underline{6}}$

Example 4 Simplify $2\sqrt{3} \times 5\sqrt{6}$

Ans.
$$\begin{aligned} & 2\sqrt{3} \times 5\sqrt{6} \\ = \ & 10\sqrt{18} \\ = \ & 10\sqrt{(\mathbf{9} \times 2)} \\ = \ & 10 \times \mathbf{3}\sqrt{2} \\ = \ & \underline{\underline{30\sqrt{2}}} \end{aligned}$$

Example 5 Simplify $\sqrt{3}(4 - 5\sqrt{3})$

Ans.
$$\begin{aligned} & \sqrt{3}(4 - 5\sqrt{3}) \\ = \ & 4\sqrt{3} - 5\sqrt{3}\sqrt{3} \\ = \ & 4\sqrt{3} - (5 \times 3) \\ = \ & \underline{\underline{4\sqrt{3} - 15}} \end{aligned}$$

Exercise 5 may now be attempted.

E. Rationalising a surd denominator

Students should be reminded of the difference between a numerator and a denominator and an explanation given of what rationalising a denominator means.

Example 1 Express $\frac{12}{\sqrt{6}}$ with a rational denominator.

Ans. $\frac{12}{\sqrt{6}}$ can be multiplied by 1, without changing its value.

The number '1' can be written here as $\frac{\sqrt{6}}{\sqrt{6}}$.

So $\frac{12}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$ will have the same value as $\frac{12}{\sqrt{6}}$ but will be written differently.

$$\frac{12}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{12\sqrt{6}}{6} = \underline{\underline{2\sqrt{6}}}$$

Example 2 Express $\sqrt{\frac{9}{10}}$ with a rational denominator.

Ans. $\sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} = \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \underline{\underline{\frac{3\sqrt{10}}{10}}}$

Exercise 6 Q1 and Q2 may now be attempted.

Exercise 6 Q3 contains eight examples appropriate to grades A/B.

Example 3 Express $\frac{1}{2+\sqrt{3}}$ with a rational denominator.

Ans. Here, multiply by $\frac{2-\sqrt{3}}{2-\sqrt{3}}$ to rationalise the denominator.

So, $\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = \underline{\underline{2-\sqrt{3}}}$

Notice - no √ term in denominator

Exercise 6 Q3 may be attempted now (for extension to grades A/B).

F. Simplify expressions using the laws of indices

Basically, there are 6 rules for the students to learn. They should be lead through them, doing examples of each type, before attempting Exercise 10, containing miscellaneous examples.

The rules are:

Rule 1 $a^m \times a^n = a^{m+n}$

Rule 2 $a^m \div a^n = a^{m-n}$

Rule 3 $(a^m)^n = a^{mn}$

Rule 4 a

Rule 5 $a^{-m} = {}^1/_{a^m}$

Rule 6 $a^{m/n} = \sqrt[n]{a^m}$

Examples

Rule 1

$3^7 \times 3^4 = 3^{7+4} = 3^{11}$

$4x^2 \times 5x^5 = 20x^{2+5} = 20x^7$

Rule 2

$3^7 \div 3^4 = 3^{7-4} = 3^3$

$\mathbf{6}a^8 \div \mathbf{3}a^6 = \mathbf{2}a^{8-6} = 2a^2$

Rule 3

$(6^3)^5 = 6^{3 \times 5} = 6^{15}$

$(x^2y^3)^4 = x^{2\times4}y^{3\times4} = x^8y^{12}$

Exercise 7 may now be attempted.

Rule 4

$(\text{Any number})^0 = 1$

$(10246)^0 = 1$

Rule 5 –ve power = $^1/$+ve power

$3^{-2} = {}^1/_{3^2} = {}^1/_9$

${}^4/_{x^{-3}} = {}^4/_{1/x^3} = 4x^3$

$a^3(a^2 - a^{-4}) = a^{3+2} - a^{3-4} = a^5 - a^{-1} = a^5 - {}^1/_a$

Exercise 8 may now be attempted.

Rule 6

$x^{3/5} = \sqrt[5]{x^3}$ $\qquad$ $\sqrt[3]{w^2} = w^{2/3}$ $\qquad$ $25^{-1/2} = 1/25^{1/2} = 1/\sqrt{25} = 1/5$

Exercise 9 may now be attempted.

Exercise 10, containing Miscellaneous Examples, comes next, there are examples in this exercise which are appropriate to grades A/B.

Example 1

$(a^{-1/3})^3 = a^{-1} = 1/a$

Example 2

$8w^{3/4} \div 2w^{-1/4} = 4w^{3/4-(-1/4)} = 4w^1 = 4w$

Example 3

If $m = 64$, find the value of $2m^{2/3}$.

Ans. $2m^{2/3} = 2(64^{2/3}) = 2[\sqrt[3]{(64)^2}] = 2(4^2) = 2 \times 16 = 32$

Example 4

Express with positive indices: $(p^{4/3})^{-3/4}$

Ans. $(p^{4/3})^{-3/4} = p^{4/3 \times -3/4} = p^{-1} = 1/p$

Example 5

Simplify :- $\dfrac{x^{-3} \times x^3}{x^{-1}}$

Ans. $\dfrac{x^{-3} \times x^3}{x^{-1}} = \dfrac{x^{-3+3}}{1/x} = \dfrac{x^0}{1/x} = \dfrac{1}{1/x} = x$

Exercise 10 may now be attempted.

The idea in Exercise 11 is to express the numerator in index form.

Illustrate by two examples on the board

Example 1 $\dfrac{9}{x^3} = 9 \times 1/x^3 = 9x^{-3}$

Example 2 $\dfrac{9}{4\sqrt{x}} = \dfrac{9}{4x^{1/2}} = \dfrac{9x^{-1/2}}{4}$

Exercise 11 may now be attempted. *This exercise is appropriate to grades A/B

The checkup exercise may then also be attempted.

QUADRATIC FUNCTIONS

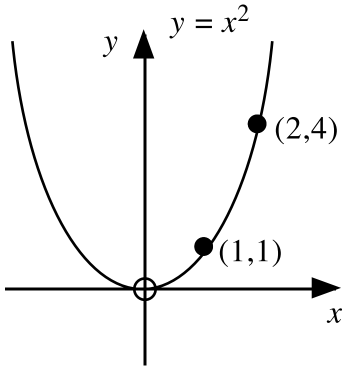
A. Quadratic graphs

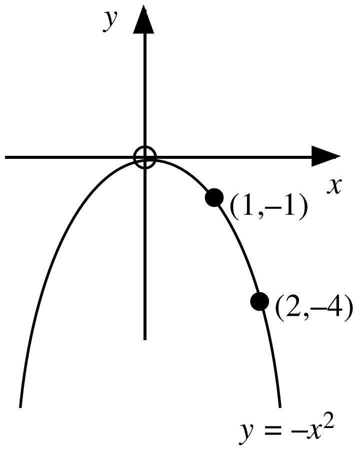
Students should be made aware of the importance of recognising the basic parabolas associated with

$y = x^2$ and $y = -x^2$.

From this, develop with the students the graphs associated with :–

$y = 2x^2$, $y = 3x^2$, $y = {}^1/_2x^2$,
$y = -2x^2$, $y = -3x^2$, $y = -{}^1/_2x^2$, etc.,





by plotting 2 or 3 points on the graph and drawing a smooth parabola shape through them.

Students should then be encouraged to develop further the idea of graphing the functions $y = x^2 \pm 1$, $y = x^2 \pm 2$, etc., by 'moving' the basic parabola $y = x^2$ up or down.
Similar development of $y = -x^2 \pm 1$, $y = -x^2 \pm 2$, etc. should then follow.
A class set of graphic calculators could be used to allow students to investigate the transformation of quadratic graphs.

Exercise 1 may now be attempted.

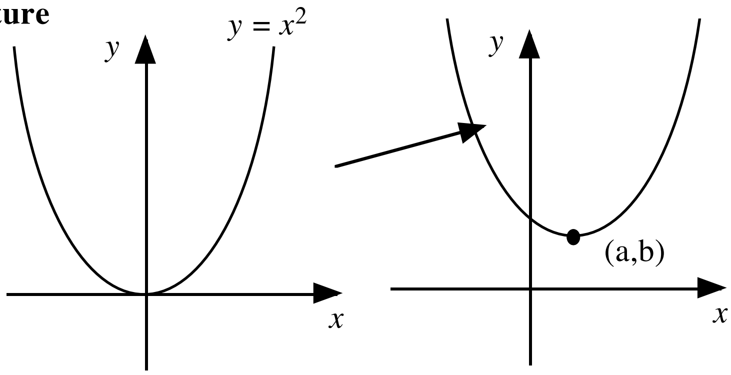
B. Turning points - their nature

Go over the idea of moving the turning point away from the origin to a point (a,b).

$y = x^2$

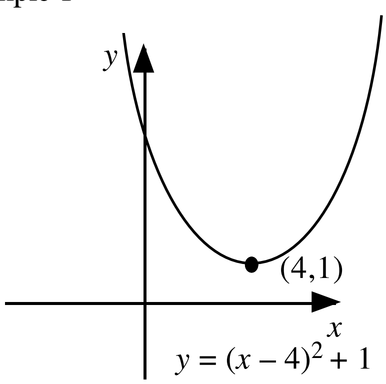
becomes $\boxed{y - b = (x - a)^2}$

=> $y = (x - a)^2 + b$

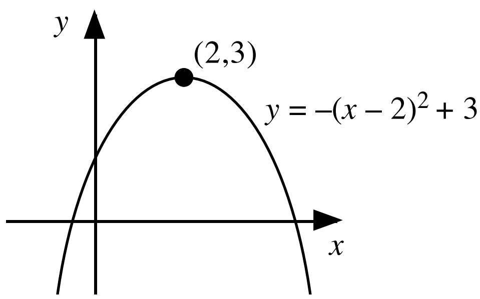


Similarly, $y = -x^2$ => $y - b = -(x - a)^2$ => $y = -(x - a)^2 + b$.

Example 1



Example 2



Exercise 2 may now be attempted.

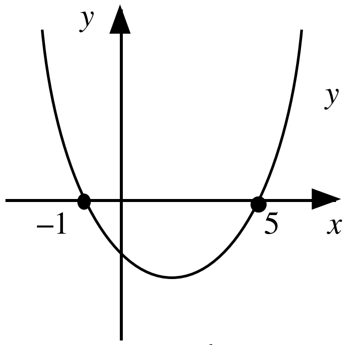
C. Solving quadratic equations graphically

The difference between a 'quadratic equation' and a 'quadratic expression' should be explained to students. The 'roots' of a quadratic equation should be defined graphically. Again, graphic calculators provide an efficient means of reinforcing the teaching of this topic, as many will not only draw the functions but also show a table of values.

Example 1

Roots (from graph) are at

$x = -1, 5$



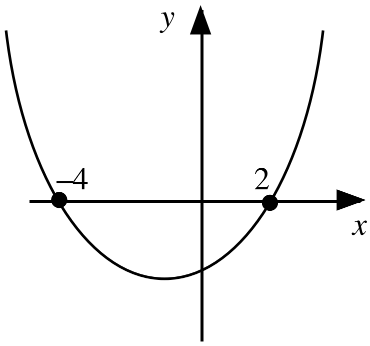
$y = x^2 - 4x - 5$

Example 2

$y = x^2 + 2x - 8$

then draw parabola (or pick out roots from table)

x	–5	–4	–3	–2	–1	0	1	2	3
y	7	0	–5	–8	–9	–8	–5	0	7



Exercise 3 may now be attempted.

D. Solving quadratic equations algebraically

Factorisation should be revised:

(a) Common factors $x^2 - 3x$, $8x^2 + 12x$, etc.

(a) Difference of 2 squares $x^2 - 4$, $4x^2 - 25$, etc.

(a) Trinomials $x^2 + 6x + 8$, $x^2 - x - 12$, etc.

Students should then be shown how to use factorisation to solve quadratic equations (up to simple trinomals with <u>1</u>x^2 at most in the trinomial).

Examples:

$2x^2 - 6x = 0$
$2x(x - 3) = 0$
$\Rightarrow 2x = 0$ or $x - 3 = 0$
$\Rightarrow x = 0$ or 3

$x^2 - 9 = 0$
$(x - 3)(x + 3) = 0$
$x - 3 = 0$ or $x + 3 = 0$
$x = 3$ or -3

$x^2 - 2x - 15 = 0$
$(x - 5)(x + 3) = 0$
$x - 5 = 0$ or $x + 3 = 0$
$x = 5$ or -3

Exercise 4 Questions 1–3 may now be attempted.

Factorisation of trinomial quadratic equation of the form: $ax^2 + bx + c = 0 \quad (a \neq 1)$.
The same technique as in (Maths 1 Int 2) should be used.

Examples

$$2x^2 + 9x - 5 = 0$$
$$(2x - 1)(x + 5) = 0$$
$$2x - 1 = 0 \text{ or } x + 5 = 0$$
$$2x = 1 \text{ or } x + 5 = 0$$
$$x = {}^1/_2 \text{ or } -5$$

$$12x^2 + x - 6 = 0$$
$$(3x - 2)(4x + 3) = 0$$
$$3x - 2 = 0 \text{ or } 4x + 3 = 0$$
$$3x = 2 \text{ or } 4x = -3$$
$$x = {}^2/_3 \text{ or } -{}^3/_4$$

Examples of quadratics not in the form $ax^2 + bx + c = 0$.

$$x(x - 5) = 14$$
$$x^2 - 5x = 14$$
$$x^2 - 5x - 14 = 0$$
$$(x - 7)(x + 2) = 0$$
$$x = 7 \text{ or } -2$$

$$(2x - 1)(x + 3) = 15$$
$$2x^2 - x + 6x - 3 = 15$$
$$2x^2 + 5x - 18 = 0$$
$$(2x + 9)(x - 2) = 0$$
$$x = -{}^9/_2 \text{ or } 2$$

Exercise 4 Questions 3, 4 may now be attempted.

E. Quadratic formula

Teachers/lecturers may wish to go through the proof of the quadratic formula.

Proof

$$ax^2 + bx + c = 0$$
$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$
$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$
$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 \pm 4ac}}{2a}$$
$$x = \frac{\pm b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{\pm b \pm \sqrt{b^2 \pm 4ac}}{2a}$$

Example 1 Solve $x^2 - 8x + 12 = 0$

Method 1 factorising

—> $(x - 6)(x + 2) = 0$

$x = -2$ or 6

Method 2 quadratic formula

$$x = \frac{\pm b \pm \sqrt{b^2 \pm 4ac}}{2a}$$

$$x = \frac{8 \pm \sqrt{64 - 48}}{2}$$

$$x = \frac{8 \pm 4}{2} = 6 \text{ or } -2$$

Example 2 Solve $3x^2 + 2x - 6 = 0$

Method 1 – Does not work!

Method 2

$$x = \frac{\pm 2 \pm \sqrt{4 + 72}}{6}$$

$$x = \frac{\pm 2 \pm \sqrt{76}}{6}$$

$$x = 1{\cdot}12 \text{ or } -1{\cdot}79$$

Students should be made aware of the following possible errors when using the formula.

(i) make certain to assign a, b, c the correct sign

(ii) the $-b$

(iii) the effect of a negative value for c on $b^2 - 4ac$

(iv) pressing '=' on calculator before doing final division or calculation.

Exercise 5 may now be attempted.

The checkup exercise may also be attempted.

FURTHER TRIGONOMETRY

A. Basic sine, cosine and tangent graphs

Exercise 1 provides step by step instructions for the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ to be drawn by hand. Access to graphic calculators may mean that staff wish to omit this exercise and use the graphic calculators to remind students of the graphs.

B. Sketching trigonometric graphs

Students should be encouraged to memorise sketches of these 3 functions as shown on page 26.

Students should then investigate the effect:

(i) 'a' has in $y = \text{a}\sin x°$, $y = \text{a}\cos x°$ and $y = \text{a}\tan x°$
(encourage them to sketch the sine, cosine or tangent graph as appropriate and <u>then</u> fill in the scales on the axes).

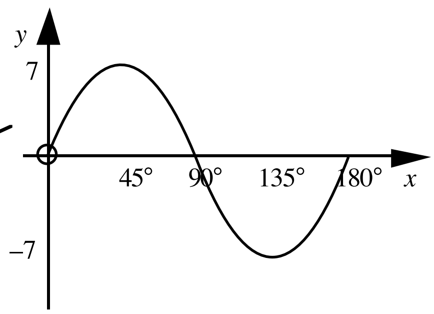
(ii) 'b' has in $y = \sin(\text{b}x)$, $y = \cos(\text{b}x)$, $y = \tan(\text{b}x)$
i.e. period becomes $\frac{360}{b}$, $\frac{360}{b}$ and $\frac{180}{b}$.

A variety of examples should be discussed, such as:

$$y = 20\sin x, \quad y = \cos 3x, \quad y = \tan 2x, \quad y = 2{\cdot}5\sin 3x, \quad y = -0{\cdot}8\cos 4x$$

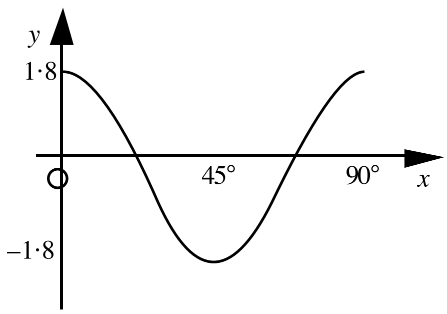
(a) Students must be able to recognise the function from a given graph.

$y = 7\sin 2x$



(b) Students must be able to sketch the graph given the function.

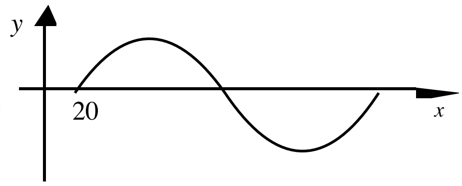
$y = 1{\cdot}8\cos 4x$

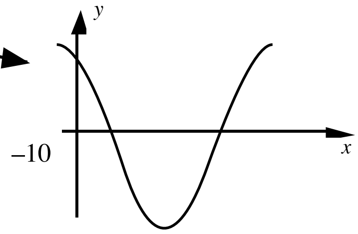


Exercise 2A may now be attempted.

Students should investigate graphs of the form
$y = \sin(x - c)$, $y = \cos(x - c)$ and $y = \tan(x - c)$
and conclude that c 'slides' the basic curve by 'c' horizontally.

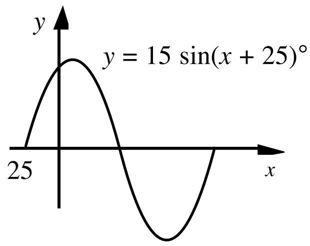
i.e. $y = \sin(x - 20)$ slides 20° right
$y = 10\cos(x + 10)$ slides 10° left





Students should also be shown how to sketch curves such as:

$y = 15\sin(x + 25)°$
and $y = 0{\cdot}6\cos(x - 15)°$

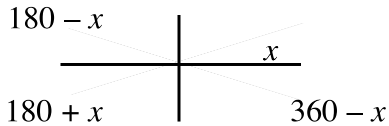


Exercise 2B may now be attempted.

Solving trigonometric equations

The graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ can be used to build up the mnenomics for solving equations.

S(in)$^+$	A(ll)$^+$
T(an)$^+$	C(os)$^+$



Examples with positive values

$\sin x = 0{\cdot}407$
$x = 24°$ or $(180 - 24)$
$= 24°$ or $156°$

$5\cos x° = 4$
$\cos x° = 4/5 = 0{\cdot}8$
$x = 36{\cdot}9°$ or $(360 - 36{\cdot}9)$
$= 36{\cdot}9°$ or $323{\cdot}1°$

$2\tan x° - 3 = 0$
$2\tan x° = 3$
$\tan x° = 3/2 = 1{\cdot}5$
$x = 56{\cdot}3°$ or $(180 + 56{\cdot}3)$
$= 56{\cdot}3°$ or $236{\cdot}3°$

Exercise 3, questions 1 and 2, may now be attempted.

Examples with negative values.

Example 1

$\cos = -0{\cdot}602 \longrightarrow$ rule (forget negative sign; look up inv cos 0·602 => 53°)
(use 53° to find the two correct answers)
($x = 180 - 53$ or $180 + 53$)
($x = 127°$ or $233°$)
(score out the 53° – it is not in answer)

Example 2

$5\sin x° = -3$

$\sin x = -\,{}^{3}/_{5}$

(If $\sin x = {}^{3}/_{5}$ => 36·9)

$x = 180 + 36{\cdot}9$ or $360 - 36{\cdot}9$

$x = 216{\cdot}9°$ or $323{\cdot}1°$

Example 3

$4\tan x° + 3 = 0$

$4\tan x° = -3$

$\tan x = -\,{}^{3}/_{4}$

(If $\tan x = {}^{3}/_{4}$ => 36·9)

$x = 180 - 36{\cdot}9$ or $360 - 36{\cdot}9$

$x = 143{\cdot}1°$ or $323{\cdot}1°$

Exercise 3, questions 3 to 5, may now be attempted.

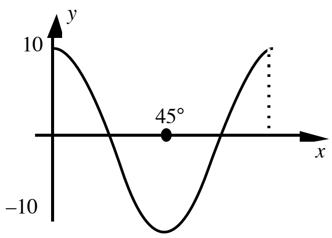
D. Period of a trig function

$y = 10\sin\underline{\underline{3}}x$ goes through three complete cycles between 0° and 360°

$$\Rightarrow \text{Period} = \frac{360}{3} = 120°$$

This could be illustrated with graphic calculators.

Example

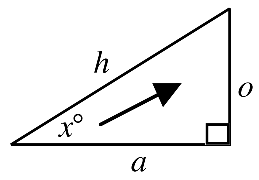


=> Period of $y = 10\cos 4x°$ is $360 \div 4 = 90°$

Exercise 4 may now be attempted.

E. Trigonometric identities

The two standard identities could be developed using the triangle opposite.



A. $\sin^2 x^\circ + \cos^2 x^\circ$

$= \left(\frac{o}{h}\right)^2 + \left(\frac{a}{h}\right)^2$

$= \frac{o^2 + a^2}{h^2}$

$= \frac{h^2}{h^2}$

$= 1$

B. $\frac{\sin x}{\cos x} = \frac{o}{h} \div \frac{a}{h}$

$= \frac{o}{h} \times \frac{h}{a}$

$= \frac{o}{a}$

$= \tan x^\circ$

Alternatively, a graphical approach can be taken using graphic calculators.
For example, draw $y = (\sin x)^2 + (\cos x)^2$ and notice that it is the same graph as $y = 1$.

Similarly, draw $y = \sin x \div \cos x$ and notice that it is the same graph as $y = \tan x$.

These identities should be learned.

$$\sin^2 x^\circ + \cos^2 x^\circ = 1$$
$$\tan x^\circ = \frac{\sin x^\circ}{\cos x^\circ}$$

Example 1 Simplify $5\sin^2 x^\circ + 5\cos^2 x^\circ$

Example 2 Prove that $\frac{1 - \cos^2 x^\circ}{3\sin^2 x^\circ} = \frac{1}{3}$

Exercise 5 may now be attempted.

The checkup exercise may also be attempted.

STUDENT MATERIALS

CONTENTS

ALGEBRAIC OPERATIONS

By the end of this set of exercises, you should be able to

(a) reduce an algebraic fraction to its simplest form

(b) apply the four rules to algebraic fractions

(c) change the subject of a formula

(d) simplify surds

(e) rationalise a surd denominator

(f) simplify expressions using the laws of indices.

ALGEBRAIC OPERATIONS

A. Reducing algebraic fractions to their simplest form

Exercise 1

1. Simplify these fractions:

(a) $\frac{6}{10}$ (b) $\frac{3}{9}$ (c) $\frac{21}{28}$ (d) $\frac{33}{44}$

(e) $\frac{100}{400}$ (f) $\frac{4a}{5a}$ (g) $\frac{2p}{3p}$ (h) $\frac{5v}{5w}$

(i) $\frac{rs}{rt}$ (j) $\frac{ab}{ac}$ (k) $\frac{a}{ad}$ (l) $\frac{m}{m^2}$

(m) $\frac{a^2}{4a}$ (n) $\frac{8z}{z^2}$ (o) $\frac{x^2}{x}$ (p) $\frac{d}{d^2}$

(q) $\frac{5x^2}{6x}$ (r) $\frac{3v}{6w}$ (s) $\frac{8xy}{4x}$ (t) $\frac{a^2}{3ab}$

(u) $\frac{2pq}{6p}$ (v) $\frac{5b}{5b^2}$ (w) $\frac{xyz}{xz}$ (x) $\frac{3ef}{7ef}$

(y) $\frac{5pq}{2pq^2}$ (z) $\frac{8x^2y}{4xy^2}$ (aa) $\frac{(x+1)^2}{(x+1)^3}$ (ab) $\frac{(x-5)^2}{(x-5)^5}$

2. Factorise either the numerator or the denominator, then simplify:

(a) $\frac{2x+8}{4}$ (b) $\frac{3x+6}{9}$ (c) $\frac{2x-8}{2}$ (d) $\frac{5x-10}{25}$

(e) $\frac{2}{2x+6}$ (f) $\frac{4}{4x-12}$ (g) $\frac{5}{5x+15}$ (h) $\frac{8}{4x-10}$

(i) $\frac{x^2-xy}{x}$ (j) $\frac{pq+p}{p}$ (k) $\frac{v^2+v}{v}$ (l) $\frac{a-a^2}{a}$

3. Factorise the numerator and/or the denominator, then simplify:

(a) $\frac{2a+6}{a+3}$ (b) $\frac{3c+9}{c+3}$ (c) $\frac{d-2}{3d-6}$ (d) $\frac{g^2+g}{g+1}$

(e) $\frac{2x+2y}{5x+5y}$ (f) $\frac{3p+3q}{7p+7q}$ (g) $\frac{4-4w}{1-w}$ (h) $\frac{x^2-xy}{8x-8y}$

(i) $\frac{x^2-1}{x-1}$ (j) $\frac{y^2-9}{y+3}$ (k) $\frac{a^2-25}{a-5}$ (l) $\frac{w+10}{w^2-100}$

(m) $\frac{x^2-1}{x^2+2x+1}$ (n) $\frac{3v^2-5v-2}{v^2-4}$ (o) $\frac{2y^2+y-1}{2y^2+5y-3}$ (p) $\frac{6x^2-13x+6}{3x^2+10x-8}$

B. Multiplying, dividing, adding and subtracting algebraic fractions

Exercise 2

1. Simplify these fractions by multiplying:

(a) $\frac{1}{3} \times \frac{3}{5}$ (b) $\frac{2}{3} \times \frac{1}{6}$ (c) $\frac{5}{6} \times \frac{3}{5}$ (d) $\frac{3}{10} \times \frac{20}{3}$

(e) $\frac{1}{6} \times 12$ (f) $\frac{9}{11} \times \frac{33}{9}$ (g) $\frac{7}{10} \times \frac{5}{1}$ (h) $8 \times \frac{2}{3}$

(i) $\frac{a}{b} \times \frac{c}{d}$ (j) $\frac{x}{y} \times \frac{v}{w}$ (k) $\frac{m}{n} \times \frac{m}{n}$ (l) $\frac{a}{b} \times \frac{b}{a}$

(m) $\frac{x}{2} \times \frac{x}{5}$ (n) $x \times \frac{x}{7}$ (o) $\frac{a}{5} \times \frac{a}{5}$ (p) $\frac{n}{3} \times \frac{3}{n}$

(q) $\frac{a}{6} \times \frac{6}{d}$ (r) $x^2 \times \frac{1}{x}$ (s) $\frac{a^2}{c} \times \frac{c}{a}$ (t) $\frac{a^3}{9} \times \frac{9}{a}$

2. Change these divisions to multiplications and simplify:

(a) $\frac{3}{7} \div \frac{6}{7}$ (b) $\frac{2}{3} \div \frac{8}{3}$ (c) $\frac{3}{8} \div \frac{9}{4}$ (d) $\frac{9}{10} \div \frac{18}{5}$

(e) $\frac{x}{2} \div \frac{x}{3}$ (f) $\frac{a}{8} \div \frac{a}{2}$ (g) $\frac{d}{3} \div \frac{d}{6}$ (h) $\frac{m}{10} \div \frac{m}{50}$

(i) $\frac{a^2}{3} \div \frac{a}{3}$ (j) $\frac{b^2}{6} \div \frac{b}{2}$ (k) $\frac{r^4}{6} \div \frac{r^2}{2}$ (l) $\frac{a}{b} \div \frac{a}{b}$

(m) $\frac{a^2}{d} \div \frac{a}{d^2}$ (n) $\frac{1}{w^2} \div \frac{5}{w}$ (o) $\frac{1}{a^3} \div \frac{1}{a^2}$ (p) $\frac{x^2}{y} \div \frac{2x}{d}$

(q) $\frac{a^2}{b} \div \frac{a}{b}$ (r) $\frac{2a^2}{5d^2} \div \frac{a^2}{d^2}$

3. Do the following additions and subtractions:

(a) $\frac{1}{3} + \frac{1}{4}$ (b) $\frac{1}{4} + \frac{2}{3}$ (c) $\frac{3}{4} - \frac{1}{5}$ (d) $\frac{1}{5} + \frac{1}{3}$

(e) $\frac{1}{3} - \frac{1}{5}$ (f) $\frac{4}{7} - \frac{1}{2}$ (g) $\frac{1}{2} + \frac{1}{5}$ (h) $\frac{1}{2} - \frac{1}{5}$

(i) $\frac{5}{8} - \frac{1}{4}$ (j) $\frac{7}{10} + \frac{1}{5}$ (k) $\frac{x}{3} + \frac{a}{2}$ (l) $\frac{c}{5} + \frac{d}{2}$

(m) $\frac{e}{3} - \frac{h}{4}$ (n) $\frac{m}{4} - \frac{n}{8}$ (o) $\frac{2x}{3} + \frac{k}{2}$ (p) $\frac{u}{2} - \frac{2w}{5}$

(q) $\frac{4r}{5} + \frac{s}{2}$ (r) $\frac{a}{3} - \frac{2d}{5}$ (s) $\frac{2x}{3} + \frac{3y}{2}$ (t) $\frac{3x}{4} + \frac{2u}{5}$

4. By finding a common denominator with letters, work out these additions/subtractions:

(a) $\frac{2}{x} + \frac{3}{y}$ (b) $\frac{5}{a} - \frac{2}{b}$ (c) $\frac{4}{c} + \frac{1}{d}$ (d) $\frac{1}{p} - \frac{2}{q}$

(e) $\frac{2}{v} + \frac{2}{w}$ (f) $\frac{1}{g} - \frac{1}{h}$ (g) $\frac{7}{k} + \frac{1}{n}$ (h) $\frac{1}{x} - \frac{8}{y}$

5. Add or subtract these fractions:

(a) $\frac{x+1}{3} + \frac{x+1}{2}$ (b) $\frac{x+2}{4} + \frac{x-1}{5}$ (c) $\frac{x+3}{2} + \frac{x+1}{4}$ (d) $\frac{2x-3}{5} + \frac{x+1}{3}$

(e) $\frac{x+1}{2} - \frac{x+1}{3}$ (f) $\frac{x+2}{2} - \frac{x+1}{5}$ (g) $\frac{2x+1}{2} - \frac{x+1}{4}$ (h) $\frac{x+1}{2} - \frac{x-1}{5}$

C. Changing the subject of a formula

Exercise 3A

This exercise has a mixed selection of formulae.
Change the subject of each formula to the letter shown in the brackets.
ALL WORKING and ALL STEPS SHOULD BE SHOWN.

1. $x + 2 = c$ (x) 2. $x - 4 = c$ (x) 3. $x + p = q$ (x)
4. $x - p = q$ (x) 5. $x/2 = a$ (x) 6. $x/7 = a$ (x)
7. $x/y = a$ (x) 8. $x/p = m$ (x) 9. $x/r = s$ (x)
10. $4x = 20$ (x) 11. $4x = a$ (x) 12. $gx = h$ (x)
13. $nx = t$ (x) 14. $2x + 1 = 5$ (x) 15. $2x + 1 = b$ (x)
16. $2x + c = b$ (x) 17. $ax + c = b$ (x) 18. $px + q = r$ (x)
19. $vx - w = y$ (x) 20. $D = S \text{ x } T$ (S) 21. $C = \pi d$ (d)
22. $x^2 = 16$ (x) 23. $x^2 = \text{y}$ (x) 24. $A = \pi r^2$ (r)
25. $T = D/S$ (S) 26. $A = y^2$ (y) 27. $P = 3\pi r^2$ (π)
28. $P = 5\pi r^2$ (r) 29. $h - p = q$ (h) 30. $h - p = q$ (p)
31. $2h - 5p = q$ (h) 32. $2h - 5p = q$ (p) 33. $b - c = ax$ (x)

Exercise 3B

1. Change the subject of each formula to *h*.

(a) $g = hf$ (b) $e = g + h$ (c) $k = h/f$ (d) $e = g - h$

2. Change the subject of each formula to *r*.

(a) $Q = r^2$ (b) $N = \pi r^2$ (c) $M = 2\pi r^2$ (d) $P = \pi r^2 w$

3. Change the subject of each formula to m.

(a) $A = klm$ (b) $B = Km$ (c) $C = \pi mr^2$ (d) $D = \dfrac{1}{3pm}$

4. Change the subject to x.

(a) $p = q + x$ (b) $r = s - x$ (c) $r = s - 5x$ (d) $r = 7x - 3$

(e) $m = 2(x + 1)$ (f) $m = {}^1/_2(x - 5)$ (g) $n = {}^1/_2(x + 2)$ (h) $p = {}^1/_2(x + q)$

5. Change the subject of the formula to the letter in brackets.

(a) $^P/_Q = R$ (P) (b) $t = {}^1/_s$ (s) (c) $M = {}^P/_{Q^2}$ (Q) (d) $v = \sqrt{\dfrac{w}{z}}$ (w)

(e) $d = \dfrac{e}{5f}$ (f) (f) $\dfrac{K}{mn} = T$ (n) (g) $R = \dfrac{7}{9s^2}$ (s) (h) $a^2 + b^2 = c^2$ (a)

6. Harder examples. Change the subject of the formula to the letter in brackets.

(a) $A + d = {}^V/_T$ (T) (b) $px + qx = r$ (x) (c) $ax = bx + c$ (x)

(d) $m = \dfrac{r - s}{s}$ (s) (e) $x = \dfrac{v - w}{v + w}$ (w) (f) $p = 2\sqrt{r} - 1$ (r)

D. Simplifying surds

Exercise 4

1. Express each of the following in its simplest form:

(a) $\sqrt{8}$ (b) $\sqrt{12}$ (c) $\sqrt{27}$ (d) $\sqrt{20}$ (e) $\sqrt{50}$ (f) $\sqrt{28}$

(g) $\sqrt{18}$ (h) $\sqrt{24}$ (i) $\sqrt{200}$ (j) $\sqrt{75}$ (k) $\sqrt{45}$ (l) $\sqrt{72}$

(m) $\sqrt{300}$ (n) $\sqrt{147}$ (o) $\sqrt{54}$ (p) $7\sqrt{8}$ (q) $5\sqrt{32}$ (r) $6\sqrt{40}$

2. Add or subtract the following:

(a) $3\sqrt{2} + 5\sqrt{2}$ (b) $6\sqrt{5} - 5\sqrt{5}$ (c) $8\sqrt{10} + 5\sqrt{10}$ (d) $9\sqrt{20} - 9\sqrt{20}$

(e) $\sqrt{6} - 3\sqrt{6}$ (f) $\sqrt{3} + \sqrt{3} - 3\sqrt{3}$ (g) $5\sqrt{7} - 8\sqrt{7} + 3\sqrt{7}$ (h) $10\sqrt{2} + 10\sqrt{3}$

3. Simplify:

(a) $\sqrt{8} - \sqrt{2}$ (b) $\sqrt{18} - \sqrt{2}$ (c) $\sqrt{125} + 5\sqrt{5}$ (d) $\sqrt{48} + \sqrt{12}$

(e) $\sqrt{45} + \sqrt{20}$ (f) $\sqrt{63} - \sqrt{28}$ (g) $\sqrt{50} + \sqrt{18}$ (h) $\sqrt{72} - \sqrt{32}$

Exercise 5

1. Simplify:

(a) $\sqrt{3} \times \sqrt{3}$ (b) $\sqrt{5} \times \sqrt{5}$ (c) $\sqrt{6} \times \sqrt{6}$ (d) $\sqrt{1} \times \sqrt{1}$

(e) $\sqrt{x} \times \sqrt{x}$ (f) $\sqrt{3} \times \sqrt{2}$ (g) $\sqrt{4} \times \sqrt{5}$ (h) $\sqrt{16} \times \sqrt{a}$

(i) $\sqrt{2} \times \sqrt{c}$ (j) $\sqrt{x} \times \sqrt{y}$ (k) $\sqrt{2} \times \sqrt{8}$ (l) $\sqrt{2} \times \sqrt{32}$

(m) $\sqrt{6} \times \sqrt{3}$ (n) $\sqrt{20} \times \sqrt{10}$ (o) $3\sqrt{2} \times \sqrt{2}$ (p) $3\sqrt{2} \times 2\sqrt{3}$

contd.

2. Multiply out the brackets:

(a) $\sqrt{2}(1 + \sqrt{2})$ (b) $\sqrt{3}(\sqrt{2} + 1)$ (c) $\sqrt{5}(\sqrt{5} + 1)$

(d) $\sqrt{7}(1 + \sqrt{7})$ (e) $\sqrt{2}(5 - \sqrt{2})$ (f) $\sqrt{2}(5 - 4\sqrt{2})$

(g) $(\sqrt{3} + 2)(\sqrt{3} - 1)$ (h) $(\sqrt{2} - 2)(\sqrt{2} + 1)$ (i) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$

(j) $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$ (k) $(\sqrt{3} + \sqrt{2})^2$ (l) $(\sqrt{5} - \sqrt{3})^2$

3. If $a = 1 + \sqrt{2}$ and $b = 1 - \sqrt{2}$, simplify:

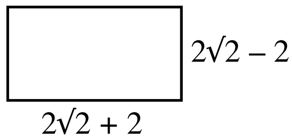
(a) $3a + 3b$ (b) $2ab$ (c) $a^2 + b^2$

4. If $r = \sqrt{5} + \sqrt{3}$ and $s = \sqrt{5} - \sqrt{3}$, simplify:

(a) $2r - 2s$ (b) $5rs$ (c) $r^2 - s^2$

5. A rectangle has sides of length $(2\sqrt{2} + 2)$ cm and $(2\sqrt{2} - 2)$ cm. Calculate:

(a) its area (b) the length of a diagonal



E. Rationalising a surd denominator

Exercise 6

1. Rationalise the denominators in the following and simplify where possible:

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{6}}$

(e) $\frac{1}{\sqrt{7}}$ (f) $\frac{10}{\sqrt{5}}$ (g) $\frac{2}{\sqrt{3}}$ (h) $\frac{3}{\sqrt{5}}$

(i) $\frac{20}{\sqrt{2}}$ (j) $\frac{6}{\sqrt{3}}$ (k) $\frac{12}{\sqrt{6}}$ (l) $\frac{3}{2\sqrt{5}}$

(m) $\frac{4}{5\sqrt{2}}$ (n) $\frac{1}{\sqrt{20}}$ (o) $\frac{1}{\sqrt{50}}$ (p) $\frac{4}{\sqrt{8}}$

2. Express each of the following in its simplest form with a rational denominator:

(a) $\frac{\sqrt{4}}{\sqrt{3}}$ (b) $\frac{\sqrt{5}}{\sqrt{2}}$ (c) $\sqrt{\frac{4}{10}}$ (d) $\sqrt{\frac{1}{11}}$ (e) $\sqrt{\frac{3}{5}}$ (f) $\sqrt{\frac{a}{b}}$

3. Simplify the following by rationalising the denominator:

(a) $\frac{1}{\sqrt{2} - 1}$ (b) $\frac{1}{\sqrt{7} - 1}$ (c) $\frac{1}{2 + \sqrt{2}}$ (d) $\frac{4}{\sqrt{5} + 1}$

(e) $\frac{3}{2 - \sqrt{3}}$ (f) $\frac{1}{\sqrt{3} - \sqrt{2}}$ (g) $\frac{2}{\sqrt{5} + \sqrt{3}}$ (h) $\frac{9}{\sqrt{5} - \sqrt{2}}$

F. Simplifying expressions using the laws of indices

Exercise 7

Rule 1 $a^m \times a^n = a^{m+n}$

1. Use Rule 1 to write down the simplest form of the products in the following:

(a) $2^3 \times 2^4$ (b) $3^5 \times 3^3$ (c) $8^2 \times 8^5$ (d) $10^{10} \times 10^{20}$

(e) $a^3 \times a^4$ (f) $b^5 \times b^3$ (g) $c^2 \times c^6$ (h) $d^5 \times d^5$

(i) $v^3 \times v^8$ (j) $x^2 \times x^{10}$ (k) $w^3 \times w$ (l) $z \times z^5$

(m) $f^4 \times f^7$ (n) $g^{10} \times g^{10}$ (o) $k \times k^{11}$ (p) $m^{100} \times m$

Rule 2 $a^m \div a^n = a^{m-n}$

2. Use Rule 2 to write down the simplest form of the quotients in the following:

(a) $2^4 \div 2^3$ (b) $3^5 \div 3^3$ (c) $8^5 \div 8^2$ (d) $10^{20} \div 10^{10}$

(e) $a^7 \div a^6$ (f) $b^5 \div b^3$ (g) $c^9 \div c^4$ (h) $d^5 \div d^5$

(i) $v^{12} \div v^{11}$ (j) $x^{10} \div x^2$ (k) $w^3 \div w$ (l) $z^2 \div z$

(m) $f^7 \div f^4$ (n) $g^{10} \div g^{10}$ (o) $k^{11} \div k$ (p) $m^{100} \div m$

(q) $\frac{x^6}{x^2}$ (r) $\frac{m^3}{m^2}$ (s) $\frac{a^5}{a}$ (t) $\frac{r^9}{r^8}$

Rule 3 $(a^m)^n = a^{mn}$

3. Simplify these. For example, $(a^3)^4 = a^{12}$.

(a) $(x^2)^3$ (b) $(y^3)^5$ (c) $(z^5)^2$ (d) $(g^2)^8$

(e) $(a^3)^7$ (f) $(b^4)^4$ (g) $(c^5)^6$ (h) $(d^3)^7$

4. Express the following without brackets, writing answers in index form:

(a) $(2^5)^2$ (b) $(7^3)^5$ (c) $(6^5)^4$ (d) $(8^3)^5$

(e) $(2^7)^7$ (f) $(3^4)^3$ (g) $(9^5)^2$ (h) $(2^5)^5$

5. Note $(ab)^m = a^m b^m$ Use this result to simplify:

(a) $(ab)^3$ (b) $(cd)^6$ (c) $(x^2y)^{10}$ (d) $(2pq)^2$

6. Use the 3 rules learned so far to simplify the following:

(a) $y^3 \times y^2$ (b) $t^4 \times t$ (c) $x^5 \div x^2$ (d) $v^7 \div v$

(e) $3x^2 \times x^3$ (f) $x^2 \times 4x^5$ (g) $2x^2 \times 6x^2$ (h) $8x^5 \div 2x^2$

(i) $6x^3 \div 2x$ (j) $x^2(x^3 + x^4)$ (k) $x^3(x^3 - 4)$ (l) $(uv)^7$

(m) $(5y)^2$ (n) $(mn^2)^8$ (o) $\frac{x^3 \times x^4}{x^2}$ (p) $\frac{u^7 \times u^2}{u^3}$

7. Find *m*. For example: if $2^m = 8$, m = 3 since $2^3 = 8$.

(a) $2^m = 16$ (b) $2^m = 32$ (c) $4^m = 64$ (d) $5^m = 625$

Exercise 8

Rule 4 $a^0 = 1$

1. Write down the values of:

(a) 2^0 (b) 12^0 (c) x^0 (d) $(32546)^0$

Rule 5 $a^{-m} = 1/a^m$

2. Write the following with positive indices. For example: $2^{-5} = 1/2^5$.

(a) 3^{-2} (b) 5^{-7} (c) a^{-4} (d) b^{-9}

(e) x^{-1} (f) $3y^{-2}$ (g) xy^{-3} (h) $\frac{1}{x^{-3}}$

(i) $\frac{1}{t^{-5}}$ (j) $\frac{6}{c^{-3}}$ (k) $\frac{1}{2y^{-2}}$ (l) $\frac{1}{7x^{-3}}$

3. Express these in a form without indices:

(a) 5^0 (b) 7^{-1} (c) 3^{-3} (d) 8^{-2} (e) $\frac{1}{2^{-3}}$ (f) $\left(\frac{2}{3}\right)^{-2}$

4. Simplify the following. For example: $x^{-2} \times x^5 = x^{-2+5} = x^3$.

(a) $a^{-2} \times a^4$ (b) $b^6 \times b^{-4}$ (c) $c^{-1} \times c^{-1}$ (d) $d^{-6} \times d^6$

(e) $e^6 \div e^{-3}$ (f) $g^{-4} \div g^4$ (g) $w^{-3} \div w^{-5}$ (h) $(x^{-2})^3$

(i) $(y^5)^{-1}$ (j) $(z^{-4})^{-4}$ (k) $(klm)^0$.

5. Express with positive indices:

(a) $4^2 \times 4^{-5}$ (b) $6^{-2} \times 6^{-3}$ (c) $2^7 \times 2^{-2}$ (d) $4^{-3} \times 5^2$

(e) $3^4 \times 2^{-3}$ (f) $(9^0)^{-8}$ (g) $(w^{-3})^{-2}$ (h) $\left(x^{-2}\right)^{-\frac{1}{2}}$

(i) $\frac{1}{2^{-2}}$ (j) $\frac{5}{2^{-3}}$ (k) $6h^{-2}$ (l) $9s^{-1}$

(m) $\frac{1}{2}k^{-1}$ (n) $\frac{3}{4}m^{-3}$

6. Multiply out the brackets:

(a) $x^5(x^2 + x^{-2})$ (b) $x^3(x - x^{-2})$ (c) $x^{-3}(x^4 + x)$ (d) $x^{-1}(x + x^2)$

(e) $x(x^2 - x^{-7})$ (f) $x^{-5}(2 - x^5)$ (g) $2x^{-2}(3 - x^3)$ (h) $3x^2(2x - x^{-2})$

7. (a) Find the value of: 3^3, 3^0, 3^{-2}, 3^1, 3^{-3}.

(b) Write in the form 4^{-p} : $1/4$, $1/16$, $1/64$.

Exercise 9

Rule 6 $a^{m/n} = \sqrt[n]{a^m}$

1. Write these in root form. For example: $x^{\frac{4}{5}} = \sqrt[5]{x^4}$

   (a) $x^{\frac{3}{4}}$ (b) $m^{\frac{3}{5}}$ (c) $r^{\frac{2}{3}}$ (d) $w^{\frac{1}{2}}$ (e) $n^{\frac{1}{3}}$ (f) $r^{-\frac{4}{3}}$

2. Write these in index form. For example: $\sqrt[5]{x^4} = x^{4/5}$

   (a) $\sqrt[3]{x^5}$ (b) $\sqrt[3]{b^4}$ (c) $\sqrt[3]{z^2}$ (d) $\sqrt[4]{w}$ (e) $\dfrac{1}{\sqrt[3]{x}}$ (f) $\sqrt{u}$

3. Evaluate the following. For example: $8^{\frac{2}{3}} = \sqrt[3]{8}^2 = (2)^2 = 4$

   (a) $9^{\frac{1}{2}}$ (b) $64^{\frac{1}{2}}$ (c) $8^{\frac{1}{3}}$ (d) $64^{\frac{1}{3}}$ (e) $100^{\frac{1}{2}}$ (f) $27^{\frac{1}{3}}$

   (g) $9^{\frac{3}{2}}$ (h) $16^{\frac{3}{4}}$ (i) $49^{\frac{3}{2}}$ (j) $16^{-\frac{1}{2}}$ (k) $27^{-\frac{1}{3}}$ (l) $25^{-\frac{3}{2}}$

   (m) $81^{\frac{3}{4}}$ (n) $81^{-\frac{3}{4}}$ (o) $\left(\dfrac{1}{4}\right)^{-\frac{1}{2}}$ (p) $\left(\dfrac{1}{8}\right)^{\frac{5}{3}}$

4. Simplify:

   (a) $\left(x^4\right)^{\frac{1}{2}}$ (b) $\left(m^9\right)^{\frac{1}{3}}$ (c) $\left(c^6\right)^{\frac{2}{3}}$ (d) $\left(n^{-15}\right)^{\frac{1}{5}}$

   (e) $\left(n^{-2}\right)^{\frac{1}{4}}$ (f) $\left(g^{-\frac{2}{3}}\right)^{-3}$ (g) $\left(b^{\frac{5}{2}}\right)^{-2}$ (h) $\left(z^{-\frac{3}{2}}\right)^{-2}$

   (i) $\left(x^{\frac{4}{5}}\right)^{\frac{1}{2}}$ (j) $\left(x^{\frac{3}{2}}\right)^{\frac{4}{3}}$ (k) $\left(5^2 + 12^2\right)^{\frac{1}{2}}$ (l) $\left(1^3 + 2^3 + 3^3\right)^{\frac{1}{2}}$

Exercise 10 ***Miscellaneous examples***

1. Simplify:

   (a) $a^{\frac{3}{2}} \times a^{\frac{1}{2}}$ (b) $b^{\frac{5}{4}} \times b^{-\frac{1}{4}}$ (c) $c^{\frac{1}{3}} \times c^{-\frac{1}{3}}$ (d) $d^{\frac{5}{2}} \div d^{\frac{1}{2}}$

   (e) $e^{\frac{3}{5}} \div e^{-\frac{1}{5}}$ (f) $\left(z^{\frac{1}{2}}\right)^2$ (g) $\left(w^{-\frac{1}{2}}\right)^2$ (h) $\left(w^{-\frac{1}{2}}\right)^0$

2. Simplify:

   (a) $2a^{1/2} \text{ x } 2a^{-1/2}$ (b) $3b^{2/3} \text{ x } 4b^{1/3}$ (c) $6c^{1/2} \text{ x } c^{1/2}$ (d) $2d^{3/2} \text{ x } d^{-1/2}$

   (e) $4e^{3/2} \div 2e^{1/2}$ (f) $8v^{3/4} \div 2v^{-1/4}$ (g) $15z^{-1/3} \div 15z^{-1/3}$

3. Multiply out the brackets:

(a) $x^{1/2}(x^{1/2} - x^{-1/2})$ (b) $x^{2/3}(x^{4/3} + x^{1/3})$ (c) $x^{-3/5}(x^{6/5} + x^{-1/5})$

4. Evaluate the following for $x = 16$ and $y = 27$:

(a) $3x^{1/2}$ (b) $4x^{3/4}$ (c) $5y^{2/3}$ (d) $8y^{-2/3}$ (e) $x^{-1/4} \text{ x } y^{1/3}$

5. Simplify the following, expressing your answers with positive indices:

(a) $p^6 \text{ x } p^{1/2}$ (b) $p^5 \div p^{-1/2}$ (c) $(p^{2/3})^{-3/2}$ (d) $(p^{-3/4})^{12}$

(e) $5p^{3/4} \div 5p^{-3/4}$ (f) $7p^{1/2} \text{ x } 7p^{-3/2}$

6. Simplify:

(a) $\dfrac{x^4 \times x^{-3}}{x}$ (b) $\dfrac{x^{-1} \times x^5}{x^{-2}}$ (c) $\dfrac{x^{-3} \times x^3}{x^{-1}}$ (d) $\dfrac{x^{\frac{1}{4}} \times x^{-\frac{3}{4}}}{x^4}$

(e) $\dfrac{x^{-\frac{1}{2}} \times x^{\frac{3}{2}}}{x}$ (f) $\dfrac{x^{\frac{3}{5}} \times x^{-\frac{3}{5}}}{x^{-1}}$

7. Multiply out the brackets:

(a) $(x^3 - 1)(x^{-3} - 1)$ (b) $(x^{-1} + 2)(x^{-1} - 2)$ (c) $(x^{1/2} + 4)(x^{-1/2} - 4)$

Exercise 11

Express the following with x in the numerator in index form:

For example :- $\dfrac{2}{5\sqrt{x}} = \dfrac{2}{5x^{\frac{1}{2}}} = \dfrac{2x^{-\frac{1}{2}}}{5} = \dfrac{2}{5}x^{-\frac{1}{2}}$

1. $\dfrac{1}{x}$ 2. $\dfrac{1}{x^2}$ 3. $\dfrac{7}{x^2}$ 4. $\dfrac{5}{x^3}$ 5. $\dfrac{1}{3x}$ 6. $\dfrac{4}{5x}$

7. $\dfrac{3}{2x^2}$ 8. $\dfrac{1}{\sqrt{x}}$ 9. $\dfrac{5}{2\sqrt{x}}$ 10. $\dfrac{3}{5x^{-3}}$ 11. $\dfrac{1}{\sqrt{(3x^6)}}$ 12. $\dfrac{2}{\sqrt{(5x^3)}}$

$\dfrac{6m^5}{m}$

$\dfrac{a^3 \text{ x } a^2}{a}$

$\dfrac{1}{4x^{-2}}$

$x^{-1/4}$ $x^{3/4}$

$$\frac{x^{-1/4} \times x^{3/4}}{x^{-1/2}}$$

Checkup for algebraic operations

1. Factorise the numerator and/or denominator if possible, then simplify:

(a) $\frac{7v}{14v}$ (b) $\frac{9a^2b}{3ab^2}$ (c) $\frac{6}{6x - 18}$ (d) $\frac{v^2 - vw}{v}$

(e) $\frac{3x + 3y}{6x + 9y}$ (f) $\frac{a - 8}{a^2 - 64}$ (g) $\frac{v^2 + 3v + 2}{v^2 - 4}$

2. Simplify:

(a) $\frac{b^3}{4} \times \frac{2}{b}$ (b) $\frac{a^2}{x} \times \frac{x}{a}$ (c) $\frac{z^2}{2} \div \frac{z}{6}$ (d) $\frac{a^2}{b} \div \frac{a}{b}$

(e) $\frac{c}{4} + \frac{c}{8}$ (f) $\frac{3u}{4} - \frac{2v}{5}$ (g) $\frac{3}{k} + \frac{1}{m}$ (h) $\frac{x + 1}{2} - \frac{x + 2}{5}$

3. Change the subject of each formula to x.

(a) $5 + x = 6$ (b) $x - a = w$ (c) $ax + m = p$ (d) $x/z = p/w$

(e) $N = 2\pi x^2$ (f) $T = 4x + 3$ (g) $w = 1/2(x + y)$ (h) $M = \frac{5}{8x^2}$

4. Express each of these in its simplest form:

(a) $\sqrt{32}$ (b) $\sqrt{1000}$ (c) $2\sqrt{45}$ (d) $\sqrt{45} - \sqrt{20}$

(e) $\sqrt{50} - \sqrt{18}$ (f) $\sqrt{72} + \sqrt{50}$ (g) $\sqrt{\frac{9}{a^2}}$ (h) $2\sqrt{8} - \sqrt{2}$

5. If $x = 1 + \sqrt{3}$ and $y = 1 - \sqrt{3}$, simplify:

(a) $2x + 2y$ (b) $5xy$ (c) $x^2 + y^2$

6. Rationalise the denominators in the following and simplify where possible:

(a) $1/\sqrt{5}$ (b) $8/\sqrt{2}$ (c) $15/\sqrt{5}$ (d) $\sqrt{2}/\sqrt{6}$

7. Write in their simplest form:

(a) $5^6 \times 5^8$ (b) $x^8 \div x^6$ (c) (d) $(a^2)^3$

(e) $(4a^3b)^2$ (f) (g) $(p^2)^0$ (h) $x^3(x^2 - x)$

8. Write with positive indices:

(a) 5^{-2} (b) ab^{-3} (c) $(y^{-3})^{-2}$ (d)

9. Write these in root form:

(a) $b^{1/2}$ (b) $c^{-3/2}$

10. Write these in index form:

(a) $\sqrt[3]{x^4}$ (b) $1/\sqrt{a^3}$

11. Evaluate:

(a) $36^{3/2}$ (b) $32^{-2/5}$ (c) $(x^8)^{1/2}$ (d) $(y^{7/2})^{-2}$

(e) $a^{-3/2}(a^{1/2} - a^{-5/2})$ (f) $2s^{1/2} \div 4s^{-1/2}$ (g)

QUADRATIC FUNCTIONS

By the end of this unit, you should be able to:

(a) recognise quadratics of the form $y = kx^2$ and
$y = (x + a)^2 + b$; a, b $\in$ Z, from their graphs

(b) identify the nature and coordinates of the turning points and the equation of the axis of symmetry of a quadratic of the form

$$y = k(x + a)^2 + b; \quad a, b \in Z, \; k = \pm 1$$

(c) know the meaning of 'root of an quadratic equation' and solve a quadratic equation graphically

(d) solve quadratic equations by factorisation

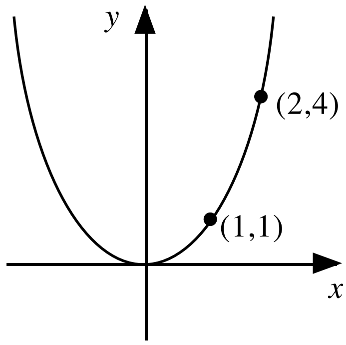
(e) solve quadratic equations by using the quadratic formula

A. Recognising quadratics from their graphs

Exercise 1

1. Each of the following graphs represents a simple parabola of the form $y = kx^2$.
   Find k each time and hence write down the equation representing each parabola.

(a)



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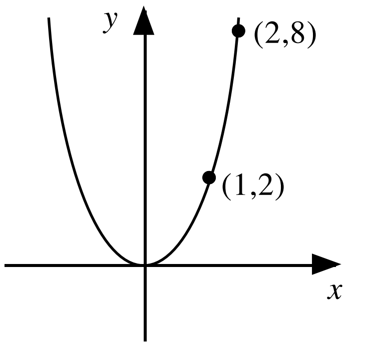

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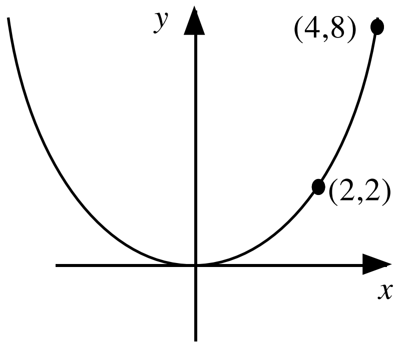

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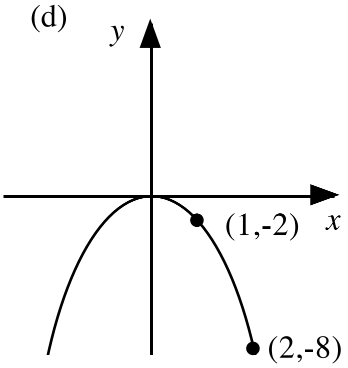

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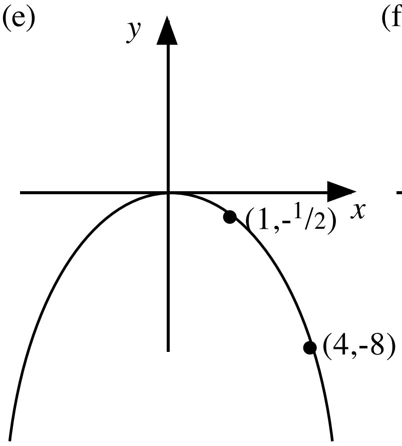

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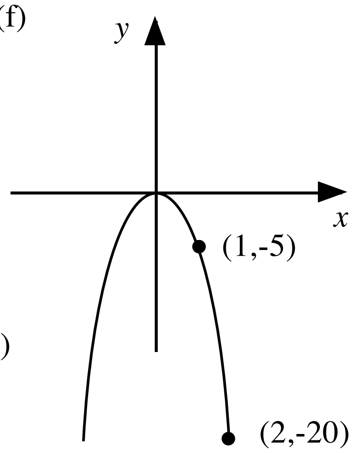
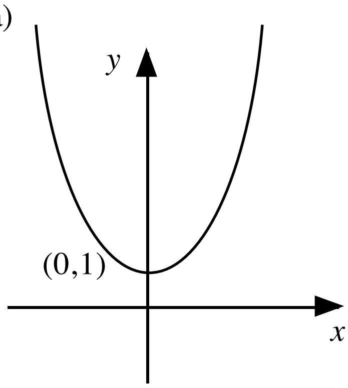
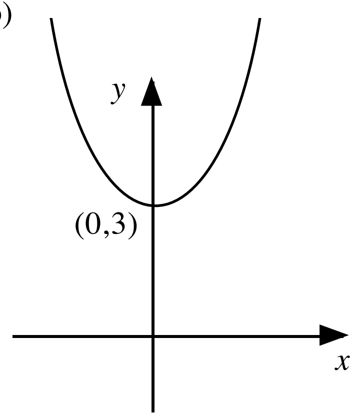


2. Each of the following parabolas can be represented by the equation
   $y = x^2 + b$ or $y = -x^2 + b$, where b is an integer. Write down their equations.

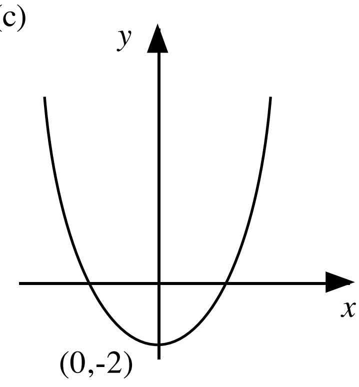
(a)



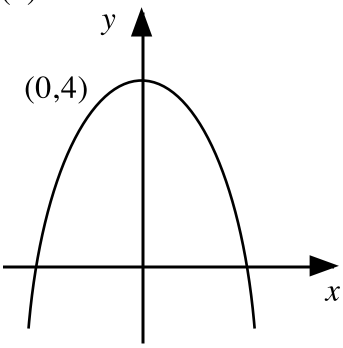
(b)



(c)



(d)



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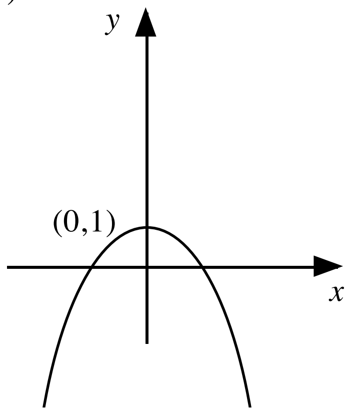
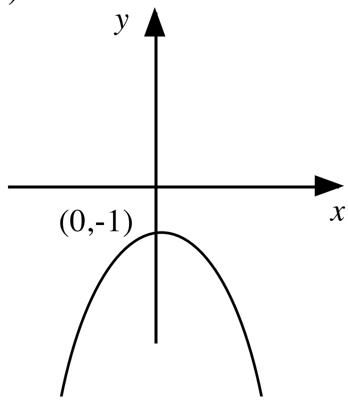
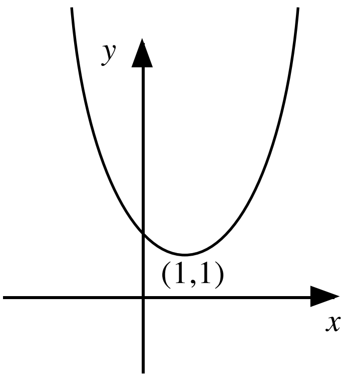


(f)

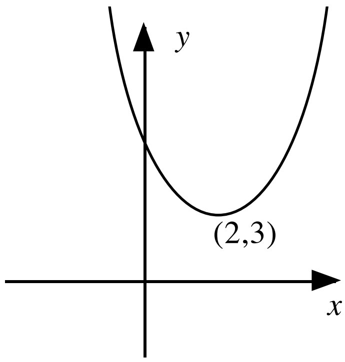


3. Each of the following parabolas can be represented by an equation of the form $y = (x + \text{a})^2 + \text{b}$, (where a and b are integers). Write down the equation of each one.

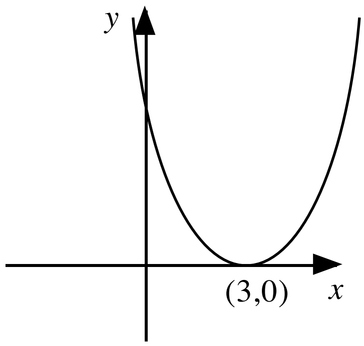
(a)



(b)



(c)



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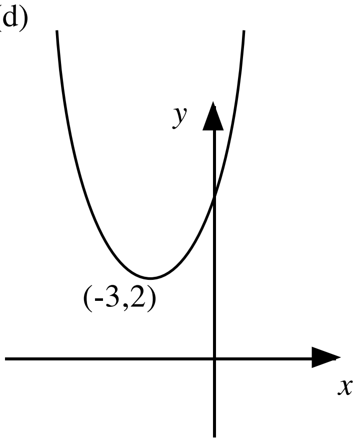

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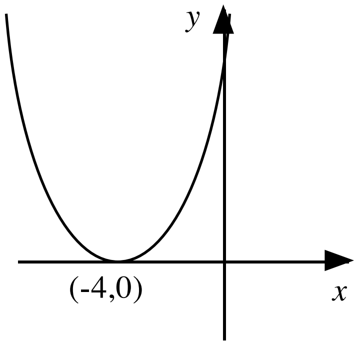

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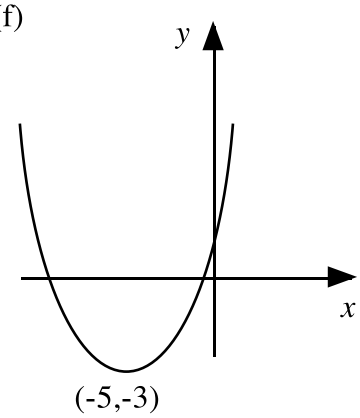
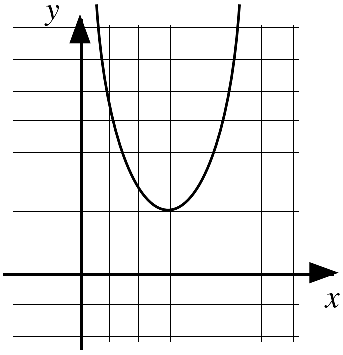


(g)



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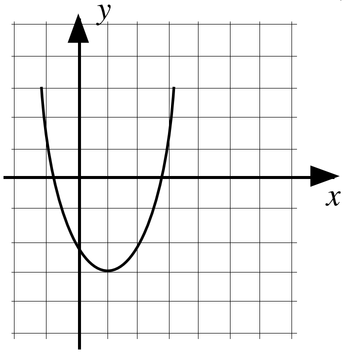
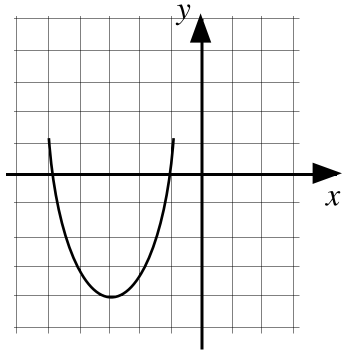
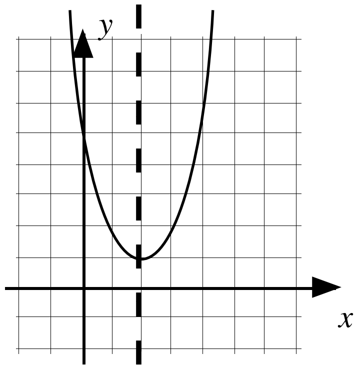


(i)



B. The nature and coordinates of turning points

Exercise 2



1. (a) Write down the coordinates of the <u>minimum</u> turning point of this parabola.

   (b) Write down the equation of the axis of symmetry. (the dotted line).

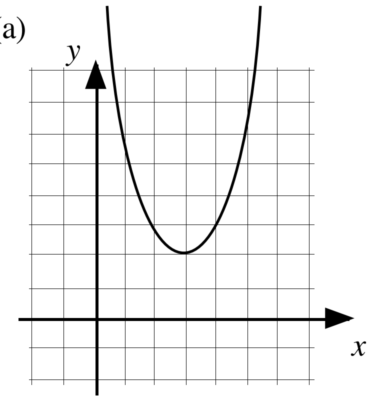
   (c) The equation of the parabola is of the form

   $$y = (x + \text{a})^2 + \text{b}.$$

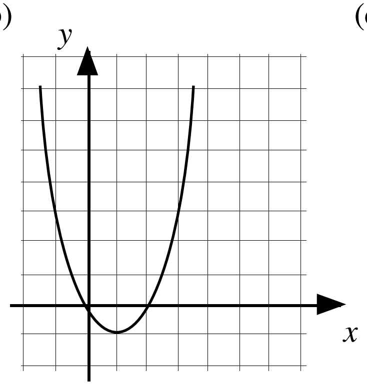
   Find the values of a and b and hence write down the equation of the parabola.

2. For each of the following parabolas:
   (i) write down the coordinates of its minimum turning point.
   (ii) write down the equation of its axis of symmetry.
   (iii) write down the equation of the function represented by the parabola.

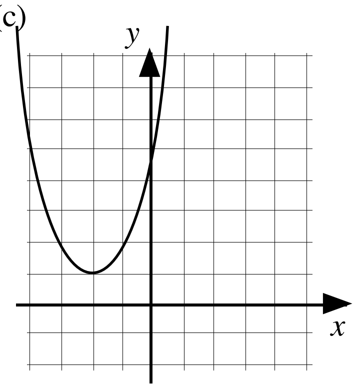
(a)



(b)



(c)



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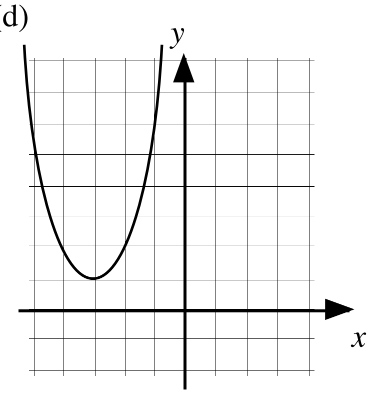

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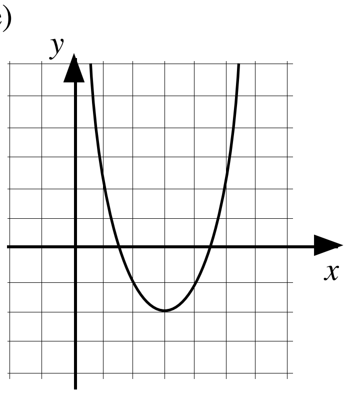
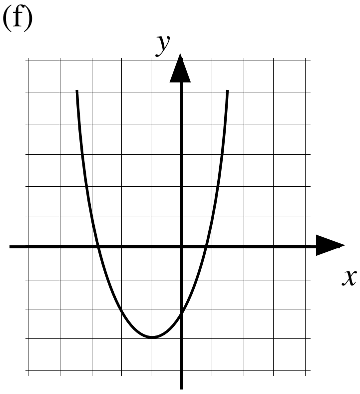


(f)



3. Without making a sketch, write down the coordinates of the minimum turning points and the equation of the axes of symmetry of these parabolas.

(a) $y = (x - 4)^2 + 1$ (b) $y = (x - 2)^2 + 7$ (c) $y = (x - 8)^2 + 3$

(d) $y = (x + 1)^2 + 2$ (e) $y = (x - 1)^2 - 3$ (f) $y = (x + 3)^2 - 7$

(g) $y = (x - 5)^2$ (h) $y = (x + 2)^2$ (i) $y = x^2 + 3$

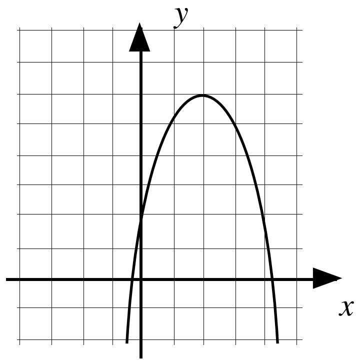
4. This parabola is of the form

$$y = -(x + \mathrm{a})^2 + \mathrm{b}$$

(a) Write down the coordinates of the <u>maximum</u> turning point.

(b) Write down the equation of the axes of symmetry.

(c) Find the values of a and b and hence write down the equation of the function represented by the parabola.



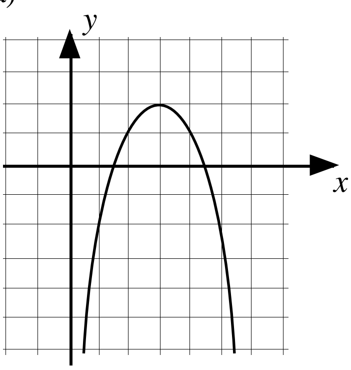
5. For each of the following parabolas:

(i) write down the coordinates of its maximum turning point.

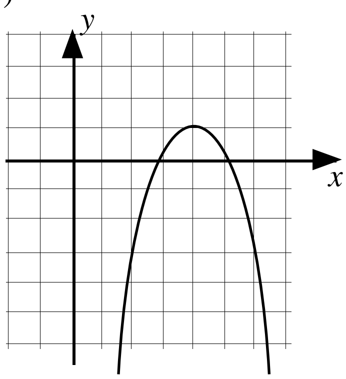
(ii) write down the equation of its axis of symmetry.

(iii) write down the equation of the function represented by the parabola.

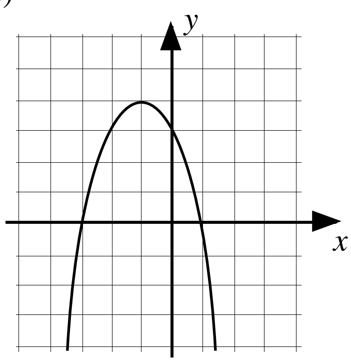
(a)



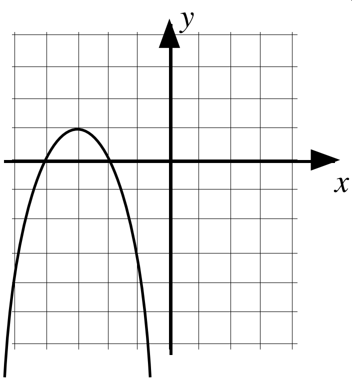
(b)



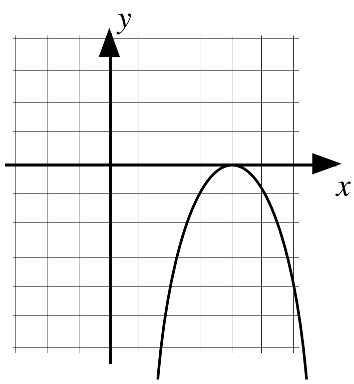
(c)



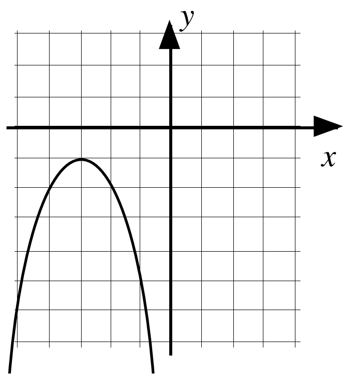
(d)



(e)



(f)



6. Without a sketch, write down the coordinates of the maximum turning points and the equation of the axes of symmetry of these parabolas.

(a) $y = -(x - 2)^2 + 6$ (b) $y = -(x - 5)^2 + 1$ (c) $y = -(x - 6)^2 - 2$

(d) $y = -(x + 1)^2 + 7$ (e) $y = -(x + 4)^2 - 5$ (f) $y = -(x + 3)^2$

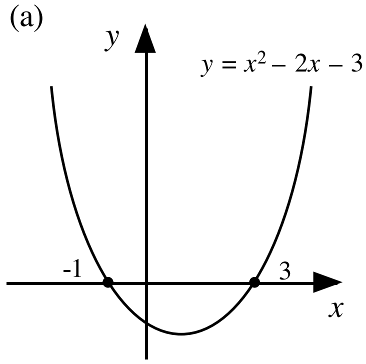
(g) $y = 7 - (x - 1)^2$ (h) $y = 1 - (x - 8)^2$ (i) $y = -2 - (x + 5)^2$

C. Solve quadratic equations graphically

Exercise 3

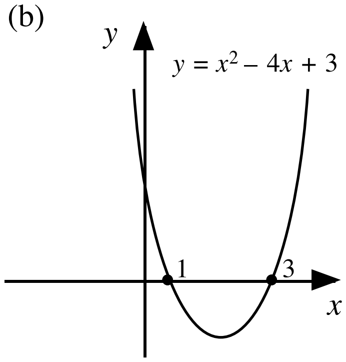
1. In each of the following, the graph has been sketched for you.
   Solve the quadratic equation associated with it.

(a)



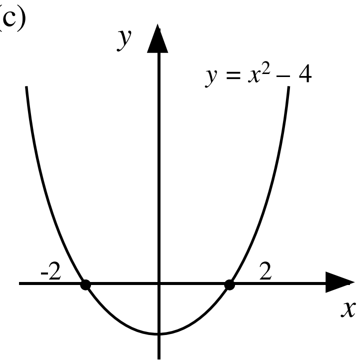
Solve $x^2 - 2x - 3 = 0$

(b)



Solve $x^2 - 4x + 3 = 0$

(c)

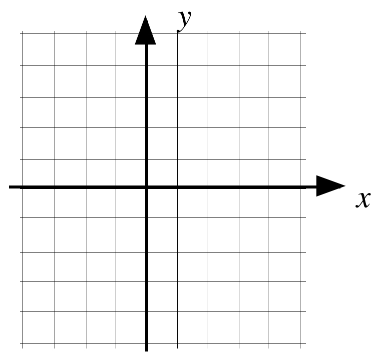


Solve $x^2 - 4 = 0$

2. For the quadratic function $y = x^2 - x - 2$

   (a) Copy and complete this table.

x	–2	–1	0	1	2	3
$y = x^2 - x - 2$	4	…	–2	…	…	…



   (b) Draw a set of axes, plot the 6 points above and draw a smooth (parabolic) curve through them.

   (c) Use your graph to solve the quadratic equation,

   $x^2 - x - 2 = 0$ [i.e. find the 2 roots]

3. Solve each of the following quadratic equations by:

   (i) completing the table

   (ii) plotting the points and drawing the smooth parabola

   (iii) reading off the roots from the graph.

(a) $x^2 - 4x = 0$

x	–1	0	1	2	3	4	5
$y = x^2 - 4x$	5	…	–3	…	…	…	…

(b) $x^2 + x - 2 = 0$

x	–3	–2	–1	0	1	2
$y = x^2 + x - 2$	4	…	–2	…	…	…

(c) $x^2 - 6x + 8 = 0$

x	0	1	2	3	4	5
$y = x^2 - 6x + 8$	8	…	…	…	…	…

(d) $6 - x - x^2 = 0$

x	–4	–3	–2	–1	0	1	2	3
$y = 6 - x - x^2$	–6	…	4	…	…	…	…	…

(e) $x^2 - 4 = 0$

x	-3	-2	-1	0	1	2	3
$y = x^2 - 4$	5	…	…	-4	…	…	…

(f) $5 + 4x - x^2 = 0$

x	-2	-1	0	1	2	3	4	5	6
$y = 5 + 4x - x^2$	…	…	…	…	…	…	…	…	…

D. Solve quadratic equations by factorising

Exercise 4

In this exercise you are going to solve quadratic equations by factorisation.

1. By considering a common factor, factorise and solve the following:

(a) $x^2 - 4x = 0$ (b) $x^2 - 10x = 0$ (c) $8x - x^2 = 0$

(d) $x^2 + 6x = 0$ (e) $x^2 + x = 0$ (f) $x^2 - x = 0$

(g) $2x^2 - 6x = 0$ (h) $5x^2 + 15x = 0$ (i) $12x^2 - 18x = 0$

2. By considering the difference of 2 squares, factorise and solve the following quadratic equations:

(a) $x^2 - 4 = 0$ (b) $x^2 - 9 = 0$ (c) $x^2 - 25 = 0$

(d) $16 - x^2 = 0$ (e) $x^2 - 100 = 0$ (f) $x^2 = 49$

(g) $x^2 = 81$ (h) $x^2 - 9 = 0$ (i) $25x^2 - 16 = 0$

3. Factorise the following trinomials and solve the quadratic equations:

(a) $x^2 + 3x + 2 = 0$ (b) $x^2 - 5x + 6 = 0$ (c) $x^2 + 6x + 5 = 0$

(d) $x^2 - 9x + 20 = 0$ (e) $x^2 + 7x + 10 = 0$ (f) $x^2 - 6x + 9 = 0$

(g) $x^2 - 7x + 12 = 0$ (h) $x^2 - 8x + 7 = 0$ (i) $x^2 - 13x + 42 = 0$

(j) $x^2 + 3x - 10 = 0$ (k) $x^2 - 3x - 4 = 0$ (l) $x^2 + 2x - 8 = 0$

(m) $x^2 - x - 20 = 0$ (n) $x^2 + x - 12 = 0$ (o) $x^2 + 2x - 35 = 0$

(p) $x^2 + 4x - 12 = 0$ (q) $x^2 + 3x - 18 = 0$ (r) $x^2 + 21x + 20 = 0$

(s) $x^2 - 9x + 8 = 0$ (t) $x^2 - 10x - 24 = 0$ (u) $x^2 + 5x - 24 = 0$

(v) $x^2 - 2x - 24 = 0$ (w) $x^2 - 23x - 24 = 0$ (x) $x^2 - 15x + 54 = 0$

4. The following are harder and take a little longer to do.
Solve:

(a) $2x^2 + 7x + 3 = 0$	(b) $2x^2 + 5x + 3 = 0$	(c) $3x^2 + 7x + 2 = 0$
(d) $2x^2 - 9x + 9 = 0$	(e) $3x^2 + 11x + 6 = 0$	(f) $5x^2 + 11x + 2 = 0$
(g) $3x^2 - 2x - 8 = 0$	(h) $3x^2 - 5x - 2 = 0$	(i) $3x^2 + 2x - 1 = 0$
(j) $2x^2 - 7x - 4 = 0$	(k) $5x^2 + 13x - 6 = 0$	(l) $2x^2 + 9x + 10 = 0$

5. Rearrange the following into quadratic equations of the form $ax^2 + bx + c = 0$ and solve them:

(a) $x(x + 2) = 3$	(b) $x(x - 1) = 20$	(c) $x(x - 3) = 10$
(d) $x(x - 5) = 6$	(e) $x(x + 3) = 70$	(f) $x(x + 1) = 56$
(g) $(x + 1)(x + 2) = 12$	(h) $(x - 1)(x + 2) = 28$	(i) $(x + 3)(x - 1) = 5$
(j) $(x - 1)(x + 1) = 8$	(k) $(x - 2)(x + 2) = 21$	(l) $(x - 2)(x - 3) = 2$
(m) $2x^2 + 3x - 1 = x^2 - x - 4$	(n) $3x^2 + 5x - 8 = 2x^2 + 2x + 2$	
(o) $2x(x + 2) = 16$	(p) $x(2x - 3) = 20$	(q) $x(3x - 1) = 10$
(r) $x(5x + 2) = 7$	(s) $(2x - 1)(x + 3) = 30$	(t) $(x + 1)(3x - 2) = 12$

E. Solve quadratic equations using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise 5

1. Solve the following quadratic equations using the formula, correct to 2 decimal places:

(a) $x^2 + 4x + 1 = 0$	(b) $x^2 + 3x + 1 = 0$	(c) $x^2 + 6x + 2 = 0$
(d) $x^2 + 10x + 7 = 0$	(e) $x^2 - 4x + 2 = 0$	(f) $x^2 - 5x + 5 = 0$
(g) $2x^2 + 7x + 2 = 0$	(h) $3x^2 + 10x + 5 = 0$	(i) $4x^2 - 7x + 2 = 0$

2. Solve the following quadratic equations using the formula, correct to 3 decimal places:

(a) $x^2 + 2x - 2 = 0$	(b) $x^2 + x - 5 = 0$	(c) $x^2 + 3x - 6 = 0$
(d) $x^2 - 2x - 5 = 0$	(e) $x^2 - 3x - 1 = 0$	(f) $x^2 + 8x - 2 = 0$
(g) $2x^2 + 3x - 4 = 0$	(h) $3x^2 - 2x - 2 = 0$	(i) $8x^2 - 3x - 1 = 0$

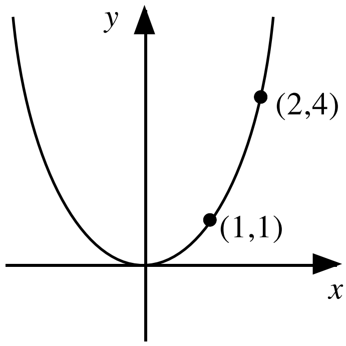
3. Rearrange the following and solve each equation giving the roots correct to 3 significant figures (3 figures of accuracy):

(a) $x(x + 5) = 2$	(b) $x(x - 3) = 1$	(c) $2x(x + 3) = 5$
(d) $3x(x - 1) = 10$	(e) $(x + 3)(x + 4) = 7$	(f) $(x - 1)^2 = 5$

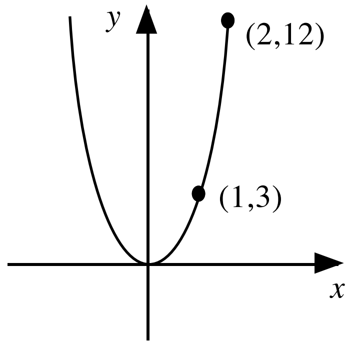
Checkup for quadratic equations

1. Write down the equations of the quadratics corresponding to these parabolas:
   (each one is of the form $y = kx^2$ or $y = (x + a)^2 + b$)

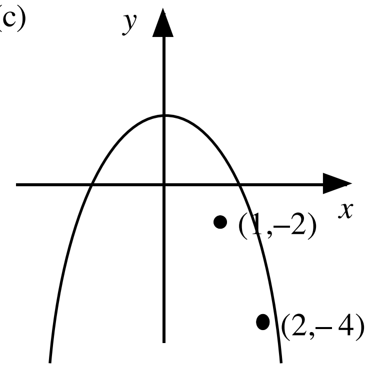
(a)



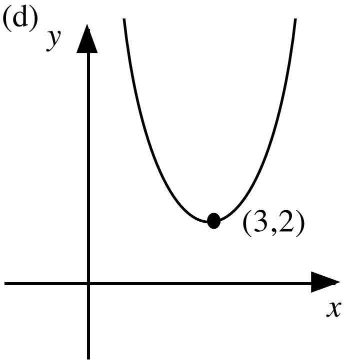
(b)



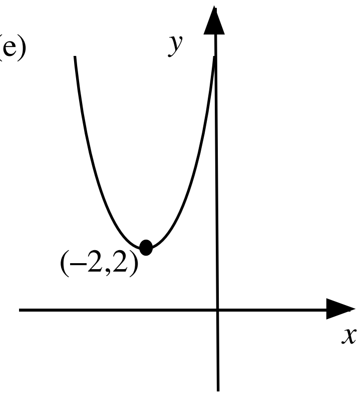
(c)



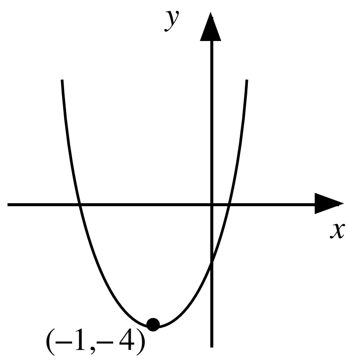
(d)



(e)

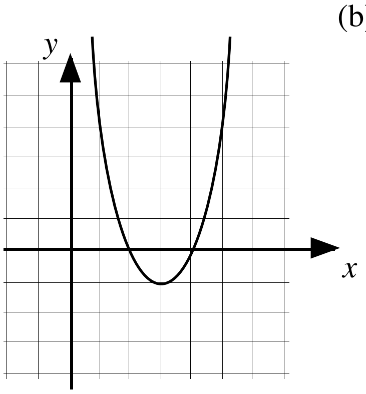


(f)

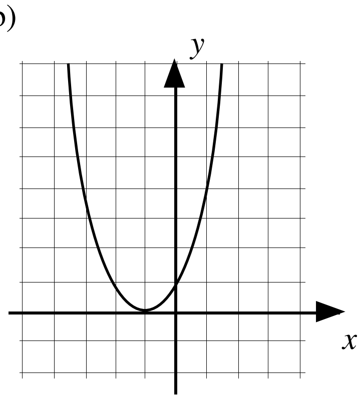


2. (i) Write down the coordinates of the turning point for each of the following, stating whether it is a maximum or a minimum.
   (ii) Write down the equation of the axes of symmetry for each one.

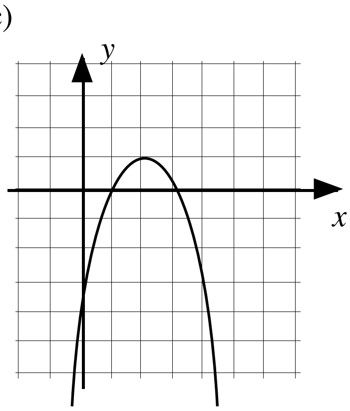
(a)



(b)



(c)



3. Write down the coordinates of the turning point of each of the following.
State whether each is a maximum or a minimum.
Give also the equation of the axes of symmetry.

(a) $y = (x - 21)^2 + 5$ (b) $y = 3(x + 2)^2 - 1$ (c) $y = -2(x - 3)^2 + 2$

4. Solve each of the following quadratic equations by
   (i) completing the table.
   (ii) plotting the points and drawing the smooth parabola.
   (iii) reading off the roots from the graph.

(a) $x^2 - 2x - 8 = 0$

x	–3	–2	–1	0	1	2	3	4	5
$y = x^2 - 2x - 8$	7	…	…	–8	…	…	…	…	…

(b) $x^2 + 2x - 3 = 0$

x	–4	–3	–2	–1	0	1	2
$y = x^2 + 2x - 3$	…	…	–3	…	…	…	…

5. Solve the following quadratic equations by factorising:

(a) $x^2 - 7x = 0$ (b) $x^2 - 9 = 0$ (c) $x^2 + 8x + 12 = 0$

(d) $6x^2 + 9x = 0$ (e) $25 - x^2 = 0$ (f) $x^2 - x - 30 = 0$

(g) $4x^2 - 9 = 0$ (h) $x^2 - 7x + 10 = 0$ (i) $2x^2 + 7x - 15 = 0$

(j) $x(x + 5) = 14$ (k) $(x - 1)(x + 2) = 18$ (l) $(x - 2)^2 = 16$

6. Solve the following quadratic equations using the formula.
(Give the answers correct to 2 decimal places):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(a) $x^2 + 6x + 3 = 0$ (b) $x^2 - 4x + 1 = 0$ (c) $3x^2 + 10x + 6 = 0$

(d) $x^2 + 5x - 2 = 0$ (e) $x^2 - 6x - 4 = 0$ (f) $2x^2 - x - 5 = 0$

FURTHER TRIGONOMETRY

By the end of this set of exercises, you should be able to

(a) recognise the graphs of sine, cosine and tangent functions

(b) sketch and identify other trigonometric functions

(c) solve simple trigonometric equations (in degrees)

(d) define the period of trigonometric functions, either from their graphs or from their equations

(e) simplify trigonometric expressions using $\sin^2 x + \cos^2 x = 1$ and $\tan x = \dfrac{\sin x}{\cos x}$

A. Sine, cosine and tangent graphs

Exercise 1

You may have drawn the sine, cosine and tangent graphs as part of the introduction to trigonometry in Maths 2 Intermediate 2. If you have retained the graphs, you may miss out questions 1 to 3 of Exercise 1 below.

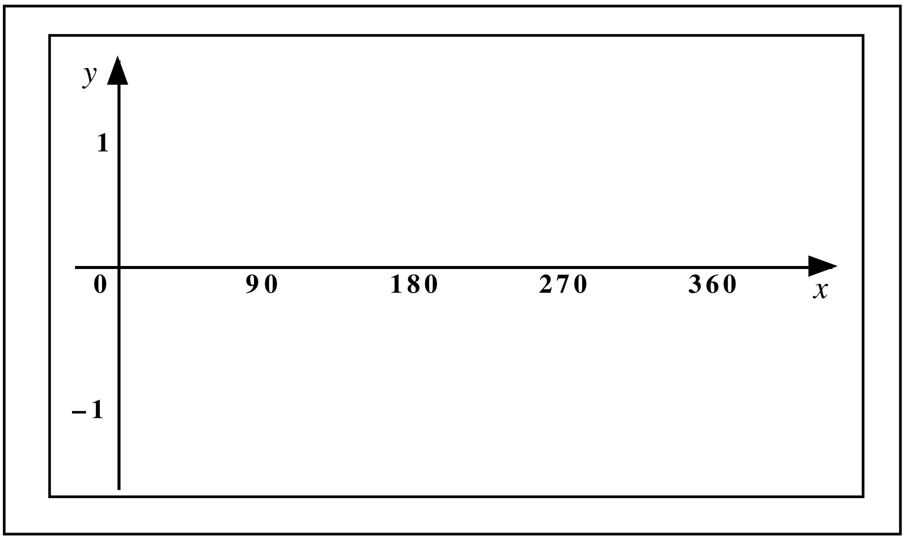
1. The Sine Graph

   (a) Make a copy of this table and use your calculator to help fill it in, giving each answer correct to 2 decimal places.

x	0°	20°	40°	60°	80°	90°	100°	120°	140°	160	180°
$\sin x°$	0·00	0·34	0·64	0·87	0·98	1·00	…	…	…	…	…

x	200°	220°	240°	260°	270°	280°	300°	320°	340°	360
$\sin x°$	…	…	…	…	…	…	…	…	…	…

   (b) Use a piece of 2 mm graph paper to draw a set of axes as illustrated below.



   (c) Plot as accurately as possible the 21 points from your table.

   (d) Join them up smoothly to create the graph of the function $y = \sin x°$.

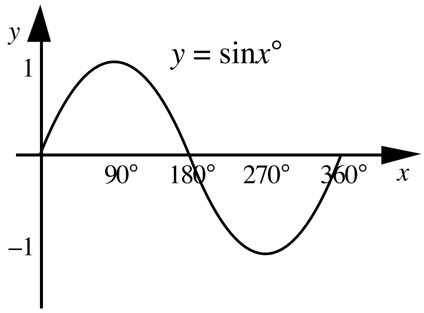
2. Repeat question 1 (a) to (d) for the function $y = \cos x°$

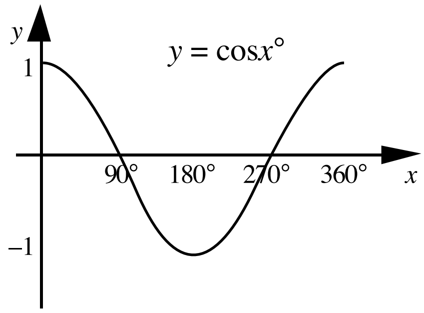
3. Repeat for the graph of $y = \tan x°$ (a different scale will be required for the vertical axis).

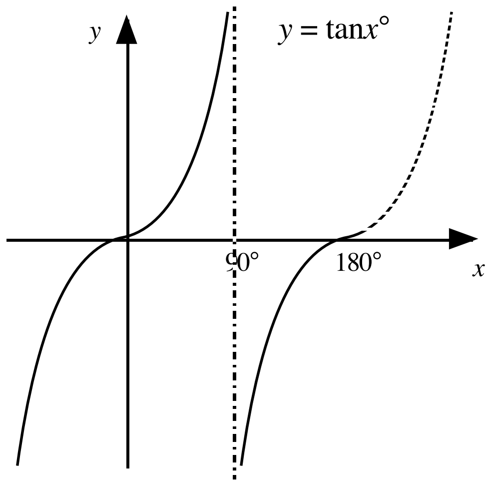
4. Study your three graphs carefully. You should now be able to sketch the sine, cosine (and tangent) graphs fairly quickly (about 30 seconds) indicating the main points where the graphs cut the x and y axes and the general shape of each graph.

   Try them now.

Sketches of the three trigonometric graphs, for comparison:







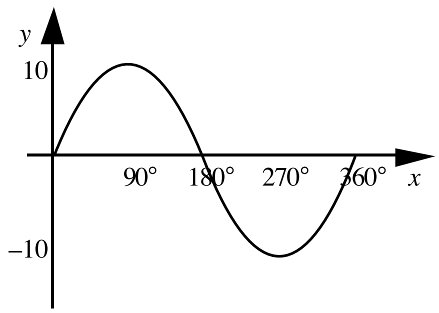
Check your graphs are similar to those shown above.

B. Sketching and identifying trigonometric functions

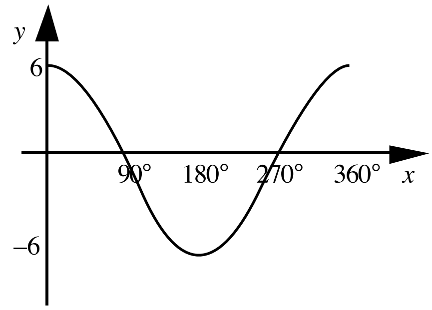
Exercise 2A

1. Write down the equations of the trigonometric functions represented by the following graphs:

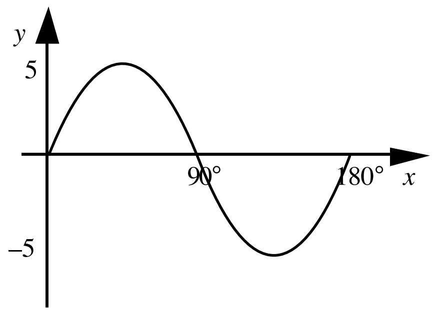
(a)



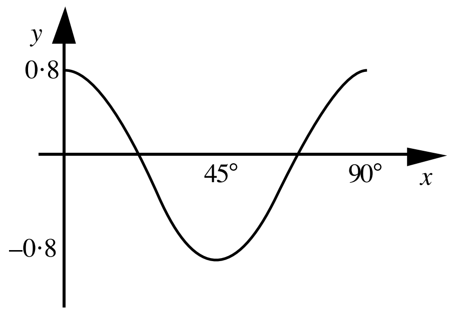
(b)



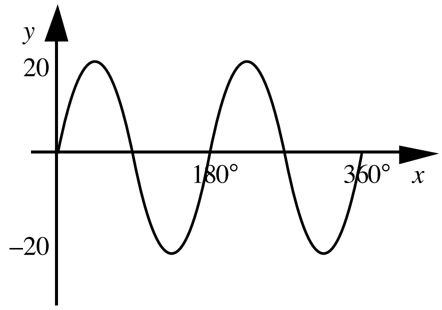
(c)



(d)



(e)



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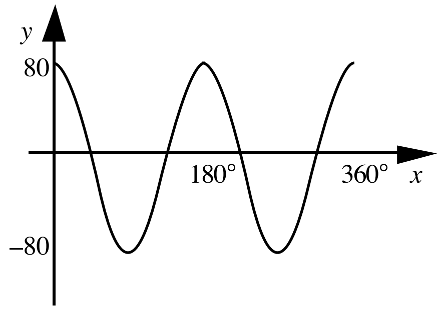
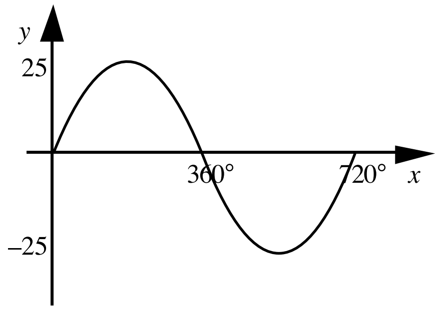
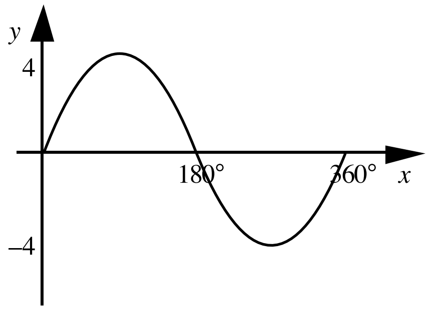


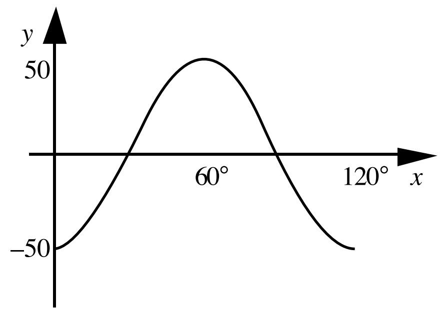
(g)



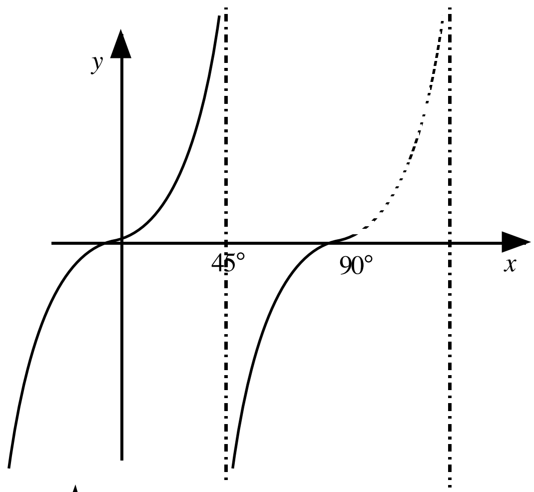
(h)



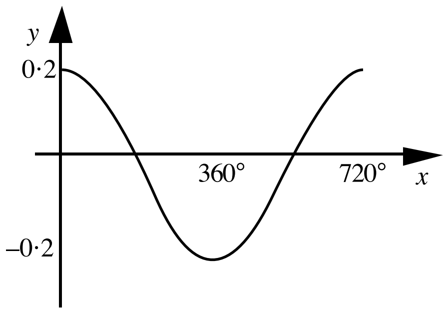
(i)



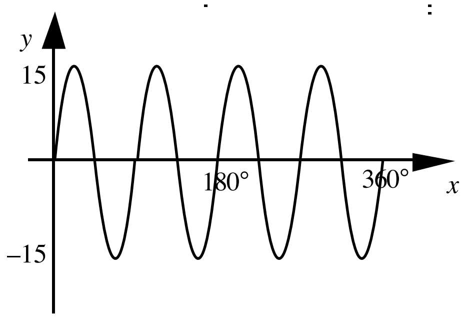
(j)



(k)



(l)



2. Make neat sketches of the following trigonometric functions, clearly indicating

   (i) the shape of the graph (draw one 'cycle' of it only)
   (ii) the important values on the horizontal axis
   (iii) the maximum and minimum values of the function.

cont'd ...

(a) $y = 3\sin x°$ (b) $y = 4\cos x°$ (c) $y = \tan 3x°$
(d) $y = 10\sin 3x°$ (e) $y = 12\cos 2x°$ (f) $y = 0{\cdot}7\sin 4x°$
(g) $y = 1{\cdot}2\cos 4x°$ (h) $y = 30\sin 6x°$ (i) $y = 100\cos 5x°$
(j) $y = -\sin x°$ (k) $y = -6\sin {}^1/_2 x°$ (l) $y = -20\cos 3x°$

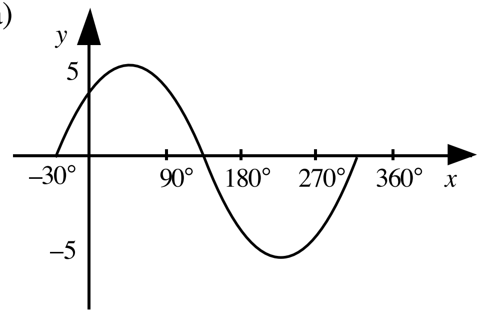
3. Make neat sketches of the following over the given range of values:

(a) $y = 60\sin 2x°$ $0 \le x \le 360$ (b) $y = 2{\cdot}5\cos 3x°$ $0 \le x \le 360$
(c) $y = 40\sin 4x°$ $0 \le x \le 180$ (d) $y = -2\cos 6x°$ $0 \le x \le 180$
(e) $y = -15\sin 8x°$ $0 \le x \le 180$ (f) $y = 1{\cdot}8\cos 30x°$ $0 \le x \le 12$

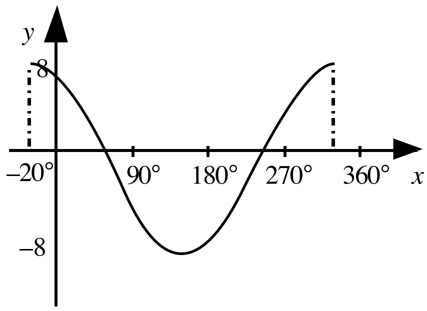
Exercise 2B

1. Write down the equations of the trigonometric functions in the form $y = k\sin(x - a)°$ or $y = k\cos(x - a)°$ represented by the following graphs:

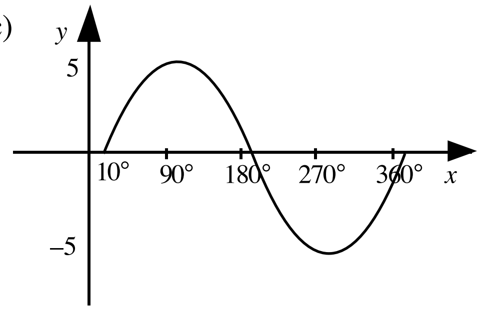
(a)



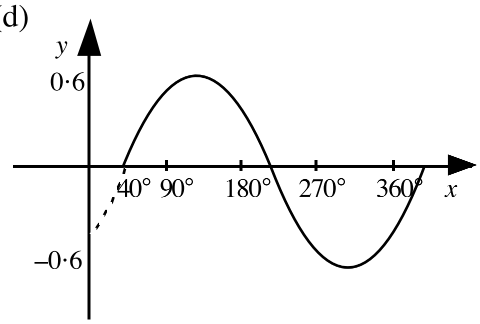
(b)



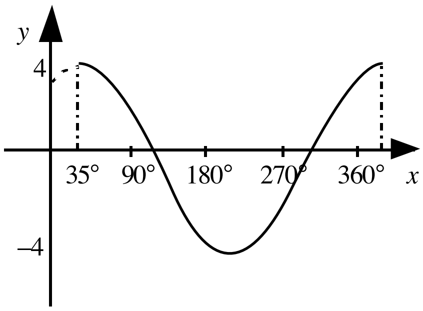
(c)



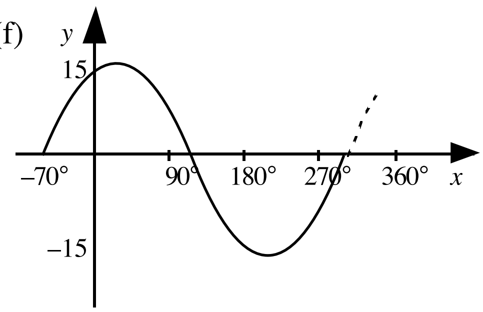
(d)



(e)



(f)



2. Make neat sketches of the following trigonometric functions, showing clearly:
(i) where each graph cuts the x - axis. (ii) the maximum and minimum values.

(a) $y = 10\sin(x - 15)°$ (b) $y = 1{\cdot}4\cos(x - 20)°$ (c) $y = 15\sin(x + 10)°$
(d) $y = 2{\cdot}4\cos(x + 30)°$ (e) $y = 300\sin(x - 80)°$ (f) $y = \tan(x - 10)°$

C. Solving trigonometric equations

S (in)	A (ll)
T (an)	C (os)

Exercise 3

1. Find the <u>two</u> solutions for each of the following in the range $0 \le x \le 360$:
(Give each answer correct to the nearest whole degree).

(a) $\sin x° = 0{\cdot}500$ (b) $\cos x° = 0{\cdot}707$ (c) $\tan x° = 0{\cdot}869$
(d) $\cos x° = 0{\cdot}940$ (e) $\tan x° = 1{\cdot}280$ (f) $\sin x° = 0{\cdot}574$
(g) $\sin x° = 0{\cdot}990$ (h) $\tan x° = 6{\cdot}314$ (i) $\cos x° = 0{\cdot}391$
(j) $\cos x° = 0{\cdot}985$ (k) $\sin x° = 0{\cdot}866$ (l) $\tan x° = 1{\cdot}732$

2. Rearrange each of the following and solve them in the range $0 \le x \le 360$.
(Give your answers correct to 1 decimal place).

(a) $2\cos x° - 1 = 0$ (b) $5\sin x° - 4 = 0$ (c) $10\tan x° - 7 = 0$
(d) $1 - 3\sin x° = 0$ (e) $5 - 6\cos x° = 0$ (f) $3\tan x° - 5 = 0$

3. Find the <u>two</u> solutions for each of the following in the range $0 \le x \le 360$:
(Give each answer correct to the nearest whole degree).

(a) $\sin x° = -0{\cdot}500$ (b) $\cos x° = -0{\cdot}707$ (c) $\tan x° = -0{\cdot}384$
(d) $\cos x° = -0{\cdot}292$ (e) $\tan x° = -1{\cdot}000$ (f) $\sin x° = -0{\cdot}866$
(g) $\tan x° = -4$ (h) $\sin x° = -0{\cdot}174$ (i) $\cos x° = -0{\cdot}927$

4. Rearrange each of the following and solve them in the range $0 \le x \le 360$.
(Give your answers correct to 1 decimal place).

(a) $4\sin x° + 1 = 0$ (b) $5\cos x° + 3 = 0$ (c) $3\tan x° + 1 = 0$
(d) $7 + 8\cos x° = 0$ (e) $0{\cdot}4\sin x° + 0{\cdot}3 = 0$ (f) $5\tan x° + 8 = 0$

5. Solve the following mixture of trigonometric equations in the range $0 \le x \le 360$.
(Give your answers correct to 1 decimal place).

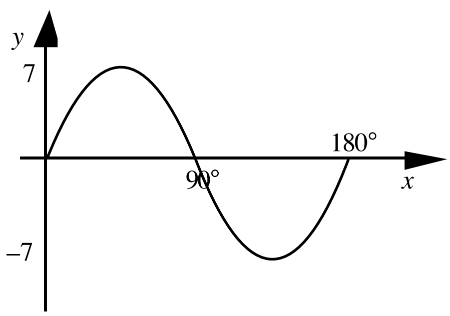
(a) $\sin x° = 0{\cdot}323$ (b) $\cos x° = -0{\cdot}9$ (c) $\tan x° = 0{\cdot}678$
(d) $\cos x° = {}^1/_4$ (e) $\sin x° = -0{\cdot}707$ (f) $\tan x° = -2$
(g) $\sin x° = {}^3/_5$ (h) $\cos x° = -0{\cdot}111$ (i) $\tan x° = {}^5/_8$
(j) $8\sin x° + 5 = 0$ (k) $6\cos x° + 3 = 0$ (l) $1 - 5\tan x° = 0$
(m) $20\sin x° - 17 = 0$ (n) $15 - 25\cos x° = 0$ (o) $8\tan x° + 7 = 0$
(p) $5\sin x° + 3 = 2\sin x° + 5$ (q) $7\cos x° - 1 = \cos x° + 4$
(r) $10\tan x° + 8 = 3\tan x° + 4$ (s) $6\sin x° + 11 = 3\sin x° + 10$

D. The period of a trigonometric function

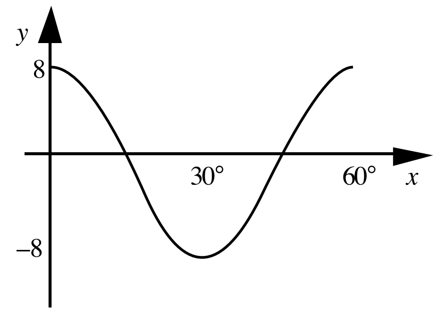
Exercise 4

1. Determine the **period** and the **maximum** and **minimum** values of these trig functions.

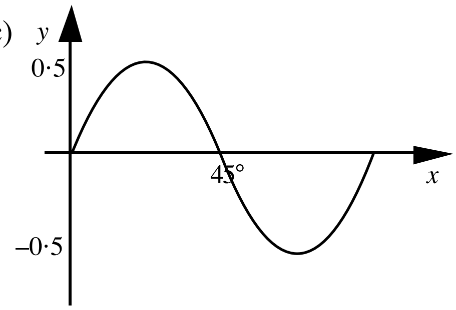
(a)



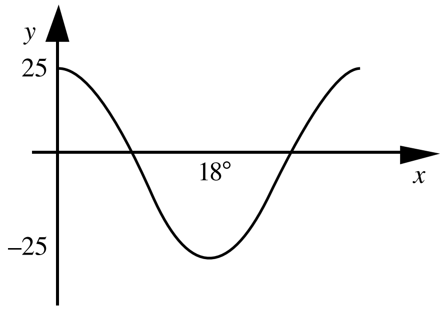
(b)



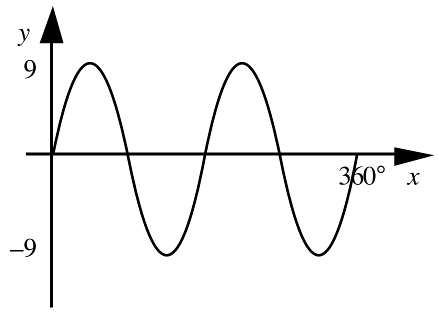
(c)



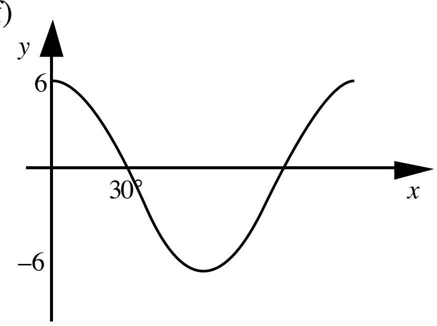
(d)



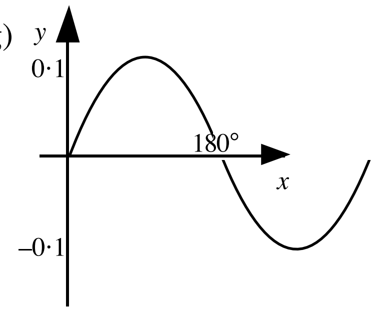
(e)



(f)



(g)



2. Determine the **period** and the **maximum** and **minimum** values of these trig functions.

(a) $y = 5\sin 2x°$ (b) $y = 3\cos 3x°$ (c) $y = 10\tan 4x°$

(d) $y = 2{\cdot}2\cos 2x°$ (e) $y = 30\sin 6x°$ (f) $y = –5\cos 30x°$

(g) $y = 50\sin 90x°$ (h) $y = –4\cos {}^{1}/_{2}x°$ (i) $y = 18\sin {}^{1}/_{4}x°$

(j) $y = 0{\cdot}9\cos 60x°$ (k) $y = {}^{1}/_{2}\sin 5x°$ (l) $y = {}^{3}/_{4}\cos 9x°$

(m) $y = 11\sin 180x°$ (n) $y = 8\sin 1{\cdot}5x°$ (o) $y = 40\cos 2{\cdot}5x°$

E. Trigonometric identities

Remember :- $\sin^2 x + \cos^2 x = 1$; and $\tan x = \dfrac{\sin x}{\cos x}$

Exercise 5

1. Simplify the following using the above 2 identities:

   (a) $2\sin^2 x° + 2\cos^2 x°$ (b) $5\cos^2 x° + 5\sin^2 x°$

   (c) $\dfrac{3\sin x°}{\cos x°}$ (d) $\dfrac{5\sin x°}{2\cos x°}$

2. Write down a simple expression, identical to:

   (a) $1 - \sin^2 x°$ (b) $1 - \cos^2 x°$ (c) $\tan x° \cos x°$ (d) $\dfrac{\sin x°}{\cos x°}$

3. Simplify:

   (a) $\dfrac{1 - \cos^2 x°}{\sin^2 x°}$ (b) $\dfrac{1 - \sin^2 x°}{2\cos^2 x°}$ (c) $\dfrac{\sin^2 x°}{\cos^2 x°}$

   (d) $\dfrac{1 - \sin^2 x°}{\cos x°}$ (e) $\dfrac{1 - \cos^2 x°}{5\sin x°}$ (f) $\tan^2 x°(1 - \sin^2 x°)$

4. Prove the following trigonometric identities:

   (a) $3 - 3\sin^2 x° = 3\cos^2 x°$ (b) $5 - 5\cos^2 x° = 5\sin^2 x°$

   (c) $\sqrt{1 - \cos^2 x°} = \sin x°$ (d) $\tan x°\sqrt{1 - \sin^2 x°} = \sin x°$

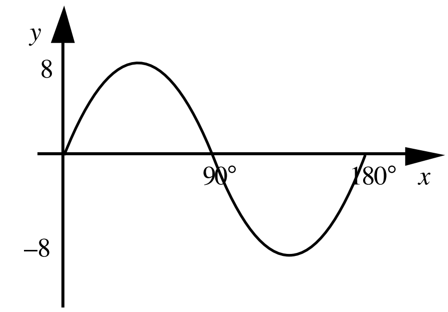
   (e) $\dfrac{1 - \cos^2 x°}{1 - \sin^2 x°} = \tan^2 x°$ (f) $\dfrac{1 - \sin^2 x°}{1 - \cos^2 x°} = \dfrac{1}{\tan^2 x°}$

Checkup for further trigonometry

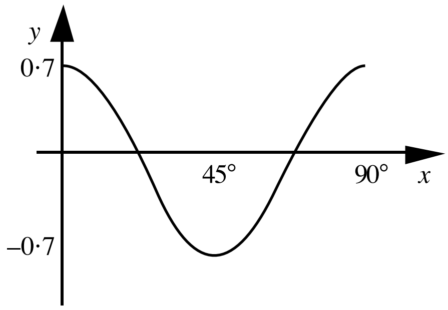
1. Make a sketch of the sine, cosine and tangent graphs, indicating all their main features.

2. Write down the equations of the trigonometric functions associated with the following graphs:

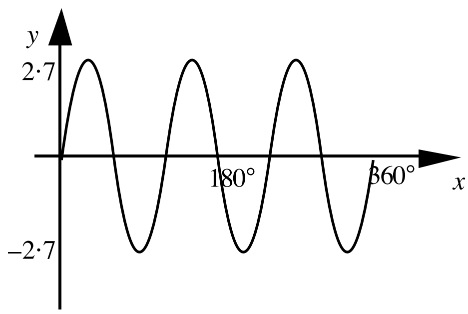
(a)



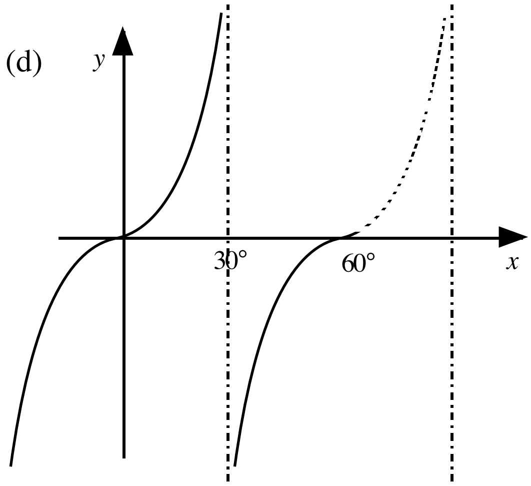
(b)



(c)



(d)



3. Make neat sketches of the following, indicating all the main points and features:

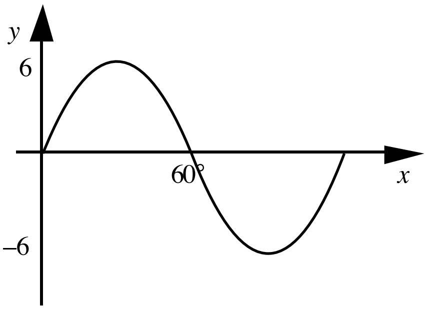
(a) $y = 20\sin 4x°$ $\quad 0 \le x \le 90$
(b) $y = 1{\cdot}6\cos 2x°$ $\quad 0 \le x \le 360$
(c) $y = -8\sin 8x°$ $\quad 0 \le x \le 90$
(d) $y = \tan 2x°$ $\quad 0 \le x \le 90$

4. Find the two solutions for each of the following in the range $0 \le x \le 360$:
(Give your answers correct to 1 decimal place).

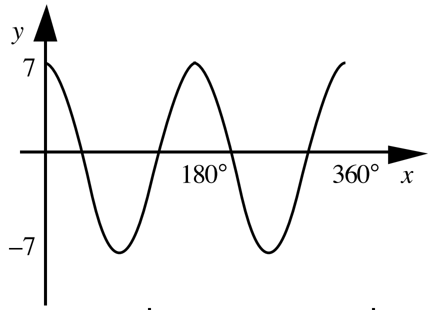
(a) $\sin x° = 0{\cdot}911$
(b) $\cos x° = 0{\cdot}444$
(c) $\tan x° = 3$
(d) $\cos x° = -0{\cdot}605$
(e) $\tan x° = -0{\cdot}8$
(f) $\sin x° = -4/5$
(g) $2\sin x° - 1 = 0$
(h) $8\cos x° + 6 = 0$
(i) $4\tan x° - 3 = 0$
(j) $3\cos x° - 2 = 0$
(k) $1 + 4\sin x° = 0$
(l) $5\tan x° = 3\tan x° - 2$

5. What are the periods of the following trigonometric graphs and functions?

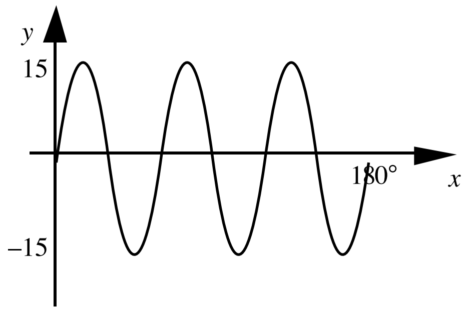
(a)



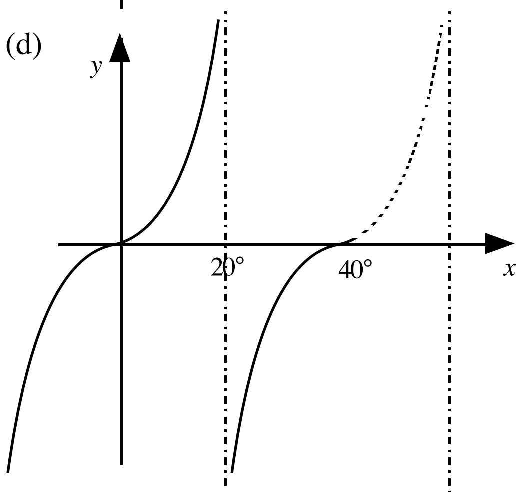
(b)



(c)



(d)



(e) $y = 10\sin 10x°$

(f) $y = 2{\cdot}3\cos 30x°$

(g) $y = -4\sin 9x°$

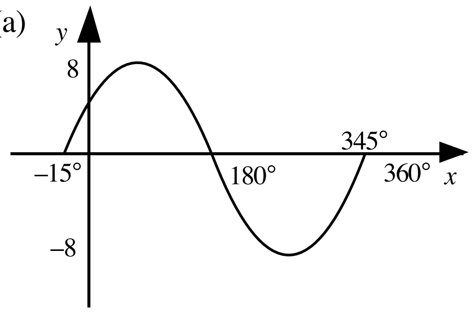
(h) $y = 5\tan 4x°$

6. (a) Simplify: (i) $6 - 6\sin^2 x°$ (ii) $\dfrac{\cos x°}{\sin x°}$

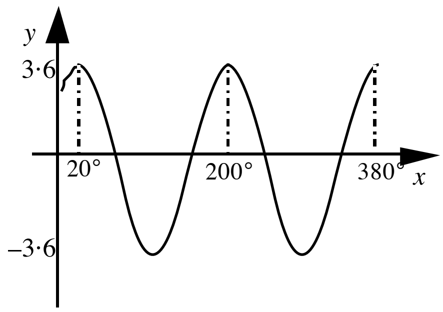
(b) Prove these identities :- (i) $\dfrac{1-\cos^2 x°}{\sin^2 x°} = 1$ (ii) $(1 - \sin x°)(1 + \sin x°) = \cos^2 x°$

7. Write down the equations of the following trigonometric graphs:

(a)



(b)



8. Sketch the following trigonometric graphs, indicating their main features:

(a) $y = 18\cos(x + 30)°$ $0 \le x \le 360$ (b) $y = 2\sin(x - 10)°$ $0 \le x \le 360$

SPECIMEN ASSESSMENT QUESTIONS

1. Factorise fully:

   (a) $\frac{12x^2y}{18xy^3}$ (b) $\frac{4a-8}{2}$ (c) $\frac{p^3-pq}{p}$ (d) $\frac{w^2-25}{w-5}$ (e) $\frac{a^2-2a-3}{a^2+2a+1}$

2. Simplify:

   (a) $\frac{v^2}{9} \times \frac{36}{v^5}$ (b) $\frac{c}{a^3} \div \frac{2c^2}{a}$ (c) $\frac{5}{p} + \frac{3}{q}$ (d) $\frac{5t}{z} - \frac{t}{2z}$ (e) $\frac{x+1}{2} + \frac{x+3}{3}$

3. Change the subject of each formula to the letter in the bracket:

   (a) $p - 2q = r$ [p] (b) $aw + g = h$ [w] (c) $\frac{a}{x} = \frac{x}{9}$ [x] (d) $^1/_4(m+n) = V$ [m]

4. Express each of these in its simplest form:

   (a) $\sqrt{28}$ (b) $3\sqrt{12}$ (c) $\sqrt{\frac{9}{a^4}}$ (d) $5\sqrt{18} - \sqrt{32}$

5. Rationalise the denominators and simplify:

   (a) $^{10}/_{\sqrt{5}}$ (b) $^{\sqrt{12}}/_{\sqrt{2}}$ (c) $\frac{p}{\sqrt{q}}$

6. Simplify, giving answers with positive indices:

   (a) $b^5 \div b^3$ (b) $(w^3)^{-2}$ (c) $\frac{a^2 \times a^4}{a^3}$ (d) $p^{1/2} \times 3p^{-3/2}$ (e) $\frac{v^3(v^2 - v^{-3})}{v^4}$

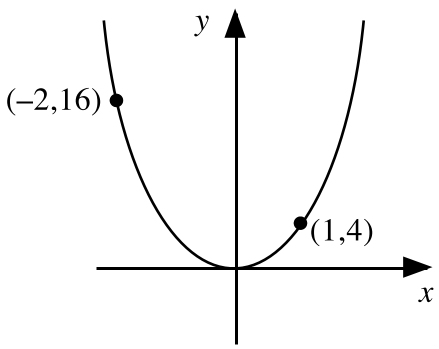
   (f) $8x^{1/2} \div 4x^{5/2}$

7. Evaluate the following for $v = 4$ and $w = 8$:

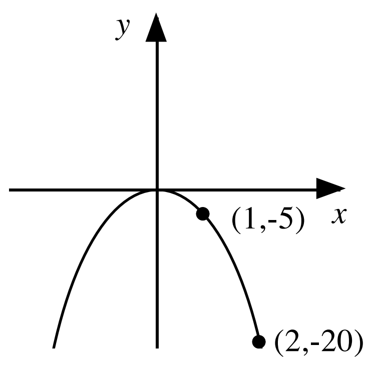
   (a) $10v^{1/2}$ (b) $2w^{2/3}$ (c) $v^{-1/2} \div w^{-1/3}$

8. Write down the equation of the quadratic functions corresponding to each of the following parabolas: (each is of the form $y = kx^2$ or $y = \pm(x + a)^2 + b$).

   (a)



   (b)



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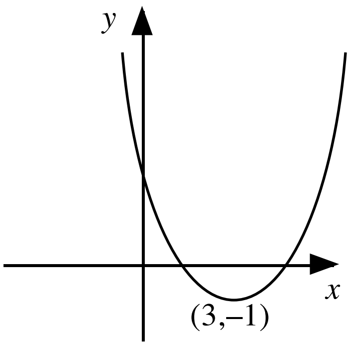
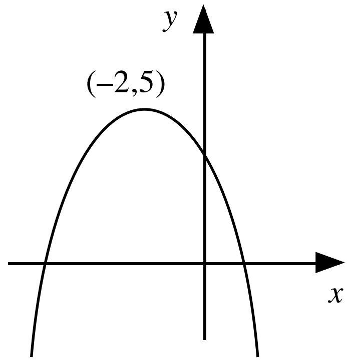
(c)



(d)



9. Write down the coordinates of the turning point of each of the following, state whether each is a maximum or minimum and give the equation of the axis of symmetry.

(a) $y = (x - 2)^2 + 7$ (b) $y = -(x + 1)^2 - 3$

10. (a) Complete the following table for the function, $y = x^2 - x - 12$.

x	–4	–3	–2	–1	0	1	2	3	4	5	6
$y = x^2 - x - 12$	8	…	…	–10	…	…	…	–6	…	…	…

(b) Plot the points and sketch the parabola.

(c) Write down the roots of the quadratic equation $x^2 - x - 12 = 0$

11. Solve the following quadratic equations by factorising:

(a) $2x^2 - 6x = 0$ (b) $x^2 - 16 = 0$ (c) $x^2 + 7x + 10 = 0$

(d) $x^2 - 2x - 15 = 0$ (e) $x(x - 2) = 24$ (f) $(x - 1)(x + 2) = 28$

12. Solve the following, correct to 2 decimal places, using the quadratic formula:

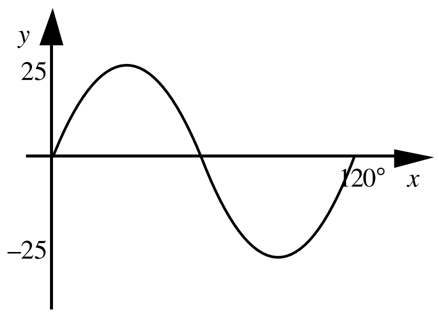
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(a) $x^2 + 5x + 1 = 0$ (b) $x^2 - 2x - 7 = 0$

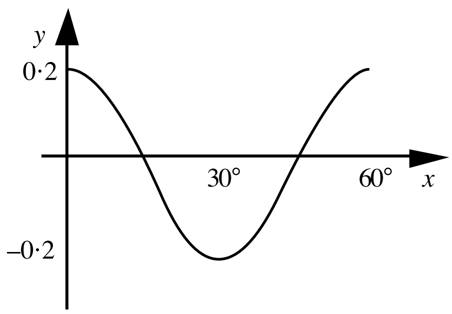
(c) $3x^2 + x - 1 = 0$

13. Write down the equations of the trigonometric functions associated with these graphs:

(a)



(b)



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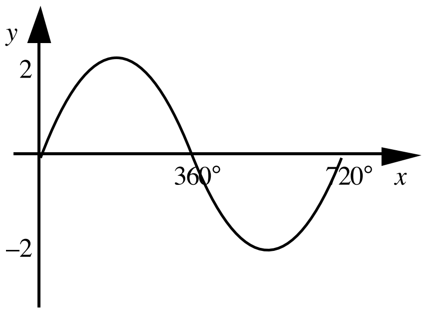
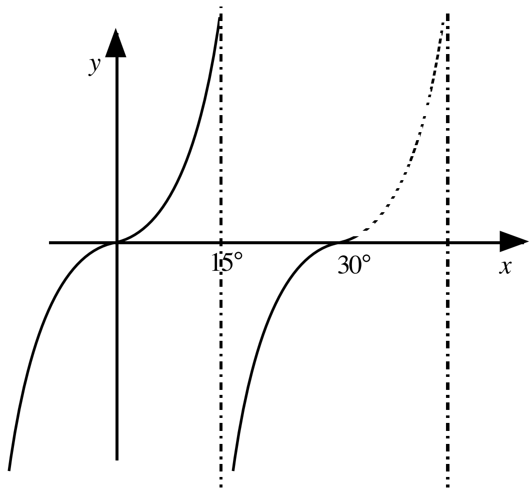
(c)



(d)



14. Sketch the following trigonometric graphs, indicating all the main features and points:

(a) $y = 11 \sin 2x°$ $0 \leq x \leq 360$ (b) $y = -5 \cos 4x°$ $0 \leq x \leq 180$

15. Find two solutions for each of the following trig equations in the range $0 \leq x \leq 360$. (Give answers correct to 1 decimal place).

(a) $\sin x° = 0{\cdot}412$ (b) $8\cos x° = 5$ (c) $10\tan x° - 7 = 0$

(d) $\cos x° = -0{\cdot}234$ (e) $\tan x° = -1$ (f) $3\sin x° + 1 = 0$

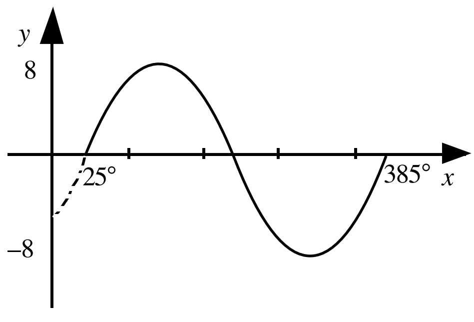
16. What are the **periods** of the following trigonometric functions:

(a) $y = 105 \sin 5x°$ (b) $y = -4 \cos 20x°$ (c) $y = \tan 3x°$

17. (a) Simplify: $\sin^2 x° - 1$

(b) Prove the identity: $\dfrac{1 - \cos^2 x}{2 - 2\sin^2 x} = \dfrac{1}{2}\tan^2 x$

18. Write down the equation of the function given by this trigonometric graph:



19. Sketch the graph of the function, $y = 0{\cdot}7 \cos(x - 10)°$ showing all the main points and features.

ANSWERS

Algebraic operations

Exercise 1

1. (a) $^3/_5$ (b) $^1/_3$ (c) $^3/_4$ (d) $^3/_4$ (e) $^1/_4$ (f) $^4/_5$ (g) $^2/_3$ (h) $^v/_w$
   (i) $^s/_t$ (j) $^b/_c$ (k) $^1/_d$ (l) $^1/_m$ (m) $^a/_4$ (n) $^8/_z$ (o) x (p) $^1/_d$
   (q) $^{5x}/_6$ (r) $^v/_{2w}$ (s) $2y$ (t) $^a/_{3b}$ (u) $^q/_3$ (v) $^1/_b$ (w) y (x) $^3/_7$
   (y) $^5/_{2q}$ (z) $^{2x}/_y$ (aa) $^1/_{(x+1)}$ (ab) $^1/_{(x-5)^3}$

2. (a) $\frac{x+4}{2}$ (b) $\frac{x+2}{3}$ (c) $x-4$ (d) $\frac{x-2}{5}$ (e) $\frac{1}{x+3}$
   (f) $\frac{1}{x-3}$ (g) $\frac{1}{x+3}$ (h) $\frac{4}{2x-5}$ (i) $x-y$ (j) $q+1$
   (k) $v+1$ (l) $1-a$

3. (a) 2 (b) 3 (c) $^1/_3$ (d) g (e) $^2/_5$
   (f) $^3/_7$ (g) 4 (h) $^x/_8$ (i) $x+1$ (j) $y-3$
   (k) $a+5$ (l) $^1/_{(w-10)}$ (m) $\frac{x-1}{x+1}$ (n) $\frac{3v+1}{v+2}$ (o) $\frac{y+1}{y+3}$
   (p) $\frac{2x-3}{x+4}$

Exercise 2

1. (a) $^1/_5$ (b) $^1/_9$ (c) $^1/_2$ (d) 2 (e) 2 (f) 3 (g) $^7/_2$ (h) $^{16}/_3$
   (i) $^{ac}/_{bd}$ (j) $^{xv}/_{yw}$ (k) $\frac{m^2}{n^2}$ (l) 1 (m) $\frac{x^2}{10}$ (n) $\frac{x^2}{7}$ (o) $\frac{a^2}{25}$ (p) 1
   (q) $^a/_d$ (r) x (s) a (t) a^2

2. (a) $^1/_2$ (b) $^1/_4$ (c) $^1/_6$ (d) $^1/_4$ (e) $^3/_2$ (f) $^1/_4$ (g) 2 (h) 5
   (i) a (j) $^b/_3$ (k) $^{r^2}/_3$ (l) 1 (m) ad (n) $^1/_{5w}$ (o) $^1/_a$ (p) $^{dx}/_{2y}$
   (q) a (r) $^2/_5$

3. (a) $^7/_{12}$ (b) $^{11}/_{12}$ (c) $^{11}/_{20}$ (d) $^8/_{15}$ (e) $^2/_{15}$
   (f) $^1/_{14}$ (g) $^7/_{10}$ (h) $^3/_{10}$ (i) $^3/_8$ (j) $^9/_{10}$
   (k) $\frac{2x+3a}{6}$ (l) $\frac{2c+5d}{10}$ (m) $\frac{4e-3h}{12}$ (n) $\frac{2m-n}{8}$ (o) $\frac{4x+3k}{6}$
   (p) $\frac{5u-4w}{10}$ (q) $\frac{8r+5s}{10}$ (r) $\frac{5a-6d}{15}$ (s) $\frac{4x+9y}{6}$ (t) $\frac{15x+8u}{20}$

4. (a) $\frac{2y+3x}{xy}$ (b) $\frac{5b-2a}{ab}$ (c) $\frac{4d+c}{cd}$ (d) $\frac{q-2p}{pq}$ (e) $\frac{2w+2v}{vw}$

(f) $\frac{h-g}{gh}$ (g) $\frac{7n+k}{kn}$ (h) $\frac{y-8x}{xy}$

5. (a) $\frac{5x+5}{6}$ (b) $\frac{9x+6}{20}$ (c) $\frac{3x+7}{4}$ (d) $\frac{11x-4}{15}$ (e) $\frac{x+1}{6}$

(f) $\frac{3x+8}{10}$ (g) $\frac{3x+1}{4}$ (h) $\frac{3x+7}{10}$

Exercise 3A

1. $x = c - 2$ 2. $x = c + 4$ 3. $x = q - p$ 4. $x = q + p$ 5. $x = 2a$
6. $x = 7a$ 7. $x = ya$ 8. $x = mp$ 9. $x = rs$ 10. $x = 5$
11. $x = a/4$ 12. $x = h/g$ 13. $x = t/n$ 14. $x = 2$ 15. $x = \frac{b-1}{2}$
16. $x = \frac{b-c}{2}$ 17. $x = \frac{b-c}{a}$ 18. $x = \frac{r-q}{p}$ 19. $x = \frac{y+w}{v}$ 20. $S = D/T$
21. $d = C/\pi$ 22. $x = 4$ or -4 23. $x = \sqrt{y}$ or $-\sqrt{y}$ 24. $r = \sqrt{(A/\pi)}$ 25. $S = D/T$
26. $y = \sqrt{A}$ (or $-\sqrt{A}$) 27. $\pi = P/3r^2$ 28. $r = \sqrt{(P/5\pi)}$ 29. $h = q + p$ 30. $p = h - q$
31. $h = \frac{q+5p}{2}$ 32. $p = \frac{2h-q}{5}$ 33. $x = \frac{b-c}{a}$

Exercise 3B

1. (a) $h = g/f$ (b) $h = e - g$ (c) $h = kf$ (d) $h = g - e$

2. (a) $r = \sqrt{Q}$ (b) $r = \sqrt{(N/\pi)}$ (c) $r = \sqrt{(M/2\pi)}$ (d) $r = \sqrt{(P/\pi w)}$

3. (a) $M = A/kl$ (b) $m = B/K$ (c) $m = c/\pi r^2$ (d) $m = 1/3pD$

4. (a) $x = p - q$ (b) $x = s - r$ (c) $x = (s - r)/5$ (d) $x = (r + 3)/7$
(e) $x = (m - 2)/2$ (f) $x = 2m + 5$ (g) $x = 2n - 2$ (h) $x = 2p - q$

5. (a) $P = QR$ (b) $s = 1/t$ (c) $Q = \sqrt{(P/M)}$ (d) $w = v^2z$
(e) $f = e/5d$ (f) $n = K/mT$ (g) $s = \sqrt{(7/9r)}$ (h) $a = \sqrt{(c^2 - b^2)}$

6. (a) $T = V/(a+d)$ (b) $x = r/(p + q)$ (c) $x = c/(a - b)$ (d) $x = r/(m + 1)$
(e) $w = (v - vx)/(x + 1)$ (f) $r = \frac{(p+1)^2}{4}$

Exercise 4

1. (a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $3\sqrt{3}$ (d) $2\sqrt{5}$ (e) $5\sqrt{2}$
(f) $2\sqrt{7}$ (g) $3\sqrt{2}$ (h) $2\sqrt{6}$ (i) $10\sqrt{2}$ (j) $5\sqrt{3}$
(k) $3\sqrt{5}$ (l) $6\sqrt{2}$ (m) $10\sqrt{3}$ (n) $7\sqrt{3}$ (o) $3\sqrt{6}$
(p) $14\sqrt{2}$ (q) $20\sqrt{2}$ (r) $12\sqrt{10}$

2. (a) $8\sqrt{2}$ (b) $\sqrt{5}$ (c) $13\sqrt{10}$ (d) 0 (e) $-2\sqrt{6}$
(f) $-\sqrt{3}$ (g) 0 (h) $10\sqrt{2} + 10\sqrt{3}$!!

3. (a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $10\sqrt{5}$ (d) $6\sqrt{3}$ (e) $5\sqrt{5}$
(f) $\sqrt{7}$ (g) $8\sqrt{2}$ (h) $2\sqrt{2}$

Exercise 5

1. (a) 3 (b) 5 (c) 6 (d) 1 (e) x (f) $\sqrt{6}$ (g) $2\sqrt{5}$ (h) $4\sqrt{a}$
(i) $\sqrt{(2c)}$ (j) $\sqrt{(xy)}$ (k) 4 (l) 8 (m) $3\sqrt{2}$ (n) $10\sqrt{2}$ (o) 6 (p) $6\sqrt{6}$

2. (a) $\sqrt{2} + 2$ (b) $\sqrt{6} + \sqrt{3}$ (c) $5 + \sqrt{5}$ (d) $\sqrt{7} + 7$ (e) $5\sqrt{2} - 2$
(f) $5\sqrt{2} - 8$ (g) $1 + \sqrt{3}$ (h) $-\sqrt{2}$ (i) 1 (j) 2
(k) $5 + 2\sqrt{6}$ (l) $8 - 2\sqrt{15}$

3. (a) 6 (b) -2 (c) 6

4. (a) $4\sqrt{3}$ (b) 10 (c) $4\sqrt{15}$

5. (a) 4 cm^2 (b) $2\sqrt{6}$ cm

Exercise 6

1. (a) $\sqrt{2}/2$ (b) $\sqrt{3}/3$ (c) $\sqrt{5}/5$ (d) $\sqrt{6}/6$ (e) $\sqrt{7}/7$
(f) $2\sqrt{5}$ (g) $2\sqrt{3}/3$ (h) $3\sqrt{5}/5$ (i) $10\sqrt{2}$ (j) $2\sqrt{3}$
(k) $2\sqrt{6}$ (l) $3\sqrt{5}/10$ (m) $2\sqrt{2}/5$ (n) $\sqrt{5}/10$ (o) $\sqrt{2}/10$ (p) $\sqrt{2}$

2. (a) $2\sqrt{3}/3$ (b) $\sqrt{10}/2$ (c) $\sqrt{10}/5$ (d) $\sqrt{11}/11$ (e) $\sqrt{15}/5$ (f) $\frac{\sqrt{(ab)}}{b}$

3. (a) $\sqrt{2} + 1$ (b) $(\sqrt{7} + 1)/6$ (c) $(2 - \sqrt{2})/2$ (d) $\sqrt{5} - 1$ (e) $6 + 3\sqrt{3}$
(f) $\sqrt{3} + \sqrt{2}$ (g) $\sqrt{5} - \sqrt{3}$ (h) $3\sqrt{5} + 3\sqrt{2}$

Exercise 7

1. (a) 2^7 (b) 3^8 (c) 8^7 (d) 10^{30} (e) a^7 (f) b^8 (g) c^8 (h) d^{10}
(i) v^{11} (j) x^{12} (k) w^4 (l) z^6 (m) f^{11} (n) g^{20} (o) k^{12} (p) m^{101}

2. (a) 2 (b) 3^2 (c) 8^3 (d) 10^{10} (e) a (f) b^2 (g) c^5 (h) 1
(i) v (j) x^8 (k) w^2 (l) z (m) f^3 (n) 1 (o) k^{10} (p) m^{99}
(q) x^4 (r) m (s) a^4 (t) r

3. (a) x^6 (b) y^{15} (c) z^{10} (d) g^{16} (e) a^{21} (f) b^{16} (g) c^{30} (h) d^{21}

4. (a) 2^{10} (b) 7^{15} (c) 6^{20} (d) 8^{15} (e) 2^{49} (f) 3^{12} (g) 9^{10} (h) 2^{25}

5. (a) a^3b^3 (b) c^6d^6 (c) $x^{20}y^{10}$ (d) $4p^2q^2$

6. (a) y^5 (b) t^5 (c) x^3 (d) v^6 (e) $3x^5$ (f) $4x^7$ (g) $12x^4$
(h) $4x^3$ (i) $3x^2$ (j) $x^5 + x^6$ (k) $x^6 - 4x^3$ (l) u^7v^7 (m) $25y^2$
(n) m^8n^{16} (o) x^5 (p) u^6

7. (a) $m = 4$ (b) $m = 5$ (c) $m = 3$ (d) $m = 4$

Exercise 8

1. (a) 1 (b) 1 (c) 1 (d) 1
2. (a) $1/3^2$ (b) $1/5^7$ (c) $1/a^4$ (d) $1/b^9$ (e) $1/x$ (f) $3/y^2$ (g) x/y^3 (h) x^3
   (i) t^5 (j) $6c^3$ (k) $1/2\,y^2$ (l) $1/7\,x^3$
3. (a) 1 (b) $1/7$ (c) $1/27$ (d) $1/64$ (e) 8 (f) $9/4$
4. (a) a^2 (b) b^2 (c) c^{-2} (d) 1 (e) e^9 (f) g^{-8} (g) w^2 (h) x^{-6}
   (i) y^{-5} (j) z^{16} (k) 1
5. (a) $1/4^3$ (b) $1/6^5$ (c) 2^5 (d) $25/4^3$ (e) $81/2^3$ (f) 1 (g) w^6 (h) x
   (i) 4 (j) 40 (k) $6/h^2$ (l) $9/s$ (m) $1/2k$ (n) $3/4m^3$
6. (a) $x^7 + x^3$ (b) $x^4 - x$ (c) $x + x^{-2}$ (d) $1 + x$ (e) $x^3 - x^{-6}$
   (f) $2x^{-5} - 1$ (g) $6x^{-2} - 2x$ (h) $6x^3 - 3$
7. (a) 27, 1, $1/9$, 3, $1/27$ (b) 4^{-1}, 4^{-2}, 4^{-3}

Exercise 9

1. (a) $\sqrt[4]{x^3}$ (b) $\sqrt[5]{m^3}$ (c) $\sqrt[3]{r^2}$ (d) $\sqrt{w}$ (e) $\sqrt[3]{n}$ (f) $1/(\sqrt[3]{r^4})$
2. (a) $x^{5/3}$ (b) $b^{4/3}$ (c) $z^{2/3}$ (d) $w^{1/4}$ (e) $x^{-1/3}$ (f) $u^{1/2}$
3. (a) 3 (b) 8 (c) 2 (d) 4 (e) 10 (f) 3 (g) 27 (h) 8
   (i) 343 (j) $1/4$ (k) $1/3$ (l) $1/125$ (m) 27 (n) $1/27$ (o) 2 (p) $1/32$
4. (a) x^2 (b) m^3 (c) c^4 (d) n^{-3} (e) $n^{-1/2}$ (f) g^2 (g) b^{-5}
   (h) z^3 (i) $x^{2/5}$ (j) x^2 (k) 13 (l) 6

Exercise 10

1. (a) a^2 (b) b (c) 1 (d) d^2 (e) $e^{4/5}$ (f) z (g) w^{-1} (h) 1
2. (a) 4 (b) $12b$ (c) $6c$ (d) $2d$ (e) $2e$ (f) $4v$ (g) 1
3. (a) $x - 1$ (b) $x^2 + x$ (c) $x^{3/5} + x^{-4/5}$
4. (a) 12 (b) 32 (c) 45 (d) $8/9$ (e) $3/2$
5. (a) $p^{13/2}$ (b) $p^{11/2}$ (c) $1/p$ (d) $1/p^9$ (e) $p^{3/2}$ (f) $49/p$
6. (a) 1 (b) x^6 (c) x (d) $x^{-9/2}$ (e) 1 (f) x
7. (a) $2 - x^3 - 1/x^3$ (b) $1/x^2 - 4$ (c) $-4x^{1/2} + 4/x^{1/2} - 15$

Exercise 11

1. x^{-1} 2. x^{-2} 3. $7x^{-2}$ 4. $5x^{-3}$ 5. $1/3x^{-1}$ 6. $4/5x^{-1}$ 7. $3/2x^{-2}$

8. $x^{-1/2}$ 9. $5/2x^{-1/2}$ 10. $3/5x^3$ 11. $\dfrac{x^{-3}}{\sqrt{3}}$ 12. $\dfrac{2x^{-3/2}}{\sqrt{5}}$

Checkup for algebraic operations

1. (a) $^1/_2$ (b) $^{3a}/_b$ (c) $^1/(x-3)$ (d) $v-w$ (e) $\frac{x+y}{2x+3y}$ (f) $^1/(a+8)$
   (g) $^{(v+1)}/(v-2)$
2. (a) $b^2/2$ (b) a (c) 3z (d) a (e) $^{3c}/_8$ (f) $^{(15u-8v)}/_{20}$
   (g) $^{(3m+k)}/_{km}$ (h) $^{(3x+1)}/_{10}$
3. (a) $x = 1$ (b) $x = w + a$ (c) $x = {}^{(p-m)}/_a$ (d) $x = {}^{pz}/_w$
   (e) $x = \sqrt{(^N/_{2\pi})}$ (f) $x = {}^{(T-3)}/_4$ (g) $x = 2w - y$ (h) $x = \sqrt{(^5/_{8M})}$
4. (a) $4\sqrt{2}$ (b) $10\sqrt{10}$ (c) $6\sqrt{5}$ (d) $\sqrt{5}$ (e) $2\sqrt{2}$ (f) $11\sqrt{2}$ (g) 3/a (h) $3\sqrt{2}$
5. (a) 4 (b) –10 (c) 8
6. (a) $^{\sqrt{5}}/_5$ (b) $4\sqrt{2}$ (c) $3\sqrt{5}$ (d) $^{\sqrt{3}}/_3$
7. (a) 5^{14} (b) x^2 (c) $6m^4$ (d) a^6 (e) $16a^6b^2$ (f) a^4 (g) 1 (h) $x^5 - x^4$
8. (a) $^1/_{5^2}$ (b) $^a/_{b^3}$ (c) y^6 (d) $(x^2)/4$
9. (a) $\sqrt{b}$ (b) $^1/\sqrt{c^3}$
10. (a) $x^{4/3}$ (b) $a^{-3/2}$
11. (a) 216 (b) $^1/_4$ (c) x^4 (d) $^1/_{y^7}$ (e) $1/_a - {}^1/_{a^4}$ (f) $^s/_2$ (g) x

Quadratic functions

Exercise 1

1. (a) $y = x^2$ (b) $y = 2x^2$ (c) $y = {}^1/_2x^2$
   (d) $y = -2x^2$ (e) $y = -{}^1/_2x^2$ (f) $y = -5x^2$
2. (a) $y = x^2 + 1$ (b) $y = x^2 + 3$ (c) $y = x^2 - 2$
   (d) $y = -x^2 + 4$ (e) $y = -x^2 + 1$ (f) $y = -x^2 - 1$
3. (a) $y = (x-1)^2 + 1$ (b) $y = (x-2)^2 + 3$ (c) $y = (x-3)^2$
   (d) $y = (x+3)^2 + 2$ (e) $y = (x+4)^2$ (f) $y = (x+5)^2 - 3$
   (g) $y = (x-3)^2 + 2$ (h) $y = (x-1)^2 - 4$ (i) $y = (x+3)^2 - 4$

Exercise 2

1. (a) (2,1) (b) $x = 2$ (c) $y = (x-2)^2 + 1$
2. (a) (i) (3,2) (ii) $x = 3$ (iii) $y = (x-3)^2 + 2$
   (b) (i) (1,–1) (ii) $x = 1$ (iii) $y = (x-1)^2 - 1$
   (c) (i) (–2,1) (ii) $x = -2$ (iii) $y = (x+2)^2 + 1$
   (d) (i) (–3,1) (ii) $x = -3$ (iii) $y = (x+3)^2 + 1$
   (e) (i) (3,–2) (ii) $x = 3$ (iii) $y = (x-3)^2 - 2$
   (f) (i) (–1,–3) (ii) $x = -1$ (iii) $y = (x+1)^2 - 3$
3. (a) (4, 1); $x = 4$ (b) (2,7); $x = 2$ (c) (8,3); $x = 8$
   (d) (–1,2); $x = -1$ (e) (1,–3); $x = 1$ (f) (–3,–7); $x = -3$
   (g) (5,0); $x = 5$ (h) (–2,0); $x = -2$ (i) (0,3); $x = 0$

4. (a) (2,6) (b) $x = 2$ (c) $y = -(x - 2)^2 + 6$

5. (a) (i) (3,2) (ii) $x = 3$ (iii) $y = -(x - 3)^2 + 2$
   (b) (i) (4,1) (ii) $x = 4$ (iii) $y = -(x - 4)^2 + 1$
   (c) (i) (–1,4) (ii) $x = -1$ (iii) $y = -(x + 1)^2 + 4$
   (d) (i) (–3,1) (ii) $x = -3$ (iii) $y = -(x + 3)^2 + 1$
   (e) (i) (4,0) (ii) $x = 4$ (iii) $y = -(x - 4)^2$
   (f) (i) (–3,–1) (ii) $x = -3$ (iii) $y = -(x + 3)^2 - 1$

6. (a) (2,6) ; $x = 2$ (b) (5,1) ; $x = 5$ (c) (6,–2) ; $x = 6$
   (d) (–1,7) ; $x = -1$ (e) (–4,–5) ; $x = -4$ (f) (–3,0) ; $x = -3$
   (g) (1,7) ; $x = 1$ (h) (8,1) ; $x = 8$ (i) (–5,–2) ; $x = -5$

Exercise 3

1. (a) $x = -1, 3$ (b) $x = 1, 3$ (c) $x = 2, -2$
2. (a) 4, 0, –2, –2, 0, 4 (b) graph (c) $x = -1, 2$
3. (a) 0, 4 (b) 1, –2 (c) 2, 4
   (d) –3, 2 (e) 2, –2 (f) 5, –1

Exercise 4

1. (a) 0, 4 (b) 0, 10 (c) 0, 8
   (d) 0, –6 (e) 0, –1 (f) 0, 1
   (g) 0, 3 (h) 0, –3 (i) 0, $^3/_2$

2. (a) 2, –2 (b) 3, –3 (c) 5, –5
   (d) 4, –4 (e) 10, –10 (f) 7, –7
   (g) 9, –9 (h) 3, –3 (i) $^4/_5$, –$^4/_5$

3. (a) –1, –2 (b) 2, 3 (c) –1, –5
   (d) 5, 4 (e) –2, –5 (f) 3
   (g) 3, 4 (h) 1, 7 (i) 6, 7
   (j) –5, 2 (k) 4, –1 (l) –4, 2
   (m) 5, –4 (n) –4, 3 (o) –7, 5
   (p) –6, 2 (q) –6, 3 (r) –1, –20
   (s) 1, 8 (t) 12, –2 (u) –8, 3
   (v) 6, –4 (w) 24, –1 (x) 6, 9

4. (a) –3, –$^1/_2$ (b) –1,– $^3/_2$ (c) –2, –$^1/_3$
   (d) 3, $^3/_2$ (e) –3, –$^2/_3$ (f) –2,–$^1/_5$
   (g) 2, –$^4/_3$ (h) 2, –$^1/_3$ (i) –1, $^1/_3$
   (j) 4, –$^1/_2$ (k) –3, $^2/_5$ (l) –2, – $^5/_2$

5. (a) 1, –3 (b) 5, –4 (c) 5, –2
   (d) 6, –1 (e) 7, –10 (f) 7, –8
   (g) 2, –5 (h) 5, –6 (i) 2, –4
   (j) 3, –3 (k) 5, –5 (l) 1, 4
   (m) –1, –3 (n) 2, –5 (o) 2, –4
   (p) 4, –$^5/_2$ (q) 2, –$^5/_3$ (r) 1, –$^7/_5$
   (s) 3, –$^{11}/_2$ (t) 2, –$^7/_3$

Exercise 5

1. (a) –0·27, –3·73 (b) –0·38, –2·62 (c) –0·35, –5·65
   (d) –0·76, –9·24 (e) 3·41, 0·59 (f) 3·62, 1·38
   (g) –0·31, –3·19 (h) –0·61, –2·72 (i) 1·39, 0·36
2. (a) 0·732, –2·732 (b) 1·791, –2·791 (c) 1·372, –4·372
   (d) 3·449, –1·449 (e) 3·303, –0·303 (f) 0·243, –8·243
   (g) 0·851, –2·351 (h) 1·215, –0·549 (i) 0·588, –0·213
3. (a) 0·372, –5·37 (b) 3·30, –0·303 (c) 0·679, –3·68
   (d) 2·39, –1·39 (e) –0·807, –6·19 (f) 3·24, –1·24

Checkup for quadratic equations

1. (a) $y = x^2$ (b) $y = 3x^2$ (c) $y = -2x^2$
   (d) $y = (x - 3)^2 + 2$ (e) $y = (x + 2)^2 + 2$ (f) $y = (x + 1)^2 - 4$
2. (a) minimum at (3, –1) ; $x = 3$ (b) minimum at (–1, 0) ; $x = -1$
   (c) maximum at (2, 1) ; $x = 2$
3. (a) minimum at (21, 5) ; $x = 21$ (b) minimum at (–2, –1) ; $x = -2$
   (c) maximum at (3,2) ; $x = 3$
4. (a) graph and $x = 4$ or $x = -2$ (b) graph and $x = 1$ or $x = -3$
5. (a) 0, 7 (b) 3, –3 (c) –2, –6 (d) 0, $-^3/_2$ (e) 5, –5 (f) 6, –5
   (g) $^3/_2$,$-^3/_2$ (h) 2, 5 (i) –5, $^3/_2$ (j) 2, –7 (k) 4, –5 (l) 6, –2
6. (a) –0·55, –5·45 (b) 3·73, 0·27 (c) –0·78, –2·55
   (d) 0·37, –5·37 (e) 6·61, –0·61 (f) 1·85, –1·35

Further trigonometry

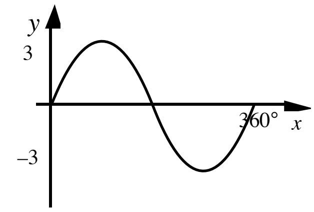
Exercise 1

1. Check Graphs – see graphs at top of page 15 for comparison.

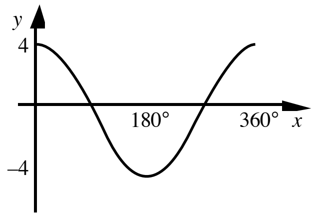
Exercise 2A

1. (a) $y = 10\sin x°$ (b) $y = 6\cos x°$ (c) $y = 5\sin 2x°$
   (d) $y = 0{\cdot}8\cos 4x°$ (e) $y = 20\sin 2x°$ (f) $y = 80\cos 2x°$
   (g) $y = 25\sin \tfrac{1}{2}x°$ (h) $y = -4\sin x°$ (i) $y = -50\cos 3x°$
   (j) $y = \tan 2x°$ (k) $y = 0{\cdot}2\cos \tfrac{1}{2}x°$ (l) $y = 15\sin 4x°$

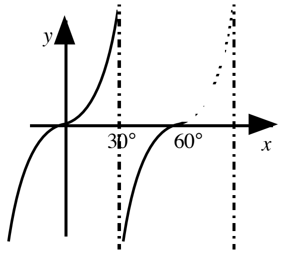
2. (a)



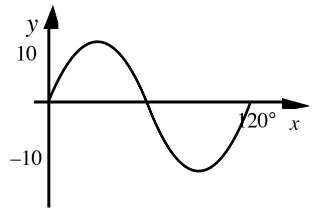
(b)



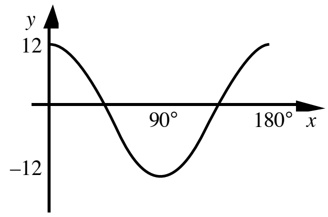
(c)



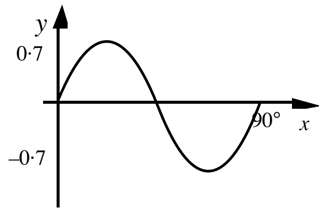
(d)



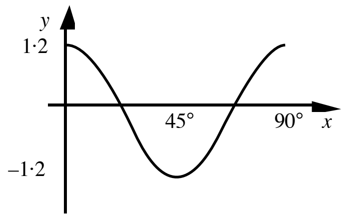
(e)



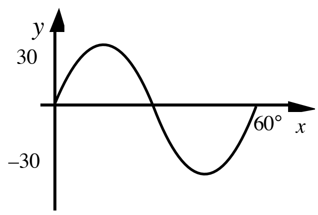
(f)



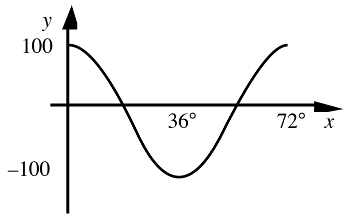
(g)



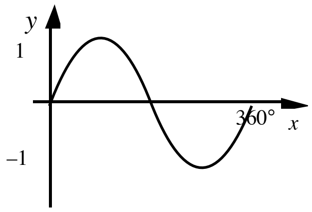
(h)



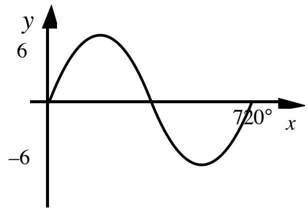
(i)



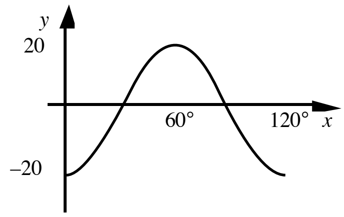
(j)

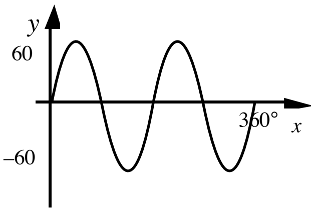


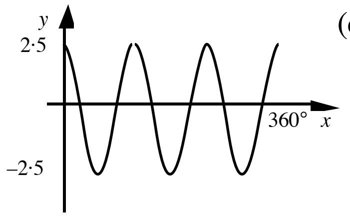
(k)

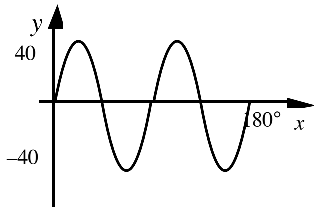


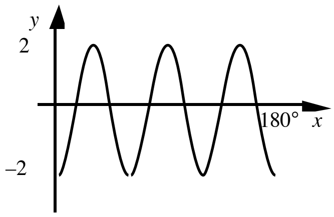
(l)

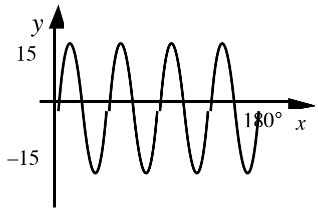


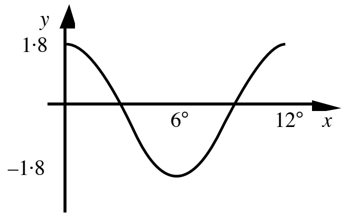
3. (a)


(b)


(c)


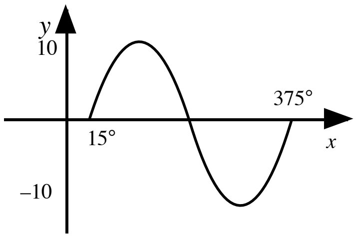
(d)


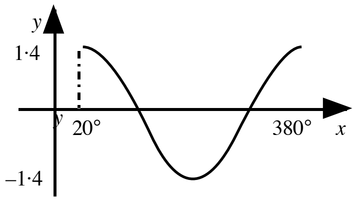
(e)


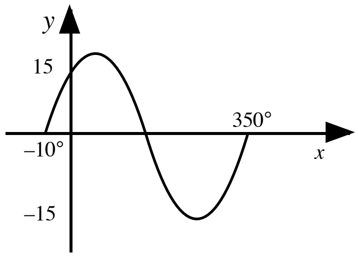
(f)


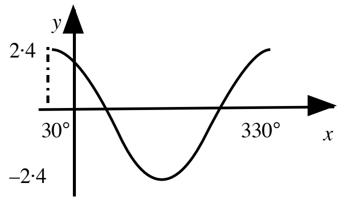
Exercise 2B

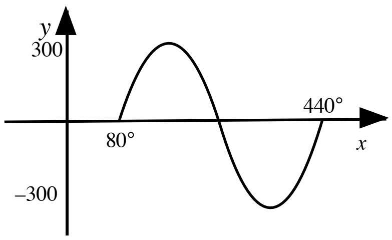
1. (a) $y = 5\sin(x + 30)°$
   (b) $y = 8\cos(x + 20)°$
   (c) $y = 5\sin(x - 10)°$
   (d) $y = 0{\cdot}6\sin(x - 40)°$
   (e) $y = 4\cos(x - 35)°$
   (f) $y = 15\sin(x + 70)°$ or $y = 15\cos(x - 20)°$

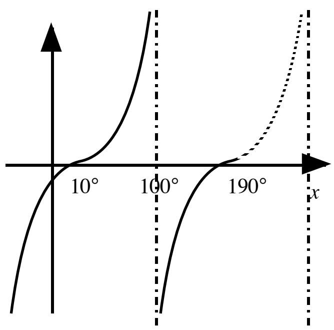
2. (a)


(b)


(c)


(d)


(e)


(f)


Exercise 3

1. (a) 30, 150 (b) 45, 315 (c) 41, 221
   (d) 20, 340 (e) 52, 232 (f) 35, 145
   (g) 82, 98 (h) 81, 261 (i) 67, 293
   (j) 10, 350 (k) 60, 120 (l) 60, 240.

2. (a) 60, 300 (b) 53·1 or 126·9 (c) 35·0, 215·0
   (d) 19·5, 160·5 (e) 33·6, 326·4 (f) 59·0, 239·0.

3. (a) 210, 330 (b) 135, 225 (c) 159, 339
   (d) 107, 253 (e) 135, 315 (f) 240, 300
   (g) 104, 284 (h) 190, 350 (i) 158, 202.

4. (a) 194·5, 345·5 (b) 126·9, 233·1 (c) 161·6, 341·6
   (d) 151·0, 209·0 (e) 228·6, 311·4 (f) 122·0, 302·0.

5. (a) 18·8, 161·2 (b) 154·2, 205·8 (c) 34·1, 214·1
   (d) 75·5, 284·5 (e) 225, 315 (f) 116·6, 296·6
   (g) 36·9, 143·1 (h) 96·4, 263·6 (i) 32·0, 212·0
   (j) 218·7, 321·3 (k) 120, 240 (l) 11·3, 191·3
   (m) 58·2, 121·8 (n) 53·1, 306·9 (o) 138·8, 318·8
   (p) 41·8, 138·2 (q) 33·6, 326·4 (r) 150·3, 330·3
   (s) 199·5, 340·5°.

Exercise 4

1.

Graph	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Period	180	60	90	36	180	120	360
Max/Min	±7	±8	±0·5	±25	±9	±6	±0·1

2.

Question	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Period	180	120	45	180	60	12	4	720
Max/Min	±5	±3	±10	±2·2	±30	±5	±50	±4

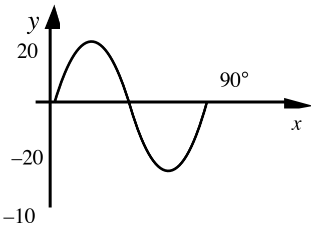
Question	(i)	(j)	(k)	(l)	(m)	(n)	(o)
Period	1440	6	72	40	2	240	144
Max/Min	±18	±0·9	±0·5	±0·75	±11	±8	±40

Exercise 5

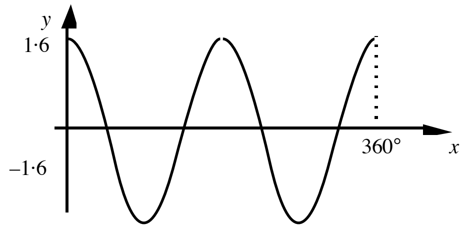
1. (a) 2 (b) 5 (c) 3tan$x$ (d) $^5/_2$tan$x$
2. (a) cos$^2x°$ (b) sin$^2x°$ (c) sin$x°$ (d) tan$x°$
3. (a) 1 (b) $^1/_2$ (c) tan$^2x°$ (d) cos$x°$ (e) $^1/_5$sin$x°$ (f) sin$^2x°$
4. All proofs.

Checkup for further trigonometry

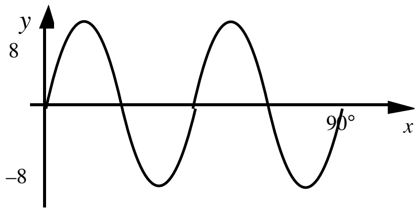
1. See sketches on page 15.
2. (a) $y = 8\sin 2x°$ (b) $y = 0{\cdot}7\cos 4x°$ (c) $y = -2{\cdot}7\sin 3x°$ (d) $y = \tan 3x°$
3. (a)



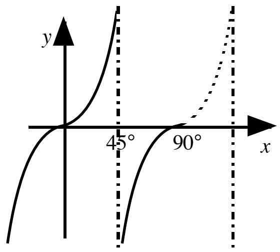
(b)



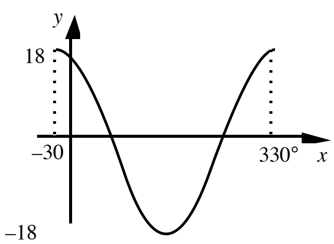
(c)



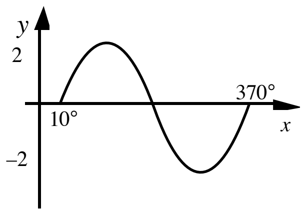
(d)



4. (a) 65·6, 114·4 (b) 63·6, 296·4 (c) 71·6, 251·6
   (d) 127·2, 232·8 (e) 141·3, 321·3 (f) 233·1, 306·9
   (g) 30, 150 (h) 138·6, 221·4 (i) 36·9, 216·9
   (j) 48·2, 311·8 (k) 194·5, 345·5 (l) 135, 315.
5. (a) 120° (b) 180° (c) 60° (d) 40°
   (e) 36° (f) 12° (g) 40° (h) 90°.
6. (a) (i) $6\cos^2 x$ (ii) $\dfrac{1}{\tan x}$ (b) Proof.
7. (a) $y = 8\sin(x + 15)°$ (b) $y = 3{\cdot}6\cos(x - 20)°$
8. (a)



(b)



Specimen assessment questions

1. (a) $\frac{2x}{3y^2}$ (b) $2a - 4$ (c) $p^2 - q$ (d) $w + 5$ (e) $\frac{a-3}{a+1}$

2. (a) $\frac{4}{v^3}$ (b) $\frac{1}{2a^2c}$ (c) $\frac{5q + 3p}{pq}$ (d) $\frac{9t}{2z}$ (e) $\frac{5x + 9}{6}$

3. (a) $p = r + 2q$ (b) $w = (h - g)/a$ (c) $3\sqrt{a}$ (d) $4V - n$

4. (a) $2\sqrt{7}$ (b) $6\sqrt{3}$ (c) $3/a^2$ (d) $11\sqrt{2}$

5. (a) $2\sqrt{5}$ (b) $\sqrt{6}$ (c) $(p\sqrt{q})/q$

6. (a) b^2 (b) $1/w^6$ (c) a^3 (d) $3/p$
   (e) $v - 1/v^4$ (f) $2/x^2$

7. (a) 20 (b) 8 (c) 1

8. (a) $y = 4x^2$ (b) $y = -5x^2$ (c) $y = (x - 3)^2 - 1$ (d) $y = -(x + 2)^2 + 5$

9. (a) (2,7); minimum; $x = 2$ (b) (–1,–3); maximum; $x = -1$

10. (a)

x	–4	–3	–2	–1	0	1	2	3	4	5	6
$y = x^2 - x - 12$	8	0	–6	–10	–12	–12	–10	–6	0	8	18

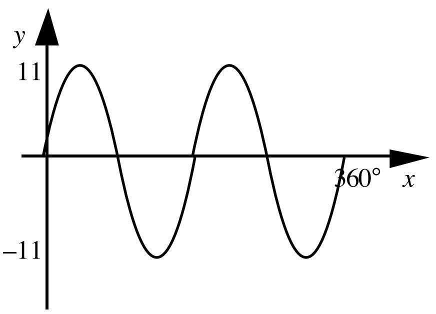
   (b) Graph
   (c) $x = -3$ or 4

11. (a) 0, 3 (b) 4, –4 (c) –2, –5 (d) –3, 5 (e) –4, 6 (f) –6, 5

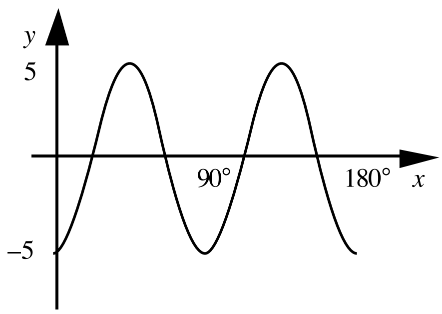
12. (a) –0·21, –4·79 (b) 3·83, –1·83 (c) 0·43, –0·77

13. (a) $y = 25\sin 3x°$ (b) $y = 0{\cdot}2\cos 6x°$
    (c) $y = -2\sin \frac{1}{2}x°$ (d) $y = \tan 6x°$

14. (a)



(b)



15. (a) 24·3°, 155·7° (b) 51·3°, 308·7° (c) 35·0°, 215·0°
    (d) 103·5°, 256·5° (e) 135°, 315° (f) 199·5°, 340·5°

16. (a) 72° (b) 18° (c) 60°

17. (a) $-\cos^2 x°$ (b) Proof

18. $y = 8\sin(x - 25)°$

19.